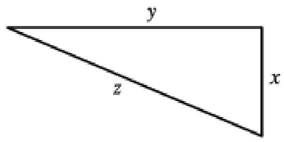
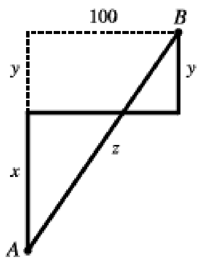
15. 

We are given that $\frac{dx}{dt} = 30$ km/h and $\frac{dy}{dt} = 72$ km/h. $z^2 = x^2 + y^2 \Rightarrow$

$$2z\,\frac{dz}{dt} = 2x\,\frac{dx}{dt} + 2y\,\frac{dy}{dt} \Rightarrow z\,\frac{dz}{dt} = x\,\frac{dx}{dt} + y\,\frac{dy}{dt} \Rightarrow \frac{dz}{dt} = \frac{1}{z}\left(x\,\frac{dx}{dt} + y\,\frac{dy}{dt}\right).$$

After 2 hours, $x = 2\,(30) = 60$ and $y = 2\,(72) = 144 \Rightarrow z = \sqrt{60^2 + 144^2} = 156$, so

$$\frac{dz}{dt} = \frac{1}{z}\left(x\,\frac{dx}{dt} + y\,\frac{dy}{dt}\right) = \frac{60(30) + 144(72)}{156} = 78 \text{ km/h.}$$

21. 

We are given that $\frac{dx}{dt} = 35$ km/h and $\frac{dy}{dt} = 25$ km/h. $z^2 = (x+y)^2 + 100^2 \Rightarrow$

$2z\,\frac{dz}{dt} = 2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$. At 4:00 PM, $x = 4(35) = 140$ and $y = 4(25) = 100 \Rightarrow$

$z = \sqrt{(140+100)^2 + 100^2} = \sqrt{67{,}600} = 260$, so

$$\frac{dz}{dt} = \frac{x+y}{z}\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = \frac{140+100}{260}(35+25) = \frac{720}{13} \approx 55.4 \text{ km/h.}$$

6. $f(x) = \tan x$. $f(0) = \tan 0 = 0 = \tan \pi = f(\pi)$. $f'(x) = \sec^2 x \geq 1$, so $f'(c) = 0$ has no solution. This does not contradict Rolle's Theorem, since $f'\left(\frac{\pi}{2}\right)$ does not exist, and so f is not differentiable on $(0, \pi)$. (Also, $f(x)$ is not continuous on $[0, \pi]$.)

24. If $3 \leq f'(x) \leq 5$ for all x, then by the Mean Value Theorem, $f(8) - f(2) = f'(c) \cdot (8-2)$ for some c in $[2, 8]$. (f is differentiable for all x, so, in particular, f is differentiable on $(2, 8)$ and continuous on $[2, 8]$. Thus, the hypotheses of the Mean Value Theorem are satisfied.) Since $f(8) - f(2) = 6f'(c)$ and $3 \leq f'(c) \leq 5$, it follows that

$$6 \cdot 3 \leq 6f'(c) \leq 6 \cdot 5 \Rightarrow 18 \leq f(8) - f(2) \leq 30.$$

17. Let $w = 1 - 3x$. Then $\frac{dw}{dx} = -3$. Also, $\frac{dy}{dx} = \frac{dy}{dw}\frac{dw}{dx}$, so

$$y' = \frac{d}{dx}\int_{1-3x}^{1} \frac{u^3}{1+u^2}\,du = \frac{d}{dw}\int_{w}^{1} \frac{u^3}{1+u^2}du \cdot \frac{dw}{dx} = -\frac{d}{dw}\int_{1}^{w} \frac{u^3}{1+u^2}\,du \cdot \frac{dw}{dx} = -\frac{w^3}{1+w^2}(-3) = \frac{3(1-3x)^3}{1+(1-3x)^2}$$

28. $\int_0^1 (3 + x\sqrt{x})\,dx = \int_0^1 (3 + x^{3/2})\,dx = \left[3x + \frac{2}{5}x^{5/2}\right]_0^1 = \left[\left(3 + \frac{2}{5}\right) - 0\right] = \frac{17}{5}$

30. $\int_0^2 (y-1)(2y+1)\,dy = \int_0^2 (2y^2 - y - 1)\,dy = \left[\frac{2}{3}y^3 - \frac{1}{2}y^2 - y\right]_0^2 = \left(\frac{16}{3} - 2 - 2\right) - 0 = \frac{4}{3}$

34. $\displaystyle\int_1^2 \frac{s^4+1}{s^2}\,ds = \int_1^2 (s^2 + s^{-2})\,ds = \left[\frac{1}{3}s^3 - \frac{1}{s}\right]_1^2 = \left(\frac{8}{3} - \frac{1}{2}\right) - \left(\frac{1}{3} - 1\right) = \frac{7}{3} + \frac{1}{2} = \frac{17}{6}$

40. $f(x) = \frac{4}{x^3}$ is not continuous on the interval $[-1, 2]$, so FTC2 cannot be applied. In fact, f has an infinite discontinuity at $x = 0$, so $\displaystyle\int_{-1}^{2} \frac{4}{x^3}\,dx$ does not exist.

