

Chapter 2

Mathematics and Economic Relations

Outline

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An economic model is a skeletal and simplified representation of the actual economy in which only primary factors and relationships relevant to our problem at hand is presented.

There are many benefits of using mathematics in the analysis within economics.



2.1 Ingredients of a Mathematical Model

If the economic model is mathematical, it will usually consist of a set of _____ designed to describe the structure of the model.

By relating a number of _____ to one another in certain ways, these equations give mathematical form to the set of analytical assumptions adopted.

Then, through application of the relevant mathematical operations to these equations, we may seek to derive a set of conclusions which logically follow from those assumptions.

In an economic model, it will consist of:

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→ *Variables*

A _____ is something whose magnitude can change and take on different values (so we use symbol, instead of using a specific number).

e.g. price _____, profit _____, revenue _____, cost _____, etc.

Variables whose solution values we seek from the model is called _____.

Variables which are assumed to be determined by forces external to the model and whose magnitudes are accepted as given is called _____.

Examples: Think of an economic model for market of rice.

Endogenous variables are _____

Exogenous variables are _____

→ *Constant*

A _____ is a magnitude that does not change.

It can be numerical, e.g. _____ .

Or, it can be represented by a symbol, e.g. _____ in case we haven't assigned the value to it yet.

If it is represented in a form of symbol, it will be called parametric constant or _____.

Note: Parameters closely resemble exogenous variables in that they are to be treated as “givens” in a model.

→ *Equations*

Equation relates variables to one another. There are three types of equations.

- A _____ equation sets up an identity between two alternate expressions that have exactly the same meaning.

The regular equals sign _____ or the identical-equality sign _____ “identically equal to” can be used interchangeably.

Example, _____.

- A _____ equation specifies the manner in which a variable behaves in response to changes in other variables.

Example,

- A _____ equation states the requirement to be satisfied.

An equilibrium condition describes the prerequisite for the attainment of equilibrium.

Example,

An optimization condition is derived from an optimization model.

Example,

Examples of economic models

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2.2 Relation and Function

Set is a collection of items/objects. Each item or object in the set must share some properties/characteristic to the group/set where it belongs.

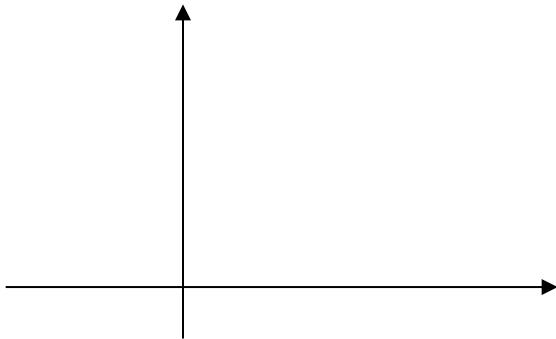
Element of one set (sets) can be associated with element of the other set, through a relation.

A **relation** represents a prescribed **mapping rule** between element in one set and element of the other set.

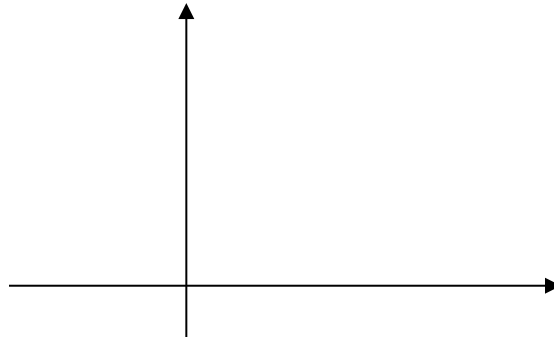
A **relation** between y and x is constituted of a collection of ordered pairs (x, y) that associates a y value with an x value.

That is, given an x value, one or more y values will be specified by that relation.

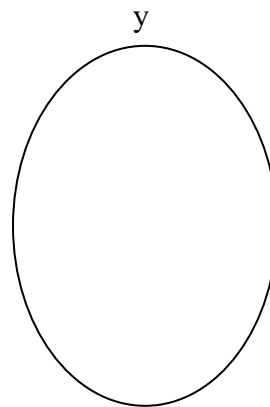
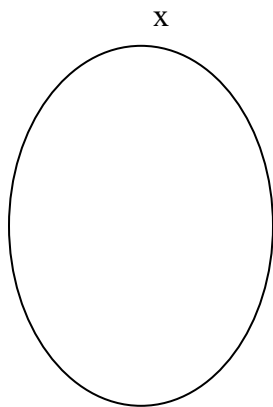
$$\{(x, y) | y = 2x\}$$



$$\{(x, y) | y \leq x\}$$



In the case of $\{(x, y) | y = 2x\}$, we have that:



For each x value, there exists only one corresponding y value. This is “One-to-One Relationship” or “One-to-One Mapping”

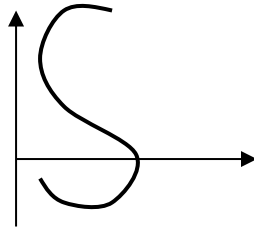
In this case, y is said to be a _____ of x , and this is denoted by _____
“ y equals f of x ”.

