Macroeconomics

Lecture 11

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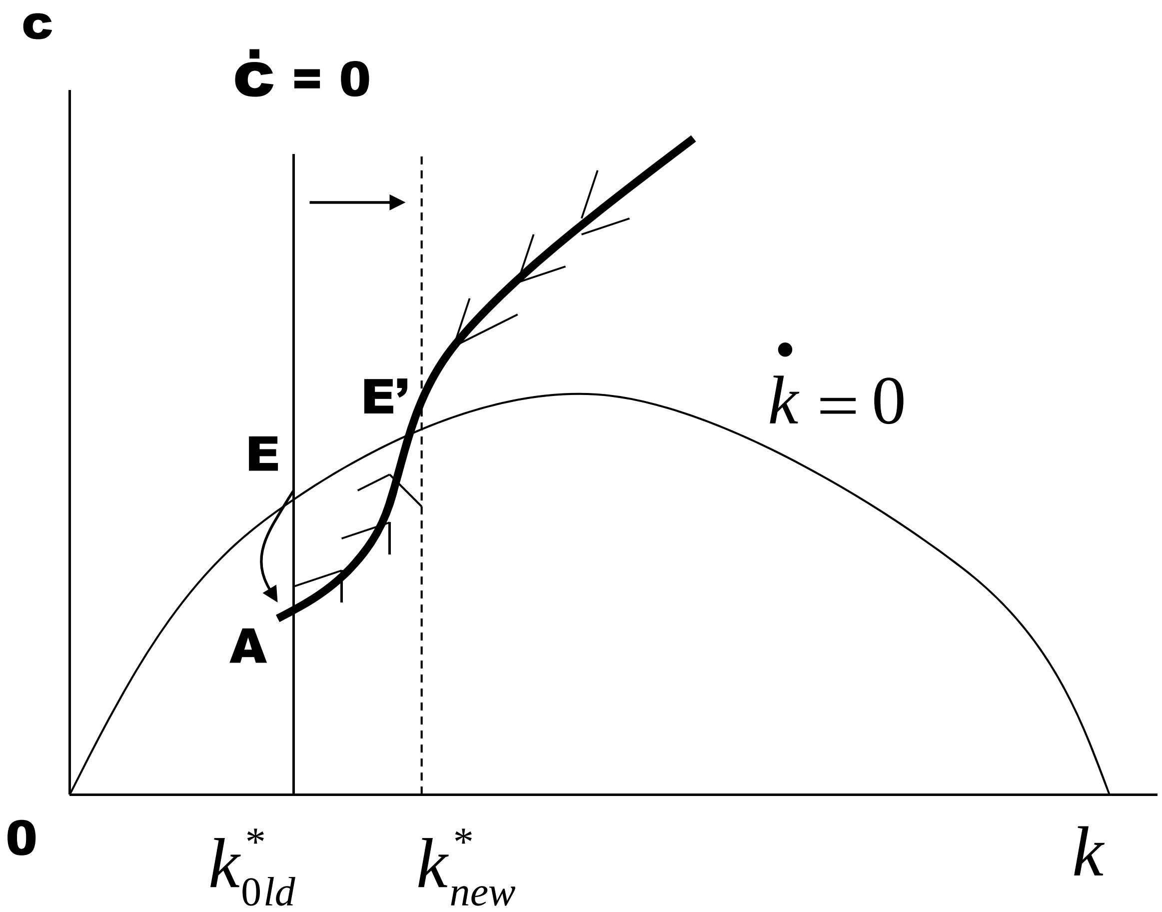
The Balanced Growth Path

- The behavior of the economy once it has converged to Point E is identical to that of the Solow economy on the balanced growth path. Capital (k), Output (y=f(k)), and consumption (c) per unit of effective labor are constant.

The Effects of a Fall in the Discount Rate (ρ)

- A fall in ρ means that, for a given k, c is lower than before. Since f”(k) is negative, the k that needed for ċ to equal to zero therefore rises. Then the ċ = 0 line shifts to the right.

