FN 201: Lecture Note 5
Time Value of Money

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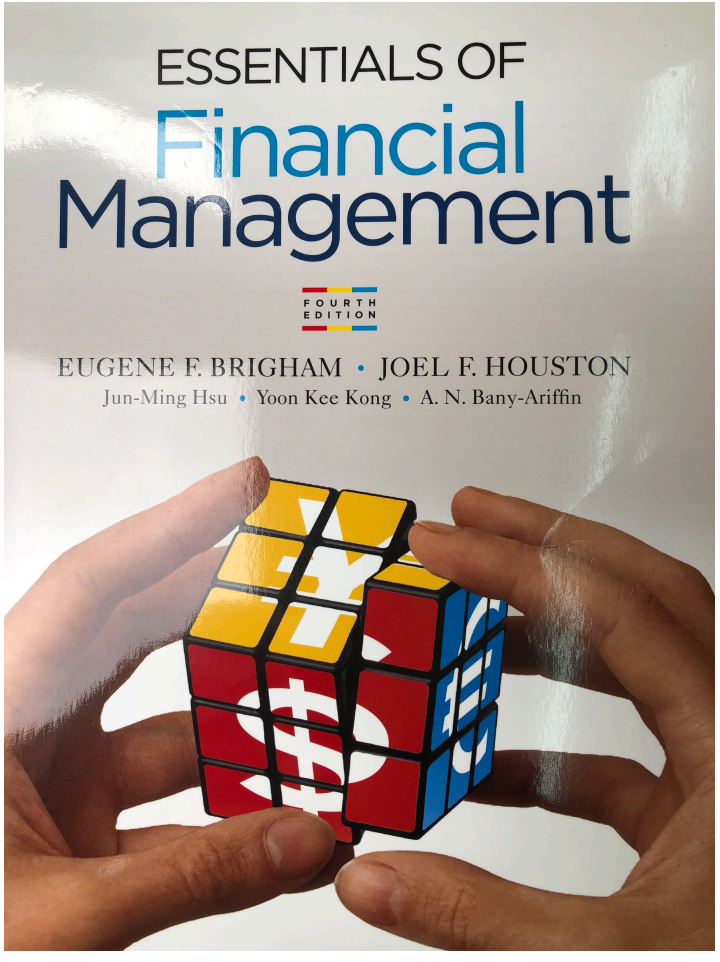
Reading

- Brigham, E.F., Houston, J.F., Hsu, JM., Kong, Y.K., Bany-Ariffin, A.N. (2018). Essentials of Financial Management. 4th Edition, Cengage Learning.

Chapter 5



Key Concepts and Skills

- Determine the future value of an investment made today
- Determine the present value of cash to be received at a future date
- Determine the future and present value of investments with multiple cash flows

Outline

1. Future Value and Compounding
2. Present Value and Discounting
3. Future and Present Values of Multiple Cash Flows
4. Valuing Level Cash Flows: Annuities and Perpetuities

Future Value vs. Present Value

Investment time line

Present cash flow

Compound

Future Value = Amount to which an investment will grow after earning interest

Present Value = Value today of a future cash flow.

Discount

Future cash flow

Basic Definitions

- Present Value – earlier money on a time line
- Future Value – later money on a time line
- Interest rate – “exchange rate” between earlier money and later money
  - Discount rate
  - Cost of capital
  - Opportunity cost of capital
  - Required return

Note (1)

Types of Interest: Simple interest vs. Compound interest

<u>Example</u>: What is the future value of $10,000 for the next 3 year if interest is 5% for one year?

Principal = 10,000			
Interest	= 500	= 500	= 500
Future Value			= 11,500

Principal = 10,000			
Interest	= 500	= 5% x (10,000 + 500)	= 5% x (10,000 + 1,025)
		= 1,025	= 1,576.25
Future Value			= 11,576.25