50. $g(x) = \int_{1-2x}^{1+2x} t \sin t \, dt = \int_{1-2x}^{0} t \sin t \, dt + \int_{0}^{1+2x} t \sin t \, dt = -\int_{0}^{1-2x} t \sin t \, dt + \int_{0}^{1+2x} t \sin t \, dt \quad \Rightarrow$

$$g'(x) = -(1-2x)\sin(1-2x) \cdot \frac{d}{dx}(1-2x) + (1+2x)\sin(1+2x) \cdot \frac{d}{dx}(1+2x)$$
$$= 2(1-2x)\sin(1-2x) + 2(1+2x)\sin(1+2x)$$

14. Let $x = 1 - u^2$. Then $dx = -2u\,du$ and $u\,du = -\frac{1}{2}\,dx$, so

$$\int u\sqrt{1-u^2}\,du = \int \sqrt{x}\left(-\tfrac{1}{2}\,dx\right) = -\tfrac{1}{2}\int x^{1/2}\,dx = -\tfrac{1}{2} \cdot \tfrac{2}{3}x^{3/2} + C = -\tfrac{1}{3}(1-u^2)^{3/2} + C.$$

16. Let $u = \sqrt{x}$. Then $du = \dfrac{1}{2\sqrt{x}}\,dx$ and $2\,du = \dfrac{1}{\sqrt{x}}\,dx$, so

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int \sin u\,(2\,du) = -2\cos u + C = -2\cos\sqrt{x} + C.$$

24. Let $u = 1 + \tan t$. Then $du = \sec^2 t\,dt$, so

$$\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}} = \int \frac{\sec^2 t\,dt}{\sqrt{1+\tan t}} = \int \frac{du}{\sqrt{u}} = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{1+\tan t} + C.$$

30. Let $u = x^2 + 1$ [so $x^2 = u - 1$]. Then $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$, so

$$\int x^3\sqrt{x^2+1}\,dx = \int x^2\sqrt{x^2+1}\,x\,dx = \int (u-1)\sqrt{u}\,\left(\tfrac{1}{2}\,du\right) = \tfrac{1}{2}\int (u^{3/2} - u^{1/2})\,du$$
$$= \tfrac{1}{2}\left(\tfrac{2}{5}u^{5/2} - \tfrac{2}{3}u^{3/2}\right) + C = \tfrac{1}{5}(x^2+1)^{5/2} - \tfrac{1}{3}(x^2+1)^{3/2} + C.$$

Or: Let $u = \sqrt{x^2+1}$. Then $u^2 = x^2 + 1 \quad \Rightarrow \quad 2u\,du = 2x\,dx \quad \Rightarrow \quad u\,du = x\,dx$, so

$$\int x^3\sqrt{x^2+1}\,dx = \int x^2\sqrt{x^2+1}\,x\,dx = \int (u^2-1)\,u \cdot u\,du = \int (u^4 - u^2)\,du$$
$$= \tfrac{1}{5}u^5 - \tfrac{1}{3}u^3 + C = \tfrac{1}{5}(x^2+1)^{5/2} - \tfrac{1}{3}(x^2+1)^{3/2} + C.$$

Note: This answer can be written as $\frac{1}{15}\sqrt{x^2+1}(3x^4 + x^2 - 2) + C$.

48. Let $u = 1 + 2x$, so $x = \frac{1}{2}(u-1)$ and $du = 2\,dx$. When $x = 0$, $u = 1$; when $x = 4$, $u = 9$. Thus,

$$\int_0^4 \frac{x\,dx}{\sqrt{1+2x}} = \int_1^9 \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\frac{du}{2} = \frac{1}{4}\int_1^9 (u^{1/2} - u^{-1/2})\,du = \frac{1}{4}\left[\tfrac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^9 = \frac{1}{4} \cdot \frac{2}{3}\left[u^{3/2} - 3u^{1/2}\right]_1^9$$
$$= \tfrac{1}{6}[(27-9) - (1-3)] = \tfrac{20}{6} = \tfrac{10}{3}$$

69. Let $u = \ln x$. Then $du = \dfrac{dx}{x}$, so $\displaystyle\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du = \tfrac{1}{3}u^3 + C = \tfrac{1}{3}(\ln x)^3 + C.$

82. Let $u = -x^2$, so $du = -2x\,dx$. When $x = 0$, $u = 0$; when $x = 1$, $u = -1$. Thus,

$$\int_0^1 xe^{-x^2}\,dx = \int_0^{-1} e^u\left(-\tfrac{1}{2}\,du\right) = -\tfrac{1}{2}\left[e^u\right]_0^{-1} = -\tfrac{1}{2}\left(e^{-1} - e^0\right) = \tfrac{1}{2}(1 - 1/e).$$