The Effects of a Fall in the Discount Rate (ρ)



Adding Government Purchases to the model

- **Assume that the government buys output at rate G(t) per unit of effective labor per unit time.**
- **Assume that the purchases are financed by lump-sum taxes of amount G(t) per unit of effective labor per unit time. The Government always runs a balanced budget**

$$\dot{k}(t) = f\left(k(t)\right) - c(t) - G(t) - (n+g)k(t) \qquad (19)$$

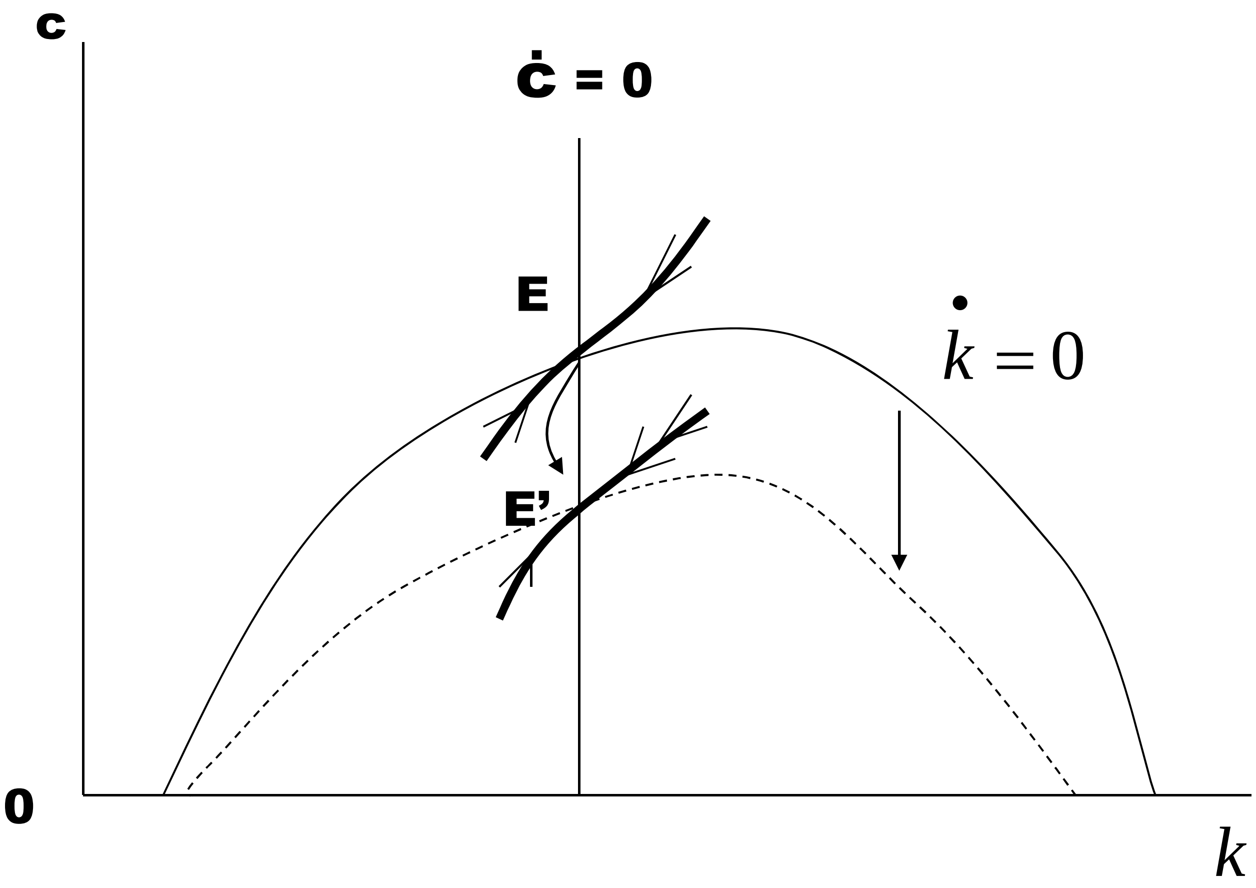
The househould's budget constraint becomes

$$\int_{t=0}^{\infty} e^{-R(t)} c(t) e^{(n+g)t} dt \le k(0) + \int_{t=0}^{\infty} e^{-R(t)} \left[w(t) - G(t)\right] e^{(n+g)t} dt \qquad (20)$$