Note (2) on Cash Flows Pattern

1. Single sum

2. Uneven cash flows

3. Annuity

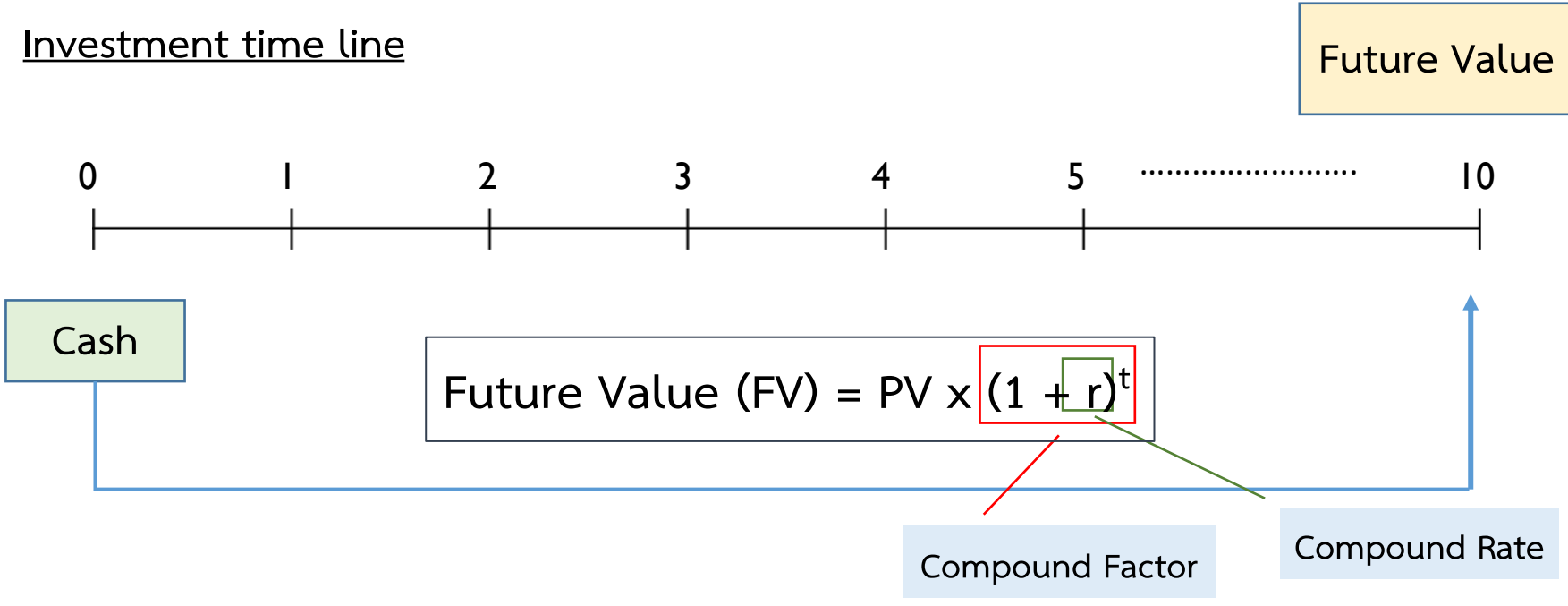
4. Perpetuity

Single Sum Cash Flows:

Present Value and Future Value

Future Value

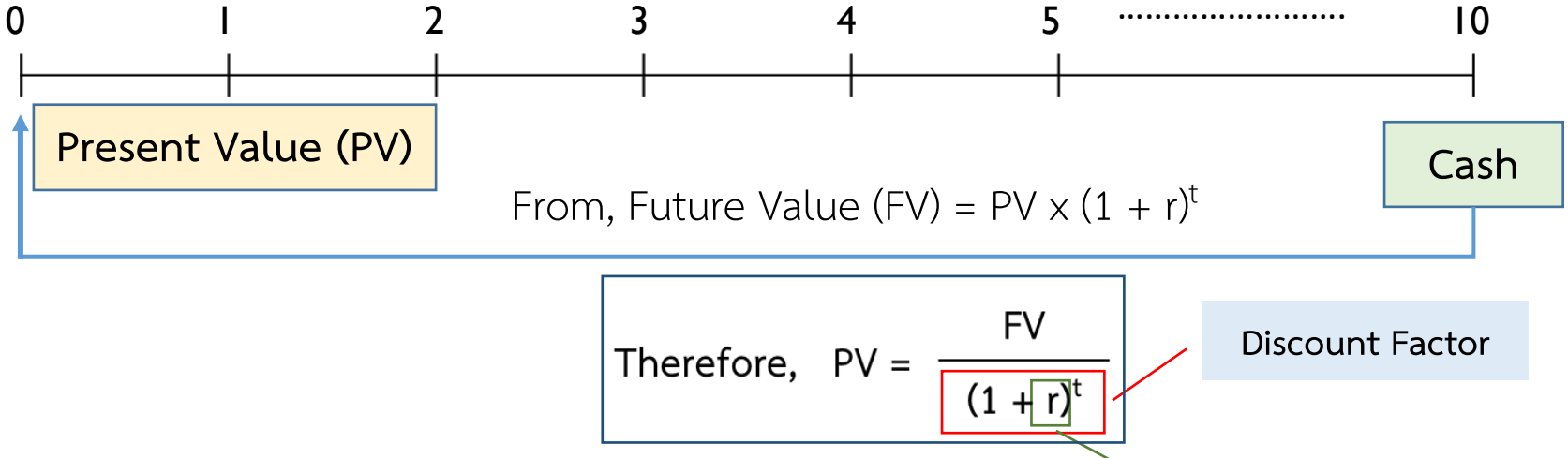
Investment time line



Example *What is the future value of $400,000 if interest is compounded annually at a rate of 5% for five year? – Using Table (FVIF)*

Present Value

Investment time line



Example *Tony plans to invest in one project which will bring back return at $450,000 at the end of 8th year, what is the present value of that project? – Using Table (PVIF)*

Example – Single Cash Flow

1. Your favorite uncle has offered you the choice of the following options. He will give you either $2,000 1 year from now or $3,000 4 years from now. Which would you choose if the discount rate is (*a*) 10 percent? (*b*) 20 percent?

2. You have just received notification that you have won the $1 million first prize in the Centennial Lottery. However, the prize will be awarded on your 100th birthday (assuming you're around to collect), 80 years from now. What is the present value of your windfall if the appropriate discount rate is 9 percent?

Example – Single Cash Flow

3. The first comic book featuring Superman was sold in 1938. In 2005, the estimated price for this comic book in good condition was about $485,000. This represented a return of 18 percent per year. For this to be true, what must the comic book have sold for when new?

4. You're trying to save to buy a new $170,000 Ferrari. You have $40,000 today that can be invested at your bank. The bank pays 6 percent annual interest on its accounts. How long will it be before you have enough to buy the car?

Annuity Cash Flows:

Present Value and Future Value

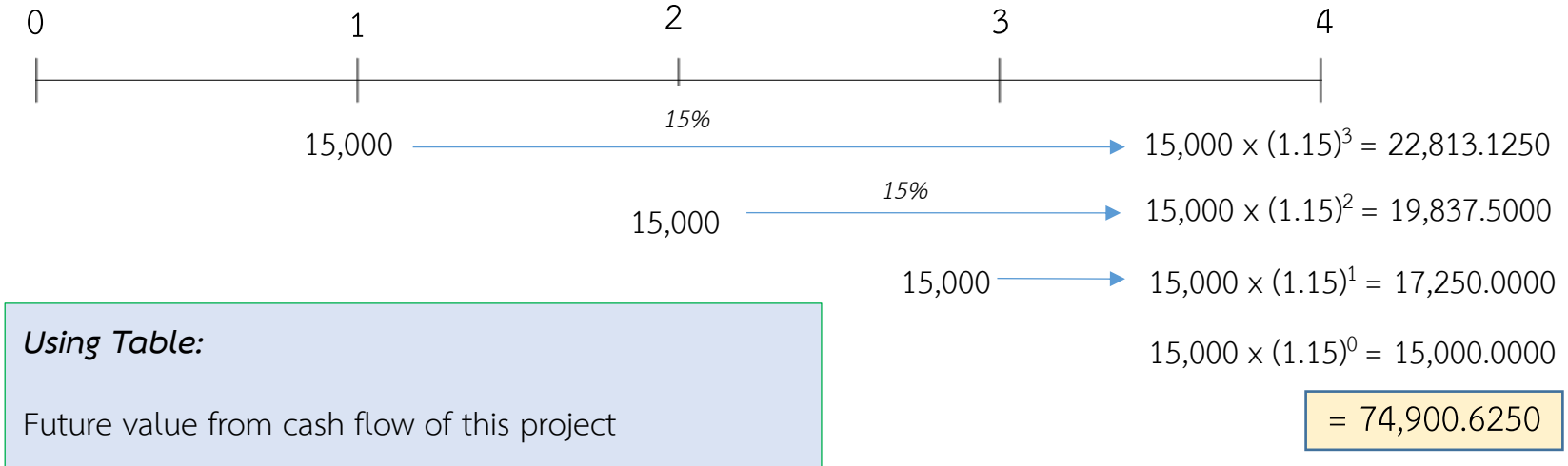
Future Value and Present Value for Annuities Due

Ordinary Annuities = Payment at the end of period

Annuities Due = Payment at the beginning of period

Future Value

Assume that BMX company receives annuity cash flows from project invested $15,000 at the end of each year until 4^{th} year with an annual growth rate 15%. Find future value at the end of 4^{th} year?



