Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score.....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c. **Please read and answer the following questions carefully and completely.**

Score.....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

$$\frac{1}{R_f} = \delta E\left[\frac{U'(C_1)}{U'(C_0)}\right]$$

$$\frac{1}{R_f} = \delta E\left[\frac{C_0}{C_1}\right]$$

$$\frac{C_1}{\delta C_0} = R_f \quad \#$$

When C_1 is high, R_f will also be high.

C_0 is high, R_f will be low.

Score.....................

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

$1 = E\left[m_{01} R_s\right]$

$$\varepsilon \approx \frac{\%\Delta \frac{C_1}{C_0}}{\%\Delta R_f} = \frac{R_f}{\frac{C_1}{C_0}} \cdot \frac{\partial \frac{C_1}{C_0}}{\partial R_f}$$

$1 = E[m_{01} R_s]$

① $1 = \Pi_s \, \delta \left(\frac{C_1}{C_0}\right)^{-1} R_f$

Total Diff ① : $0 = -\delta\left(\frac{C_1}{C_0}\right)^{-2}\Pi_s R_f \, d\left(\frac{C_1}{C_0}\right) + \Pi_s \delta\left(\frac{C_1}{C_0}\right)^{-1} dR_f$

$0 = \cancel{\delta \Pi_s}\left[\left(-\frac{C_1}{C_0}\right)^{-2} R_f \, d\left(\frac{C_1}{C_0}\right) + \left(\frac{C_1}{C_0}\right)^{-1} dR_f\right]$

$\left(\frac{C_1}{C_0}\right)^{-1} dR_f = \left(\frac{C_1}{C_0}\right)^{-2} R_f \, d\left(\frac{C_1}{C_0}\right)$

$\left(\frac{C_1}{C_0}\right)^{1} = R_f \cdot \frac{d\frac{C_1}{C_0}}{dR_f}$

$1 = \frac{R_f}{\frac{C_1}{C_0}} \frac{d\left(\frac{C_1}{C_0}\right)}{dR_f} = \varepsilon$

$\varepsilon > 1$, $SE > IE$

If in the next year intrest rate is Falling, the individual consumption level will decrease in the future because by the substitution effect when interest fall people will tend to spend more because the opportunity cost is lower and by income effect, people relatively become poorer so they spend less.

Score.....................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

$$P_i = E[m_{01} X_i]$$

$$P_i = E\left[\frac{\delta U'(C_1)}{U'(C_0)} X_i\right]$$

$$P_i = \delta \frac{C_0}{C_1} X_i$$

$\therefore$ when current consumption increase the price of this asset will increase and when future consumption increase the price of this asset will be low.

Score.....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

$$\frac{\sigma(m_{01})}{E[m_{01}]} = \frac{\sqrt{\mathrm{var}\left(e^{(\gamma-1)\ln\left(\frac{c_1}{c_0}\right)}\right)}}{E\left[e^{(\gamma-1)\ln\left(\frac{c_1}{c_0}\right)}\right]} = \frac{\sqrt{E\left[e^{2(\gamma-1)\ln\left(\frac{c_1}{c_0}\right)}\right] - E\left[e^{(\gamma-1)\ln\left(\frac{c_1}{c_0}\right)}\right]^2}}{E\left[e^{(\gamma-1)\ln\left(\frac{c_1}{c_0}\right)}\right]}$$

$$= \sqrt{E\left[e^{2(\gamma-1)\ln\left(\frac{c_1}{c_0}\right)}\right] / E\left(e^{(\gamma-1)\ln\left(\frac{c_1}{c_0}\right)}\right)^2 - 1}$$

$$= \sqrt{e^{2(\gamma-1)\mu_c + 2(\gamma-1)^2\sigma_c^2} / e^{2(\gamma-1)\mu_c + (\gamma-1)^2\sigma_c^2} - 1}$$

$$= \sqrt{e^{(\gamma-1)^2\sigma_c^2} - 1}$$

$$\approx \pm(\gamma-1)\sigma_c$$, in this case $\gamma = 0 \therefore \pm\sigma_c$ X

Hanson - Jagnathan bound is the ratio of standard deviation of the Stochastic discount factor to its mean to exceed the Sharpe ratio in this case it should be bound by ±1 standard deviation.