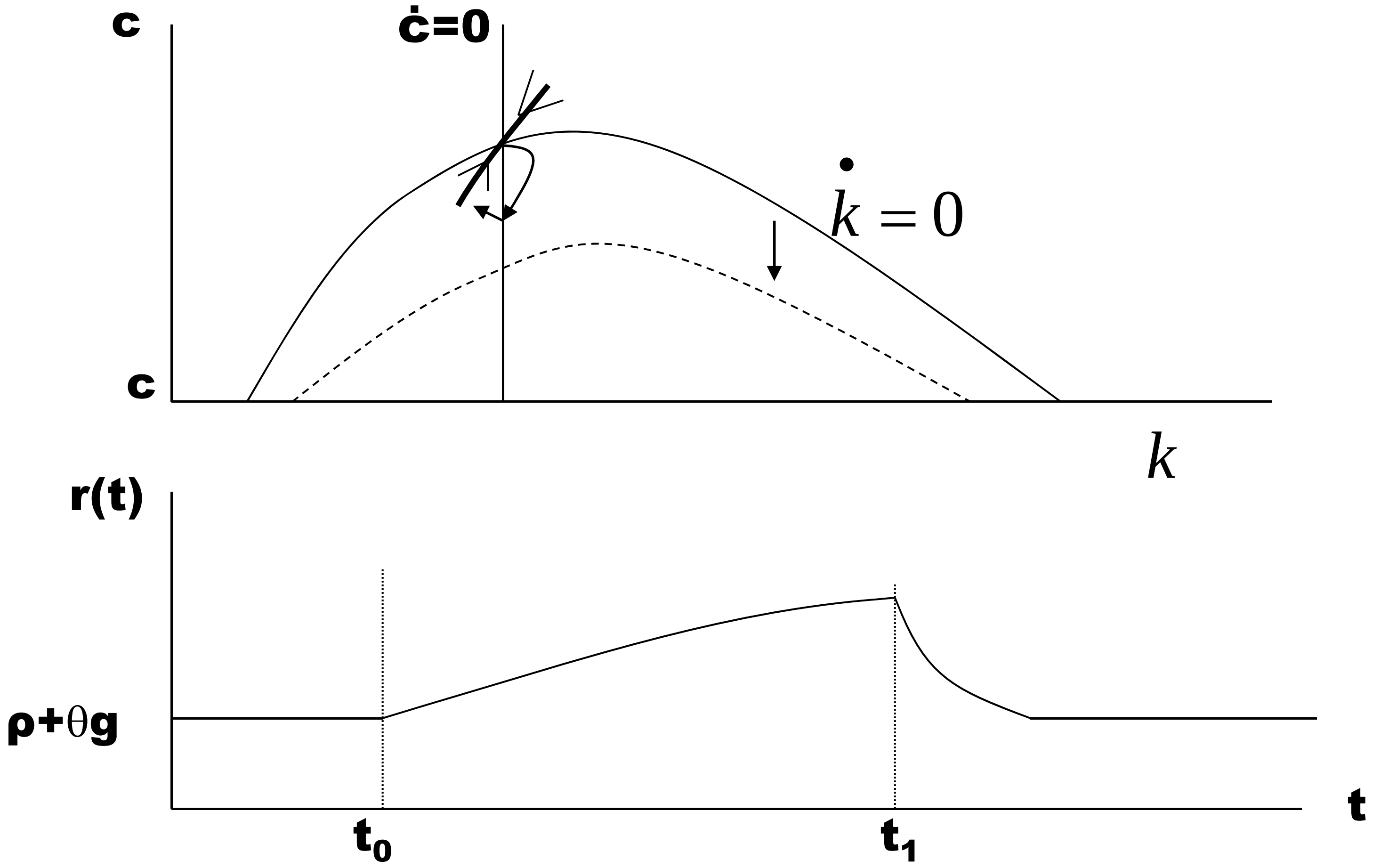
The Effects of Permanent Change in Government Purchases