We may write _____ to show that the function f is a mapping rule from set x to set y .

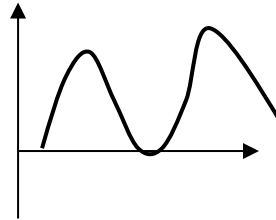
In $y = f(x)$, x is referred to as the argument of the function or “_____ Variable”
 y is called the value of the function or “_____ Variable”.

Test:

Is this following graph a function?



(a)



(b)

Note: A function needs a unique y for each x , but the converse is not required. More than one x value may legitimately be associated with the same y value. That is, function is one-to-one or many-to-one relation, but not one-to-many relation.

2.3 Types of Function

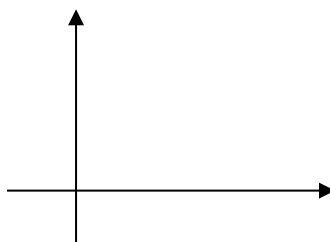
Different types of function represent different rules of mapping.

2.3.1 Constant Functions

A function whose range (the set of all values that the y variable can take) consists of only one element is called a constant function.



Example: In a national-income model, if we assume that investment is exogenously determined and equal to autonomous investment (I_0) which takes value of 100 millionbaht, the graph is:



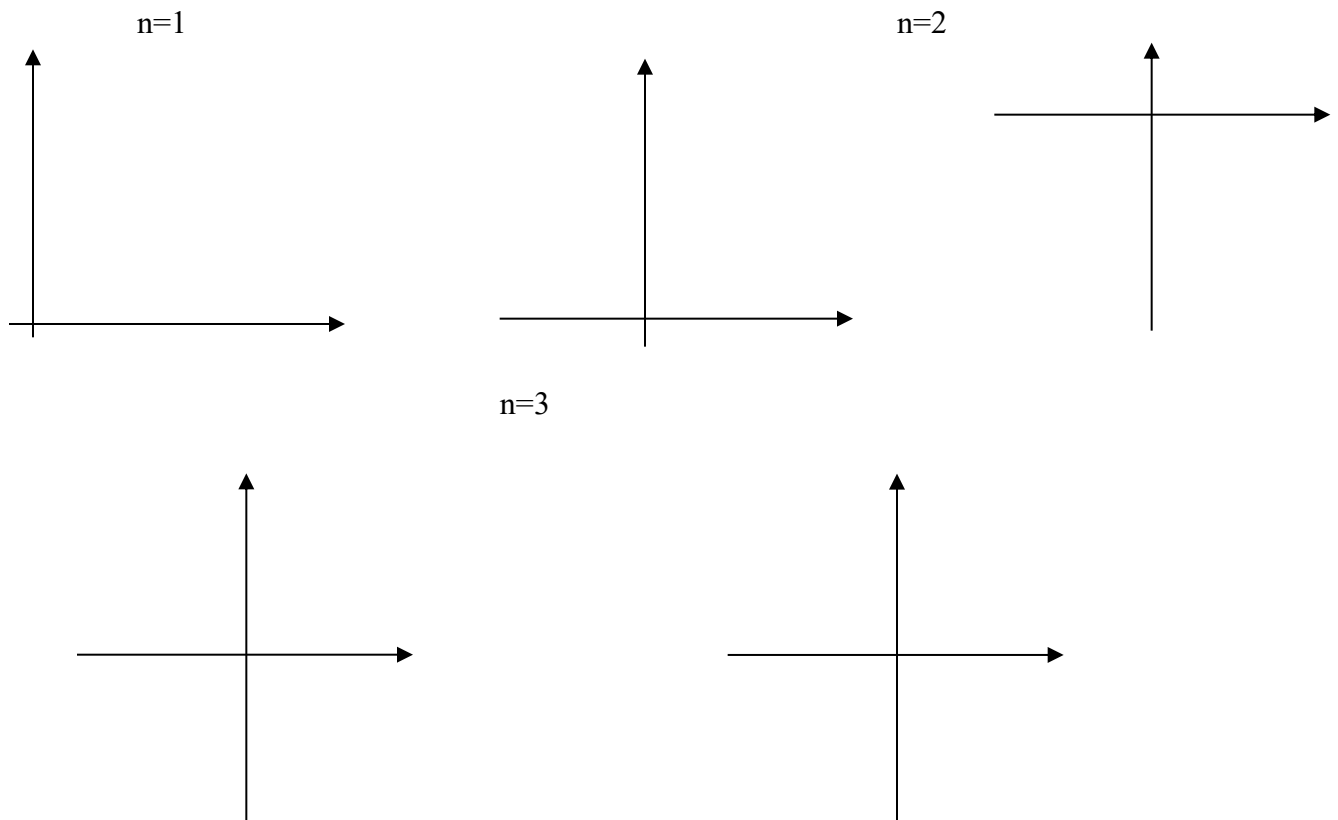
2.3.2 Polynomial Functions

A polynomial(multi-term) function of a single variable x has the general form of

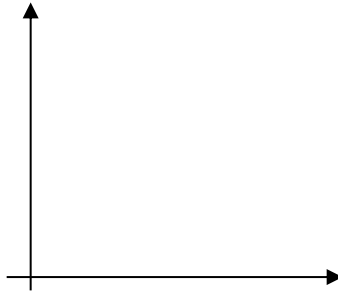
The highest power involved, the value of n is called the degree of the polynomial function.

