Decision Under Uncertainty

EE431/438

Pindyck, Robert S. and Daniel L. Rubinfeld, Microeconomics, (6th ed.), New Jersey: Prentice -Hall, 2005 : chapter 5
Nicholson, Walter, Microeconomics Theory, Basic Principles and Extensions, 1995 : chapter 9 (pp 269 - 276)

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Introduction

- Individuals and firms are linked through the supply and the demand for financial assets
- certainty case: $r = \rho^j$
- uncertainty: what should be the rate of return on a given financial asset?
- rate of return $= \dfrac{\text{payoff} - \text{price}}{\text{price}}$; price $= \dfrac{\text{payoff}}{\text{1+rate of return}}$ = present value

States of the worlds

- *States of the world* = possible outcomes of an uncertain situation, states of the world are mutually exclusive
- Example: Tomorrow can be sunny or rainy. Also, it can be warm or cold.
- We do notdefine S1= sunny, S2= rainy, S3= warm, S4= cold and allow for, say, S1 and S3 to occur at the same time.
- Instead, we define S1= sunny and warm, S2= sunny and cold, S3= rainy and warm, S4= rainy and cold

Probability

- We are in a situation of Risk if the decision maker can assign a probabilityto each state of the world
- probability: the relative frequency with which a state will occur
- example: flipping a coin
- subjective probability
- a gamble or a lottery offers *n* prizes, π_i is the probability for state *i*
- $\sum_{i=1}^{n} \pi_i = 1$

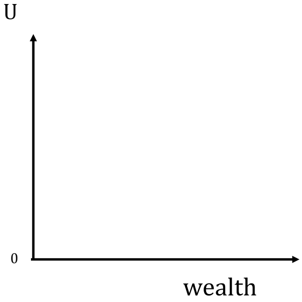
Expected Value

- To compare the return of any two gambles, we need a measure of the average payoff
- $E(X) = \sum_{i=1}^{n} \pi_i X_i$
- Expected value = “central tendency”
- Notice that the outcome is different from the expected value
- Example: $X_1 = 1000$, $\pi_1 = 0.5$, $X_2 = 200$, $\pi_2 = 0.5$

Fair Game

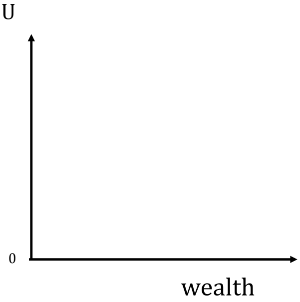
- Flipping a coin: $X_H = 100$, $X_T = 20$
- If the cost of the game is equal to its expected value, the game is called “a fair game”
- Should a price of a game be equal to its expected value ?
- Are people willing to pay “the fair price” which is equal to the game’s expected value?
- Degree of risk aversion

Expected Utility Hypothesis



- Vonn Neumann Morgenstern theorem : maximising expected utility is a reasonable goal to pursue in uncertain situations
- Risk neutral, risk lovers, risk averse

Expected Utility Hypothesis (cont)



- Risk averse: prefer a certain wealth to a fair gamble
- Certainty equivalence
- Risk premium
- Certainty Equivalence (C.E.) .... Expected value of the Gamble (C.G.) : risk averse

State Preference Model

- outcomes of any random events can be categorized into a certain number of *states of the world*
- *contingent commodity :* goods delivered only if a particular state of the world occurs, ex. 1$ in good times
- contingent commodity can be extended to include claims on any goods in the future
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	S_1	S_2	S_3	S_4
A_1	1	0	0	0
A_2	0	1	0	0
A_3	0	0	1	0
A_4	0	0	0	1

- a security pay 2 $ when S_1 occurs and 3$ if S_2 occurs is a combination of A_1 and A_2
- *Arrow-Debreu securities* or *Pure securities*

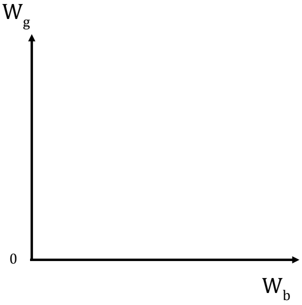
State Preference Model (Cont1)

- A complete market : exists a pure security for every state the world
- two states : good time and bad time
- $V(W_g, W_b) = \pi U(W_g) + (1 - \pi)U(W_b)$
- The price of a contingent claim which pays a dollar of wealth in good time, P_A
- The price of a contingent claim which pays a dollar of wealth in bad time, P_B

State Preference Model (Cont2)

- a fair price : $P_A = \pi_A = \pi$, $P_B = \pi_B = 1 - \pi_A = 1 - \pi$
- From utility maximising condition; MRS = $\frac{\frac{\partial V}{\partial W_g}}{\frac{\partial V}{\partial W_b}} = \qquad =$ relative price =
- Hence, when the claims are fair, $\frac{U'(W_g)}{U'(W_b)} = 1$

State Preference Model (Cont3)



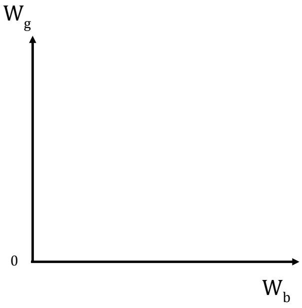
- If the price of contingent claims are fair, utility maximization will occur on the certainty line where $W_g = W_b$

State Preference Model : Insurance (Cont4)

- W_g , $W_b = W_g - A$
- Max $V(W_g, W_b) = \pi U(W_g) + (1 - \pi)U(W_b)$
- fair price : $\dfrac{P_g}{P_b} = \dfrac{\pi}{1 - \pi}$
- Utility maximizing condition : $\dfrac{U'(W_g)}{U'(W_b)} = 1$
- Buy insurance against the bad state : A units at price $(1 - \pi)$
- Wealth in the good state = ...
- Wealth in the bad state = ...
- fair price = fully insurance

State Preference Model : Insurance (Cont5)

- W_g , $W_b = W_g - A$



- Let the probability of the bad state to occur is higher
- price of insurace against the bad state is ....
- IC becomes .... (steeper/flatter)
- budget constaint : $P_g W_g + P_b W_b = W$; slope $= -\dfrac{P_b}{P_g} = \dfrac{1-\pi}{\pi}$

Mean-Variance Analysis

- Risk averse individuals dislike risk but this doesn't mean they are never prepared to take any risks. It just means they need to be compensated for the risk they take by way of a greater expected return
- Risks: deviation from expected outcome
- Expected outcome = expected return
- Risks: variance, standard deviation

Mean-Variance Analysis

- $\sigma^2 = E[(x - \mu)^2]$
- S.D = $\sigma = E[(x - \mu)^2]$