- The permanent increase in G and taxes reduce households' lifetime wealth. Thus, consumption falls immediately, and the capital stocks and real interest rate are unaffected

The Effects of Permanent Change in Government Purchases



The Effects of Temporary Change in Government Purchases



The Diamond Model

- There is turnover in population: rather than being a fixed number of infinitely-lived households, new individuals are continually being born, and old individuals are continually dying.

The Diamond Model

- Each individual lived for only two periods
- L_t individuals are born in period t. Population grows at rate n; thus
- $L_t = (1+n)L_{t-1}$.
- At time t, there are L_t individuals in the first period of their lives (young people) and $L_{t-1}=L_t/(1+n)$ individuals in their second periods (old people).

The Diamond Model

- Each individual supplies one unit of labor when he is young and divides the resulting labor income between first-period consumption and savings; in the second period, the individual consumes the savings and any interest he earns.

The Diamond Model

- Let C_{1t} and C_{2t} denote the consumption in period t of young and old individuals. The utility of an individual born at t, denote U_t, depend on C_{1t} and C_{2t+1}. Let's assume constant-relative-risk-aversion utility.

$$U_t = \frac{C_{1t}^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{C_{2t+1}^{1-\theta}}{1-\theta}, \qquad \theta > 0,\ \rho > 0. \qquad (21)$$

The Diamond Model

- This function is needed for balanced growth. Because life times are finite, we no longer have to assume $\rho > n + (1-\theta)g$ to ensure that lifetime utility does not diverge.
- Production function is $Y_t = F(K_t, A_t L_t)$. $F(.)$ has constant returns to scale. A_t grows at rate g. Markets are competitive. $r_t = f'(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$

The Diamond Model

- The initial capital stock is K_0. Thus, at t=0, the capital owned by the old and the labor supplied by young are combined to produce output. The old consume both their capital income and their existing wealth; they then die and exit the model. The Young divide their labor income, $w_t A_t$, between consumption and savings. Hence $K_{t+1}=L_t(w_t A_t - C_t)$.

Household Behavior

The second – period consumption of an individual born at t

$$C_{2t+1} = (1 + r_{t+1})(w_t A_t - C_{1t}) \qquad (22)$$

Dividing both sides by $(1 + r_{t+1})$

$$C_{1t} + \frac{1}{1 + r_{t+1}} C_{2t+1} = A_t w_t \qquad (23)$$

∴ *The present value of lifetime consumption equals initial wealth (which is zero) plus the present value of lifetime labor income.*

The individual maximizes utility (21) subject to (23)

$$L = \frac{C_{1t}^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{C_{2t+1}^{1-\theta}}{1-\theta} + \lambda \left[A_t w_t - \left(C_{1t} + \frac{1}{1 + r_{t+1}} C_{2t+1} \right) \right] \qquad (24)$$

The first-order conditions are

$$C_{1t}^{-\theta} = \lambda \qquad (25)$$

$$\frac{1}{1+\rho} C_{2t+1}^{-\theta} = \frac{1}{1+r_{t+1}} \lambda \qquad (26)$$

Substituting the first equation into the second yields

$$\frac{1}{1+\rho} C_{2t+1}^{-\theta} = \frac{1}{1+r_{t+1}} C_{1t}^{-\theta} \qquad (27)$$

Or $$\frac{C_{2t+1}}{C_{1t}} = \left[\frac{1+r_{t+1}}{1+\rho}\right]^{\frac{1}{\theta}} \qquad (28)$$

This is analogous to the Euler equation.

