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Quiz 6 EE320 (Semester 1/2019)

A firm has an order of Q units of its product and has two plants to manufacture these units. Let q_1 be the number of units to be produced at the first plant, and q_2 denote the number to be manufactured at the second plant. Suppose that the cost function, determined the level of q_1 and q_2, is given by

$$C(q_1, q_2) = TC(q_1) + TC(q_2) = 0.5\, q_1^2 + 10q_1 + 40q_2 + 200$$

1. (5 points) How many units should be produced at each plant to minimize this cost function? Use the LaGrange method.
2. (3 points) Confirm your result by checking the second-order condition if Q = 100
3. (2 points) Derive the (optimal) cost function of the multi-plant firm.

(a) min $C(q_1, q_2)$

s.t. $q_1 + q_2 = Q$ — given level of total output from the two plants combined. (Exo; parameter)

Forming LaGrange

$$\mathcal{L} = 0.5q_1^2 + 10q_1 + 40q_2 + 200 + \lambda[Q - q_1 - q_2]$$

FOC

$$\frac{\partial \mathcal{L}}{\partial q_1} = q_1 + 10 - \lambda = 0 \quad \xrightarrow{③} \quad q_1^* = 30$$

$$\frac{\partial \mathcal{L}}{\partial q_2} = 40 - \lambda = 0 \quad \xrightarrow{①} \quad \lambda^* = 40$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Q - q_1 - q_2 = 0$$

$$q_2^* = Q - q_1^* \Rightarrow q_2^* = Q - 30$$

Note that Sol^n makes sense if $Q \geq 30$ (total product must be greater than 30 units)

(If Q is less than, there might be some trouble.)

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b) Check SOC when $Q = 100$

$$q_1^* = 30 \quad , \quad q_2^* = 70 \quad (= Q - q_1^* = 100 - 30)$$

$$\bar{H} = \begin{bmatrix} \mathcal{L}_{\lambda\lambda} & \mathcal{L}_{\lambda q_1} & \mathcal{L}_{\lambda q_2} \\ \mathcal{L}_{q_1\lambda} & \mathcal{L}_{q_1 q_1} & \mathcal{L}_{q_1 q_2} \\ \mathcal{L}_{q_2\lambda} & \mathcal{L}_{q_2 q_1} & \mathcal{L}_{q_2 q_2} \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$N = 2 \rightarrow N-1 = 1$ Check the last one only

$$|\bar{H}| = \begin{vmatrix} 0 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 0 \end{vmatrix} \begin{matrix} 0 & -1 \\ -1 & 1 \\ -1 & 0 \end{matrix}$$

$$\rightarrow \{0 + 0 + 0\} - \{1 + 0 + 0\}$$

$$= (-1) \rightarrow < \rightarrow \text{minimum sol}^{n}$$

$\rightarrow$ cost is minimized

ⓒ optimal Cost function $= C(q_1^*, q_2^*)$

$$\boxed{\text{Note that } \frac{\partial C^*}{\partial Q} = MC = 40 = \lambda^*}$$

$$\text{Cost } C^* = 0.5(30)^2 + 10(30) + 40(Q-30) + 200 \quad ; Q \geq 30$$

$$= 450 + 300 + 40Q - 1200 + 200 \quad ; Q \geq 30$$

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Question 2: Suppose that household's utility function is given by: $u = (x)(y-6)$. Let the household's income be \$210, and price of goods x and goods y be set equal to \$2 and \$5, respectively.

a) (5 points) Use the LaGrange method and derive the optimal consumption bundles, i.e. Marshallian demand for x and y.
b) (3 points) Confirm your result in (a) with the second-order condition.
c) (2 points) What would happen to the maximized level of utility function if income has increased by \$10?

$\max \ u(x,y) = x(y-6)$ $\quad P_x = 2$

s.t. $2x + 5y = 210 \quad (P_x \cdot x + P_y y = M) \quad P_y = 5, \ M = 210$

$\therefore$ LaGrange $\Rightarrow \ \mathcal{L} = x(y-6) + \lambda(210 - 2x - 5y)$

(a) FOC

$[x]: \ y - 6 - \lambda(2) = 0 \xrightarrow{①} y = 2\lambda + 6$

$[y]: \ x - \lambda(5) = 0 \xrightarrow{②} x = 5\lambda$

$[\lambda]: \ 210 - 2x - 5y = 0$

$\therefore 210 - 2(5\lambda) - 5(2\lambda + 6) = 0$ Plug in Budget Equation.

$210 - 10\lambda - 10\lambda - 30 = 0$

$\lambda^* = \frac{180}{20} = 9$

$\therefore y^* = 2\lambda^* + 6 \Rightarrow 18 + 6 = 24$ units

$x^* = 45$ units $(= 5 \cdot \lambda^*)$

$(x^*, y^*, \lambda^*) = (45, 24, 9)$

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(b) $$\bar{H} = \begin{bmatrix} \mathcal{L}_{\lambda\lambda} & \mathcal{L}_{\lambda x} & \mathcal{L}_{\lambda y} \\ \mathcal{L}_{x\lambda} & \mathcal{L}_{xx} & \mathcal{L}_{xy} \\ \mathcal{L}_{y\lambda} & \mathcal{L}_{yx} & \mathcal{L}_{yy} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & -5 \\ -2 & 0 & 1 \\ -5 & 1 & 0 \end{bmatrix}$$

as there are two generic choice variables (x, y) then, we check N-1 submatrix ⇒ 2-1 = 1 sub matrix (last one $\bar{H}_3$)

$$|\bar{H}| = \begin{vmatrix} 0 & -2 & -5 \\ -2 & 0 & 1 \\ -5 & 1 & 0 \end{vmatrix} \begin{matrix} 0 & -2 \\ -2 & 0 \\ -5 & 1 \end{matrix}$$

$$= (0 + 10 + 10) - (0 + 0 + 0)$$

$$= 20 > 0$$ max sol^n confirmed.

(c) Note that $\lambda = 9 = \frac{\partial U^*}{\partial M}$ = sensitivity of optimized level of utility with respect to income

$$\therefore \Delta U^* = \frac{\partial U^*}{\partial M} \cdot \Delta M$$

$$= \lambda^* \cdot \Delta M = (9)(10)$$

maximum utility will increase by 90 if income increased by 10.