Using Table:

Future value from cash flow of this project

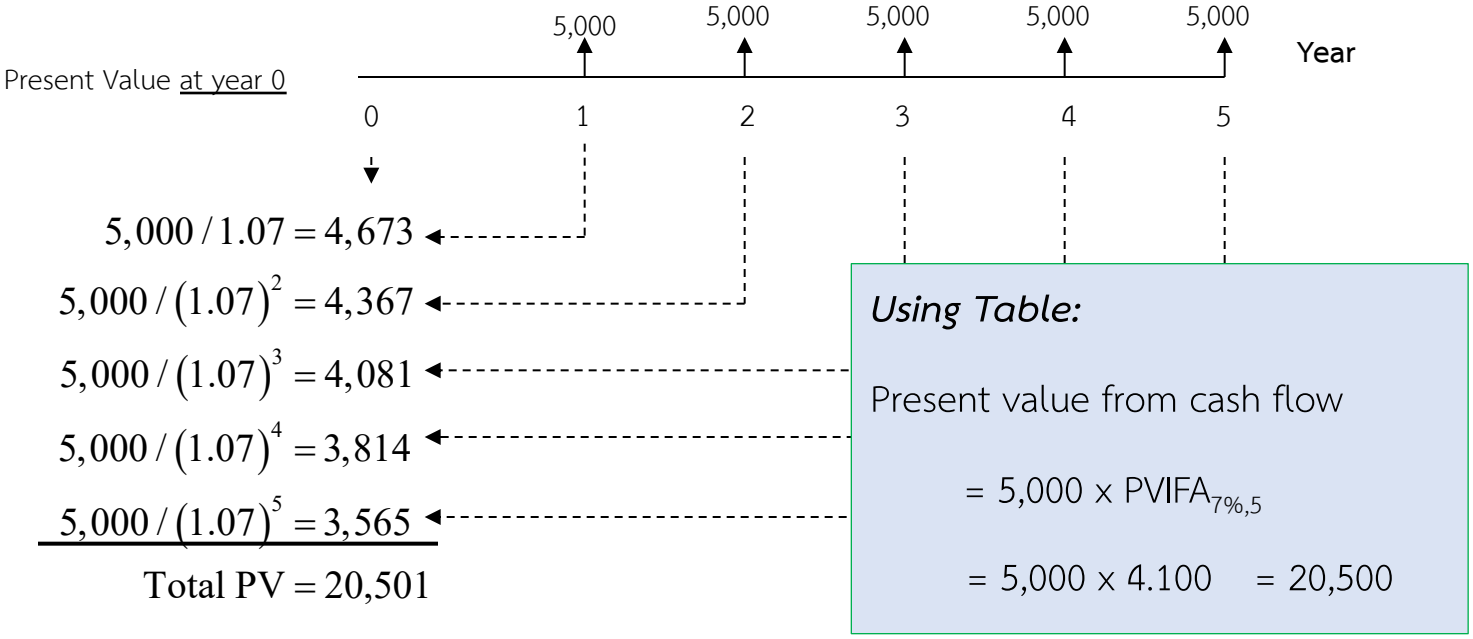
= 15,000 x $FVIFA_{15\%,4}$

= 15,000 x 4.993 = 74,895.0000

Present Values

Example

Tiburon Autos offers you "easy payments" of $5,000 per year, at the end of each year for 5 years. If interest rates are 7%, per year, what is the cost of the car?



Example – Ordinary Annuities

1. Your father is about to retire. His firm has given him the option of retiring with a lump sum of $20,000 or an annuity of $2,500 for 10 years. Which is worth more now, if an interest rate of 6 percent is used for the annuity?
2. David and Helen Zhang are saving to buy a boat at the end of five years. If the boat costs 20,000 and they can earn 10% a year on their savings, how much do they need to put aside at the end of years 1 through 5?
3. You need to have $50,000 at the end of 10 years. To accumulate this sum, you have decided to save a certain amount at the *end* of each of the next 10 years and deposit it in the bank. The bank pays 8 percent interest compounded annually for long-term deposits. How much will you have to save each year (to the nearest dollar)?

Example – Ordinary Annuities

4. A 55-year-old executive will retire at age 65 and expects to live to age 75. Assuming a 10 percent rate of return, calculate the amount he must have available at age 65 in order to receive $10,000 annually from retirement until death.