Eq (28) *and* (23) *give*

$$C_{1t}+\frac{(1+r_{t+1})^{\frac{(1-\theta)}{\theta}}}{(1+\rho)^{\frac{1}{\theta}}}C_{1t}=A_t w_t \qquad (29)$$

Or

$$C_{1t}=\frac{(1+\rho)^{\frac{1}{\theta}}}{(1+\rho)^{\frac{1}{\theta}}+(1+r_{t+1})^{\frac{(1-\theta)}{\theta}}}A_t w_t \qquad (30)$$

The interest rate determines the fraction of income the individual consumes in the first period. Letting $s(r)$ *denote the fraction of income saved. Then*

$$s(r)=\frac{(1+r)^{\frac{(1-\theta)}{\theta}}}{(1+\rho)^{\frac{1}{\theta}}+(1+r)^{\frac{(1-\theta)}{\theta}}} \qquad (31)$$

We can rewrite (30) *into*

$$C_{1t}=\left[1-s(r_{t+1})\right]A_t w_t \qquad (32)$$

The young's saving is increasing in r *iff* $(1+r)^{\frac{(1-\theta)}{\theta}}$ *is increasing in* r.

$s(r)$ *is increasing in* r *if* $\theta<1$.

The rise in r *has both the income and a substituting effects.*

The equation of motion of k

The capital stock in period t+1 is the amount saved by young individuals in period t.

$$K_{t+1} = s\left(r_{t+1}\right)L_t A_t w_t \qquad (33)$$

Dividing both side by $L_{t+1}A_{t+1}$,

$$k_{t+1} = \frac{1}{(1+n)(1+g)} s\left(r_{t+1}\right) w_t \qquad (34)$$

$\because r_t = f'(k_t)$ *and* $w_t = f(k_t) - k_t f'(k_t)$, *Hence,*

$$k_{t+1} = \frac{1}{(1+n)(1+g)} s\left(f'(k_{t+1})\right)\left[f(k_t) - k_t f'(k_t)\right] \qquad (35)$$

