EE 422 Mathematical Economics 2 (2014)

Assignment 2

The due date for this assignment is September 16.

1. Solve the following first-order differential equations; if an initial condition is given, definitize the arbitary constant: (10 points each)

   a) $\frac{dy}{dt} + y = 4;$ $\quad y(0) = 0$

   b) $\frac{dy}{dt} - 5y = 0;$ $\quad y(0) = 6$

   c) $\frac{dy}{dt} + 3y = 2;$ $\quad y(0) = 4$

   d) $\frac{dy}{dt} - 7y = 7;$ $\quad y(0) = 7$

   e) $3\frac{dy}{dt} + 6y = 5;$ $\quad y(0) = 0$

   f) $\frac{dy}{dt} + 2ty = 0$

   g) $\frac{dy}{dt} + 2ty = t;$ $\quad y(0) = \frac{3}{2}$

   h) $\frac{dy}{dt} + t^2 y = 5t^2;$ $\quad y(0) = 6$

   i) $2\frac{dy}{dt} + 12y + 2e^t = 0;$ $\quad y(0) = \frac{6}{7}$

   j) $\frac{dy}{dt} + y = t$

2. Verify that each of the following differential equations is exact, and solve by the four-step procedure: (10 points each)

   a) $2yt^3 dy + 3y^2 t^2 dt = 0$

   b) $3y^2 t dy + (y^3 + 2t)dt = 0$

   c) $t(1+2y)dy + y(1+y)dt = 0$

3. Determine, for each of the following, (1) whether the variables are separable and (2) whether the equation is linear or else can be linearized (5 points each)

a) $2tdy + 2ydt = 0$

b) $\frac{y}{y+t}dy + \frac{2t}{y+t}dt = 0$

c) $\frac{dy}{dt} = -\frac{t}{y}$

d) $\frac{dy}{dt} = 3y^2t$

4. According to Problem 3, solve (10 points each)
   - **a)** Subproblem (c) by applying a separable-variable equation
   - **b)** Subproblem (d) by applying a Bernoulli equation

5. Plot the phase line for each of the following, and discuss its qualitative implications (10 points each)

a) $\frac{dy}{dt} = y - 7$

b) $\frac{dy}{dt} = 1 - 5y$

c) $\frac{dy}{dt} = 4 - \frac{y}{2}$

d) $\frac{dy}{dt} = 9y - 11$

6. According to the Solow model, if capital is growing at the rate a (that is $K = Ae^{at}$), find the growth rate of investment.