5. Kangaroo Autos is offering free credit on a new $10,000 car. You pay $1,000 down and then $300 a month for the next 30 months. Turtle Motors next door does not offer free credit but will give you $1,000 off the list price. If the rate of interest is 10% a year, (about 0.83% a month) which company is offering the better deal?

Future Value and Present Value for Annuities Due

Ordinary Annuities = Payment at the end of period

Annuities Due = Payment at the beginning of period

Future Value and Present Value for Annuities Due

Example:

Answer the questions that follow:

Case	Amount of Annuity	Interest	Period (Years)
A	2,500	8%	10
B	500	12%	6
C	30,000	20%	5
D	11,500	9%	8
E	6,000	14%	30

a. Assume you deposit the amount of annuity, calculate the future value when it is (1) an ordinary annuity, and (2) an annuity due.

b. Assume you have the annuity return from investment, calculate the present value when it is (1) an ordinary annuity, and (2) an annuity due.

c. Compare the finding in parts a.1 vs. a.2, and b.1 vs. b.2. All else being identical, which type of annuity – ordinary or annuity due – is preferable? Why?

Present Value of Perpetuity Cash Flows

Present Value of Perpetuity Cash Flows

Regular perpetuity

Regular perpetuity

$$\text{Present Value} = \frac{\$\$\$}{r}$$

Perpetuity Due

$$\text{Present Value} = \$\$\$ + \frac{\$\$\$}{r}$$

Present Value of Perpetuity Cash Flows

Example:

1. What is the present value of a perpetuity of $80 per year if the discount rate is 11 percent?

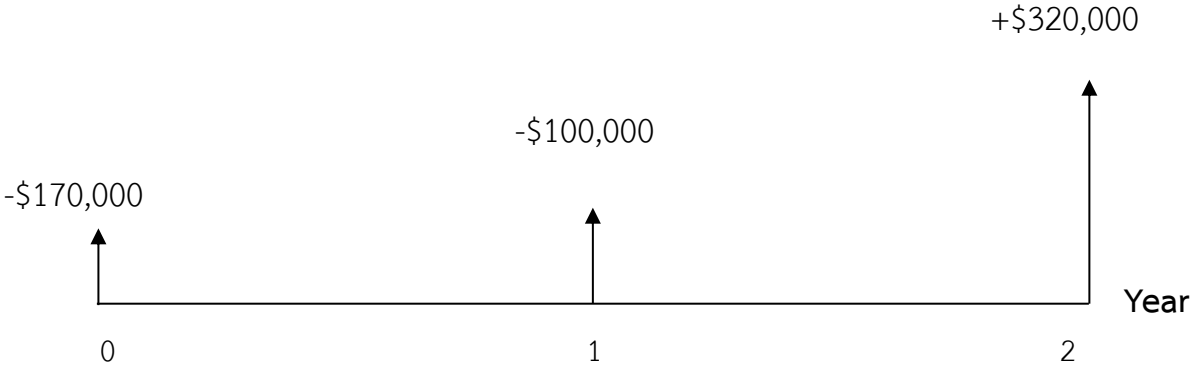
2. You want to invest in long-lived company with perpetuity dividend and risk (as presented by different values of discount rates) as below table. Which company would bring you highest present value?

Company	Annual amount	Interest
A	20,000	8%
B	100,000	10%
C	3,000	6%
D	60,000	5%

Present Value of Uneven Cash Flows

Present Value of Uneven Cash Flows

Assume that the cash flows from the construction and sale of an office building is as follows. Given a 5% required rate of return.



Present Value	
Year 0	
-170,000	= -$170,000
-100,000/1.05	= -$95,238
320,000/1.05^2	= $290,249
Total (Net) Present Value	= $25,011

Present Value of Uneven Cash Flows

Example:

1. Candy Parker has been offered an opportunity to receive the following mixed stream of revenue over the next 3 years. What is present value of this opportunity given interest rate at 15%?

Year	Annual amount
1	1,000
2	2,000
3	500

Present Value of Uneven Cash Flows

Example:

2. As a financial consultant for ABCompany, which project results in the most profitable (Hint: by calculating present value of the streams of cash flows). Assume that the firm's opportunity cost is 12%.

A		B		C	
Year	Cash Flows	Year	Cash Flows	Year	Cash Flows
1	-2000	1	10000	1-5	10000/year
2	3000	2-5	5000/year	6-10	8000/year
3	4000	6	7000		
4	6000				
5	8000				

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Question?