The case of Logarithmic Utility and Cobb-Douglas Production

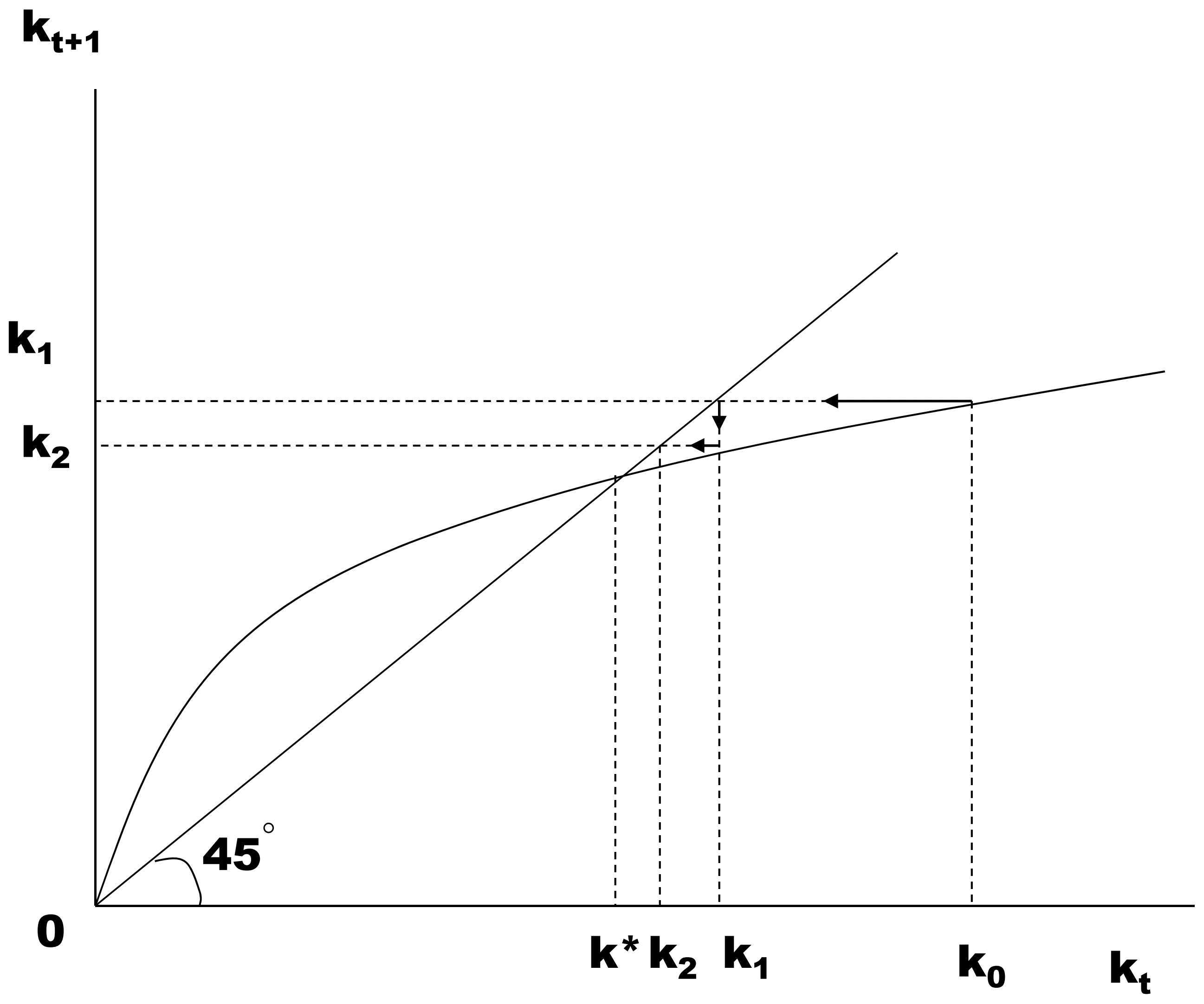
$$Logarithmic\ \ Utility\ \ and\ \ Cobb-Douglas\ \ \mathrm{Pr}oduction$$

$$When\ \theta\ is\ 1, \qquad s(r)=\frac{1}{2+\rho},$$

$$and\ \ when\ \ production\ \ is\ \ Cobb-Douglas,$$

$$f(k)\ is\ \ k^{\alpha}\ \ and\ \ \ w\ \ is\ \ (1-\alpha)k^{\alpha}.\ \ Eq\ (35)\ \ becomes$$

$$k_{t+1}=\frac{1}{(1+n)(1+g)}\frac{1}{2+\rho}(1-\alpha)k_t^{\alpha}\equiv Dk_t^{\alpha} \qquad (36)$$

