
Portfolio Return and Risk

R2, Chapter 4

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1

Agenda

- First Half: Risk and Return Review
 - Single Asset Risk and Return
 - Portfolio Risk and Return
- Second Half: Application with Excel
 - Basic Excel Calculation
 - Manipulate Data with Excel Function
 - RANGE, VLOOKUP and etc..
 - **Basic Security Analysis**



2

What are investment returns?

- Investment returns **measure the financial results** of an investment.
- Returns may be **historical** or **prospective**
- Returns can be expressed in:
 - Dollar terms.
 - Percentage terms.



3

What is investment risk?

- Typically, investment returns are **not known with certainty**.
- Investment risk pertains to the probability of earning a return **less than that expected**.
- The greater the **chance of a return far below** the expected return, the greater the risk.
- Context
 - Single Asset Risk
 - Portfolio Risk



4

Single Asset Return and Expected Return

- Simple Rate of Return of asset i^{th}

$$R_i = \frac{P_1 - P_0 + D}{P_0} \text{ or } R_i = \ln\left(\frac{P_1 + D}{P_0}\right)$$

- Expected** Rate of Return of asset i^{th} at outcome j^{th}

$$E(R_i) = \bar{R}_i = \sum_{j=1}^M P_{ij} R_{ij}$$



5

Example

- IBM Stock have a return of 14%, 10% and 6% in a good time, normal time and bad time respectively (1/3 probability in all time)

$$10\% = \left(\frac{1}{3}\right)(14\%) + \left(\frac{1}{3}\right)(10\%) + \left(\frac{1}{3}\right)(6\%)$$

- What if the probability changes to 1/4, 1/2, 1/4 ?



6

Single Asset Dispersion, or Risk

- Variance and Standard Deviation

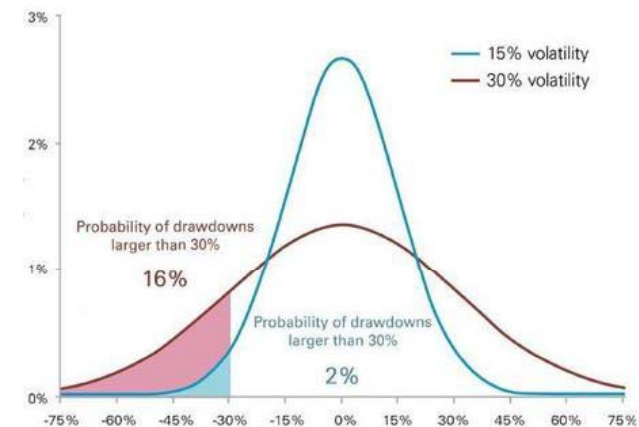
$$\sigma_i^2 = \sum_{j=1}^M (R_{ij} - \bar{R}_i)^2 P_{ij}$$

$$\sigma_i = \sqrt{\sum_{j=1}^M (R_{ij} - \bar{R}_i)^2 P_{ij}}$$



7

Probability Distribution: Which Stock is Riskier? Why?



8

Sample Historical Return & Risk

- Historical Average Return

$$\bar{r}_{\text{avg}} = \frac{\sum_{t=1}^n r_t}{n}$$

Excel's fnc.
=AVERAGE()
=STDEV()

- Historical Standard Deviation

$$s = \sqrt{\frac{\sum_{t=1}^n (r_t - \bar{r}_{\text{avg}})^2}{n-1}}$$



9

Exercise

- Single Asset Return & Risk



10

Portfolio Expected Return and Risk

- Expected Portfolio Return

$$E(R_p) = \bar{R}_p = \sum_{i=1}^N X_i \bar{R}_i$$

Excel's fnc.
=SUMPRODUCT()
=CORREL()

- Correlation Coefficient Between 2 Assets

$$\rho_{12} = \frac{\sum_{j=1}^M (R_{1j} - \bar{R}_1)(R_{2j} - \bar{R}_2)}{\sqrt{\sum_{j=1}^M (R_{1j} - \bar{R}_1)^2 \sum_{j=1}^M (R_{2j} - \bar{R}_2)^2}} = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$$

covariance



11

Portfolio Expected Return and Risk

- Portfolio Risk
 - Two-Asset Portfolio

$$\sigma_p^2 = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + 2X_1 X_2 \sigma_{12}$$

$$\sigma_p^2 = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + 2X_1 X_2 \rho_{12} \sigma_1 \sigma_2$$

- How about 3,4 or 5 assets?
- Write down a general form



12

Portfolio Expected Return and Risk - Matrix

- Consider a matrix form, let \mathbf{X} be a column vector of asset weight, \mathbf{R} be a column vector of expected return and Σ be variance covariance matrix

$$\mu_p = \mathbf{X}'\mathbf{R}$$

$$\sigma_p^2 = \mathbf{X}'\Sigma\mathbf{X}$$

- 2 Assets Portfolio Variance

$$\mu_p = \begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} E(R_1) \\ E(R_2) \end{bmatrix} \quad \sigma_p = \begin{bmatrix} X_1 & X_2 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$



13

Exercise

- Portfolio Return & Risk



14

Consider Theoretical Two-Stock Portfolios

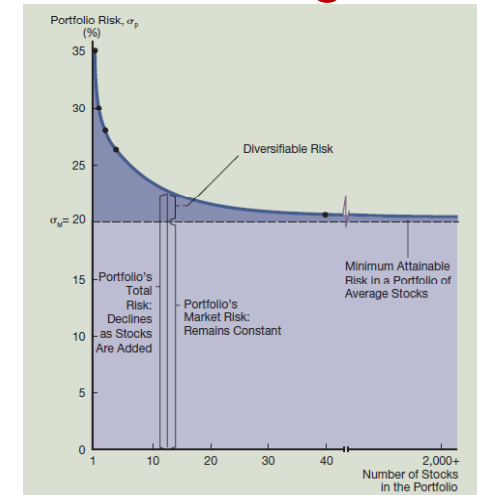
- Two stocks can be combined to form a riskless portfolio if $\rho = -1.0$.
- Risk is not reduced at all if the two stocks have $\rho = +1.0$.
- In general, stocks have $\rho \approx 0.35$, so risk is lowered but **not completely** eliminated.
- Investors typically hold many stocks.
 - By adding more stocks, σ_p would decrease but it would not be completely $\sigma_p=0$



15

Effects of Portfolio Size on Portfolio Risk for Average Stocks

σ_1 stock $\approx 35\%$
 $\sigma_{\text{Many stocks}}$ $\approx 20\%$



16

...Look Closely...

- **Single Asset Risk** = Diversification Risk + Market Risk
- **Diversification Risk**
 - A risk that can be avoided by diversification at **no cost**
 - A random event: lawsuits, strikes, successful and unsuccessful marketing programs, winning or losing a major contract and etc...
- **Market Risk, β**
 - A risk that is inevitable, to bear such risk, a premium **must** be given
 - Factors that systematically affect most firms: war, inflation, recessions, and high interest rates.



17

What Can We Learn From The Picture?

- As more stocks are added, each new stock has a smaller risk-reducing impact on the portfolio.
- σ_p falls very slowly after about 40 stocks are included. The lower limit for σ_p is about $20\% = \sigma_M$.
- By forming **well-diversified** portfolios, investors can eliminate about **half** the risk of owning a single stock.



18

Conclusion

- Risks are typically decreased when we add more stocks into a portfolio
- Risks can be managed but not virtually eliminated
- Not only ρ plays important role in risk reduction, proportion of investment also plays a crucial part
 - Up coming **“Minimum Variance Portfolio”**
- **Homework**
 - R2, Ch4, Q1-2



19