Depending on the value of the integer n , subclasses of polynomial function are:

Degree n	Type of function	name
$n=0$	$y = a_0$	Constant
$n=1$	$y = a_0 + a_1x$	Linear
$n=2$	$y = a_0 + a_1x + a_2x^2$	Quadratic
$n=3$	$y = a_0 + a_1x + a_2x^2 + a_3x^3$	Cubic

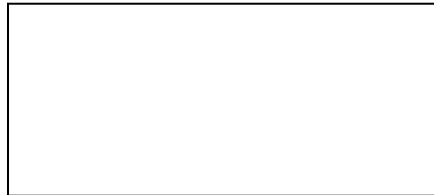


What if SES panel survey shows that the rich tend to consume less out of each additional unit of income they have earned? How should the graph for consumption function be?



2.3.3 Exponential and Logarithmic Function

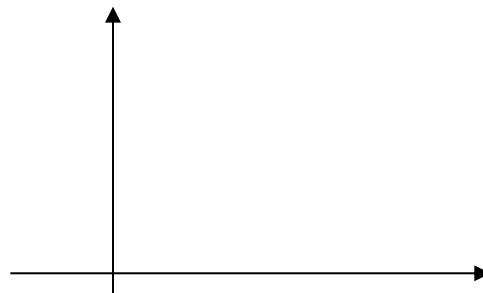
Exponential Function



Example: draw the following graphs

$$y = g(t) = b^t$$

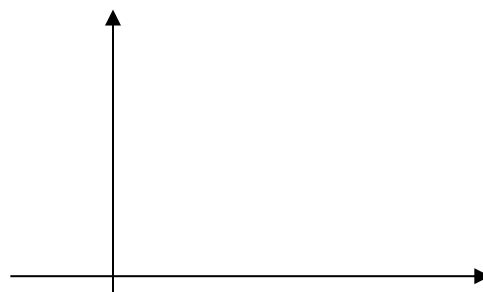
$$y = f(t) = b^{2t}$$



Example: draw the following graphs

$$y = g(t) = b^t$$

$$y = f(t) = 2b^t$$



If the base is the natural number e , we call it Natural exponential function:

$$y = e^t$$

, where $e = 2.71828\dots$

Common applications of exponential function are such as:

1. Growth calculation

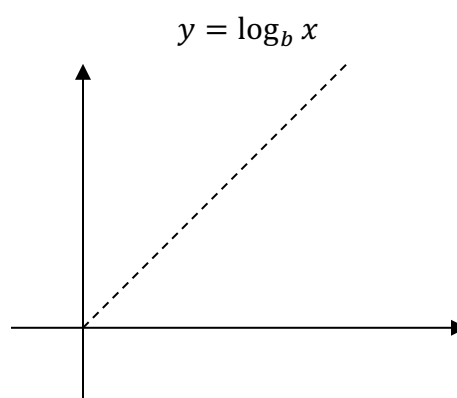
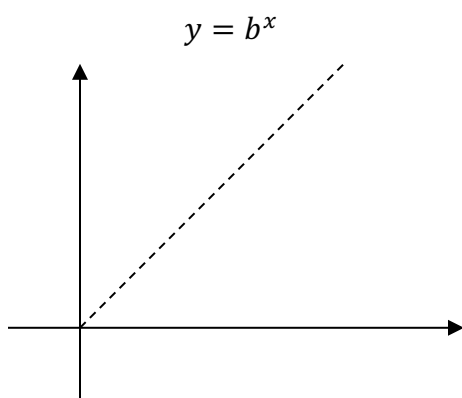
$X_t = (1 + g)X_{t-1}$, where g is growth rate, X_t is output at the t .

If we know output at the beginning period and growth rate, can we calculate output at time t ?

2. Present value

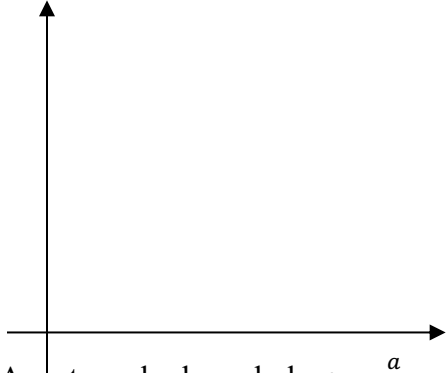
Logarithmic Function

Logarithmic Function is the inverse function of exponential function.



2.3.4 Other functions

Power Function: $y = Ax^b$



A rectangular hyperbola: $y = \frac{a}{x}$, or $xy = a$



Example:.....