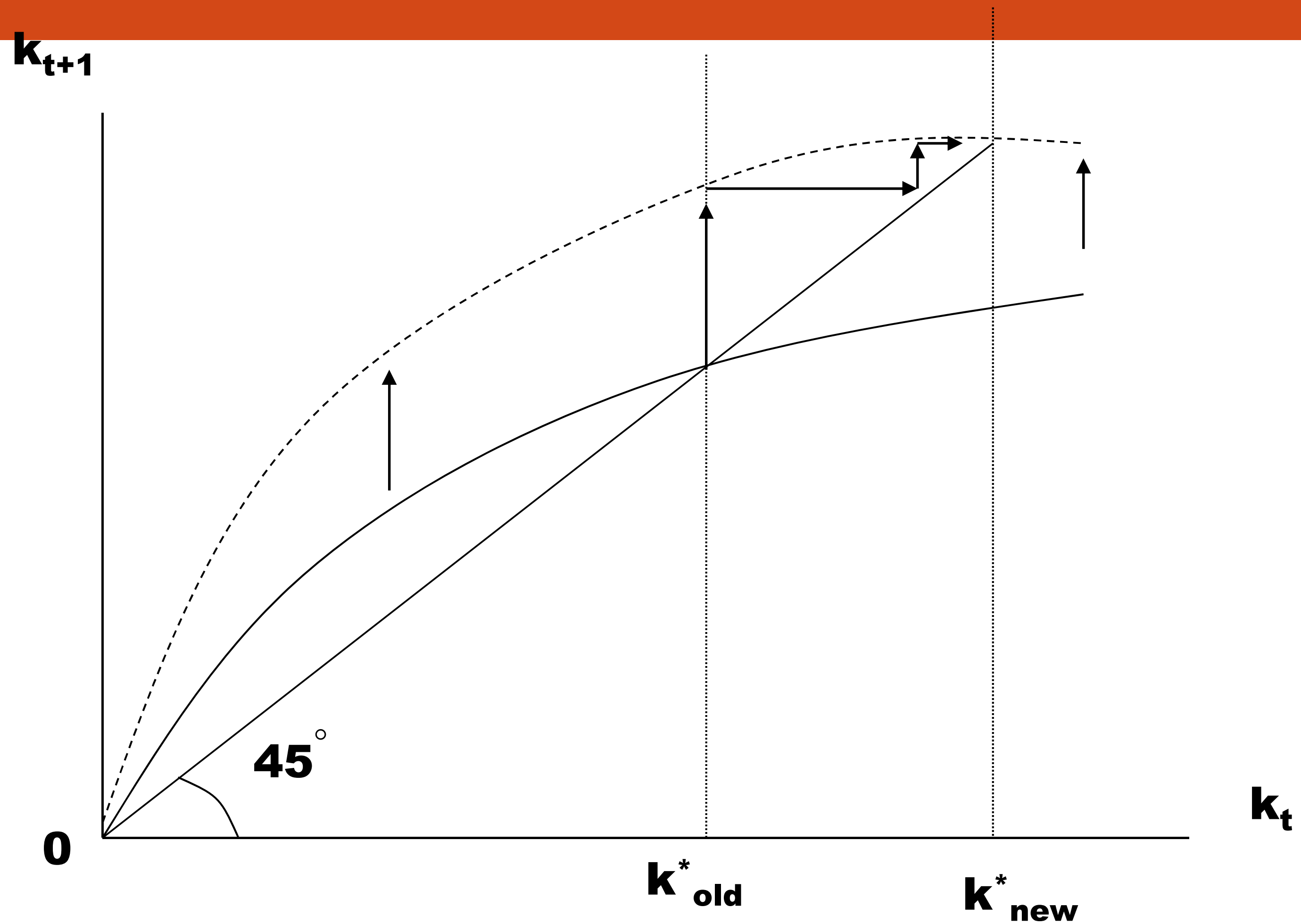


k_{t+1}
k_1
k_2
45°
0
k* k_2 k_1
k_0
k_t

The Diamond Model

- The properties of the economy once it has converged to its balanced growth paths are the same as those of the Solow and Ramsey economies on their balanced growth paths: the saving rate is constant, capital input ratio is constant and so on.

