Chapter 7

Optimization without Constraints: One Independent Variable Case

7.1 Product Market: Profit Maximization

$$¶ = TR - TC$$

$$TR = r(Q)$$

$$TC = c(Q)$$

$$¶ = r(Q) - C(Q)$$

First-Order Derivative:

$$\frac{d¶}{dQ} = r'(Q) - c'(Q) = 0$$

$$r'(Q) = c'(Q)$$

$$MR = MC \rightarrow \text{stationary point}$$

Second-Order Derivative:

$$r''(Q) - c''(Q) < 0$$

$$r''(Q) < c''(Q)$$

Perfectly Competitive Market:

Revenue function: $TR = PQ$

$$MR = \frac{dTR}{dQ} = P\frac{dQ}{dQ} = P$$

Necessary Condition: $P = MC$

Sufficient Condition: $r''(Q) = \frac{dMR}{dQ} = \frac{dP}{dQ} = 0$ (P is a constant.)

From $r''(Q) < c''(Q)$

For maximize profit, $\frac{dMC}{dQ}$ = c”(Q) > 0 → increasing MC

Monopoly Market:

$P = f(Q) = a - bQ$

Revenue function: $TR = r(Q) = P*Q$

$= [a - bQ]Q$

$= aQ - bQ^2$

$MR = \frac{dTR}{dQ}$

$= a - 2bQ$

$AR = \frac{TR}{Q}$

$= a - bQ$

7.2 The Effects of Taxes

7.2.1 Lump-Sum Tax

$TR = r(Q)$

$TC = c(Q)$

$T = t_0$

Profit after tax: $¶_N = TR - TC - T$

$= r(Q) - c(Q) - t_0$

First-Order Condition: $\frac{d¶_N}{dQ} = r'(Q) - c'(Q) = 0$

Second-Order Condition: $r''(Q) < c''(Q)$

Q_e after lump-sum tax = Q_e before lump-sum tax

7.2.2 Profit Tax

$$TR = r(Q)$$

$$TC = c(Q)$$

$$T = t¶ ; \quad 0 < t < 1$$

$$¶_N = TR - TC - t¶$$

$$¶_N = r(Q) - c(Q) - t[r(Q) - c(Q)]$$

$$= (1 - t)[r(Q) - c(Q)]$$

First-Order Condition: $\frac{d¶_N}{dQ} = (1\text{-}t)[r'(Q) - c'(Q)] = 0$

$0 < t < 1 \rightarrow (1 - t) > 0$ so $r'(Q) - c'(Q)$ must $= 0$

$$MR = MC$$

Second-Order Condition: $r''(Q) < c''(Q)$

Q_e after profit tax = Q_e before profit tax

7.2.3 Excise Tax

$$TR = r(Q)$$

$$TC = c(Q)$$

$$T = tQ ; \; t > 0$$

$$¶_N = r(Q) - c(Q) - tQ$$

First-Order Condition: $\frac{d¶_N}{dQ} = r'(Q) - c'(Q) - t$

$$= r'(Q) - [c'(Q) + t] = 0$$

$$MR = MC + t$$

Q_e after excise tax < Q_e before excise tax

7.2.4 Maximization of Tax Revenue

Assume that the government collects excise tax on a monopolist.

$$TR = b_1Q + c_1Q^2$$

$$TC = a_2 + b_2Q + c_2Q^2$$

Government levies tax t baht per unit.

$$TC = a_2 + b_2Q + c_2Q^2 + tQ$$

$$a_2 + (b_2 + t)Q + c_2Q^2$$

$$¶ = b_1Q + c_1Q^2 - [a_2 + (b_2 + t)Q + c_2Q^2]$$

First-Order Condition: $\frac{d¶}{dQ} = 0$

$$Q_e = \frac{- b_1 + (b_2 + t)}{2(c_1 - c_2)}$$

$$T = tQ$$

$$= t * \frac{- b_1 + (b_2 + t)}{2(c_1 - c_2)}$$

$$= \frac{-b_1t + b_2t + t^2}{2(c_1 - c_2)}$$

$$= \frac{-t(b_1 - b_2) + t^2}{2(c_1 - c_2)}$$

First-Order Condition: $\frac{dT}{dt} = 0$

$$= \frac{-(b_1 - b_2) + 2t}{2(c_1 - c_2)} = 0$$

$t = \underline{b_1 - b_2}$ → stationary point

Second-Order Condition: $\frac{d^2T}{dt^2} = \frac{2}{2(c_1 - c_2)} < 0$

$c_1 < 0,\ c_2 > 0 \rightarrow c_1 - c_2 < 0$

7.2.5 Maximization of Tax Revenue: Perfectly Competitive Market

Tax = t baht per unit

$$Q_d = Q_s$$

$$Q_d = a - bP \qquad (a, b > 0)$$

$$Q_s = -c + d(P-t) \qquad (c, d > 0),\ t \geq 0$$

$$a - bP = -c + d(P-t)$$

$$P_e = \frac{a + c + dt}{b + d}$$

$$Q_e = \frac{ad - bc - bdt}{b + d}$$

Tax revenue: $T = t * \frac{ad - bc - bdt}{b + d}$

$$= \frac{adt - bct - bdt^2}{b + d}$$

First-Order Condition: $\frac{dT}{dt} = \frac{ad - bc - 2bdt}{b + d} = 0$

$$t = \frac{ad - bc}{2bd}$$

Second-Order Condition: $\frac{d^2T}{dt^2} = \frac{-2bd}{b+d} < 0$