The Effect of a fall in ρ



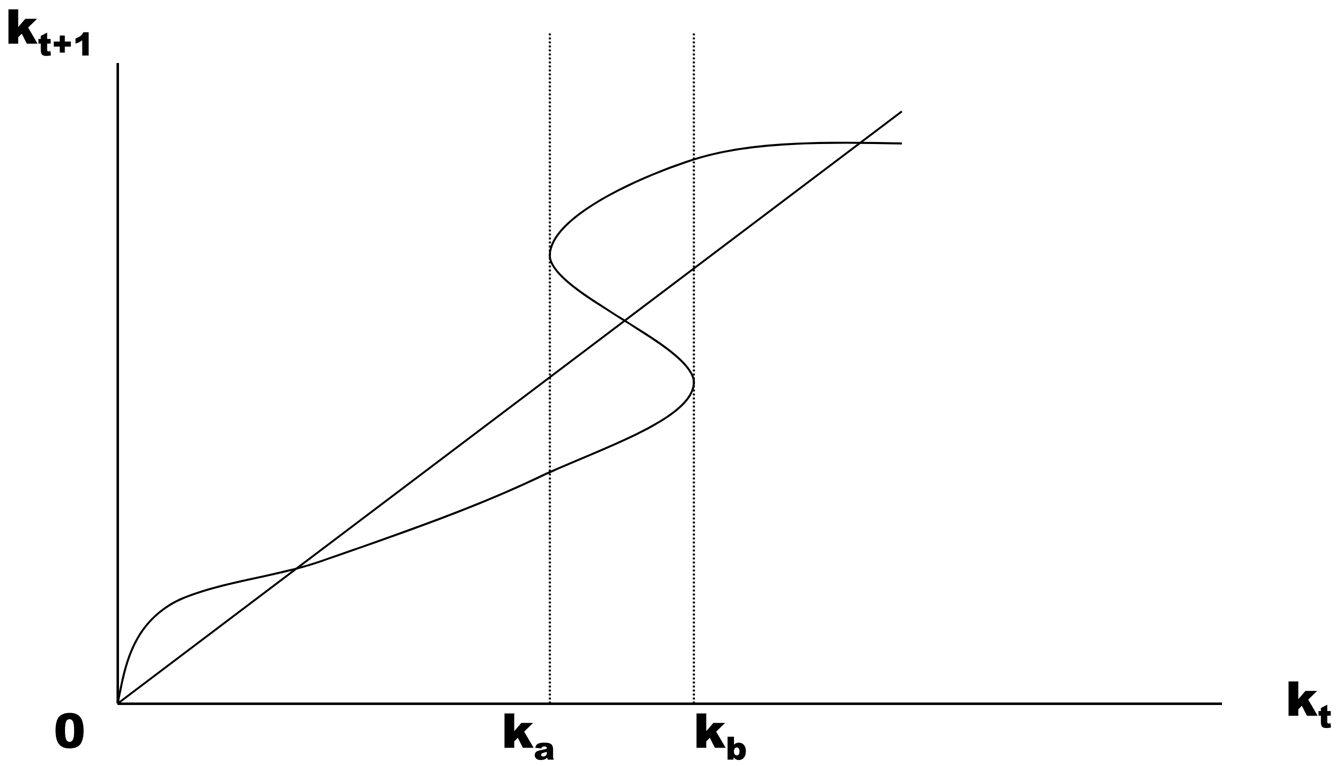
The General Case

- To understand the properties intuitively, let's rewrite Eq.(36) as

$$k_{t+1} = \frac{1}{(1+n)(1+g)} s\left(f'(k_{t+1})\right) \frac{\left[f(k_t) - k_t f'(k_t)\right]}{f(k_t)} f(k_t) \qquad (37)$$

- k_{t+1} is the product of four terms: the ratio of effective labor in period t to t+1, the fraction of that labor income that is saved, the fraction of that output that is paid to labor, and output per unit of effective labor at time t.

One possibilities for the relationship between k_t and k_{t+1}



One possibilities for the relationship between k_t and k_{t+1}

- When k_t is between k_a and k_b, there are three possible values of k_{t+1}. This can happen if saving is a decreasing function of interest rate.
- Eq. (37) does not fully determine how k evolves over time given its initial value.

One possibilities for the relationship between k_t and k_{t+1}

- This raises the possibility that "self-fulfilling prophecies" and "sunspots" can affect the behavior of the economy and that the economy can exhibit fluctuations even though there are no exogenous disturbances.