Homework

1.) Find cournot equilibrium when there are 3 firm in the market

$P = a - ba \quad ; \quad q = q_1 + q_2 + q_3$

$c_1 = c_2 = c_3 = c$

Find p^*

Find profit $\Pi_1 = \Pi_2 = \Pi_3 = ?$

Firm 1: $\Pi = PQ - c_1$

$= (a - b(q_1 + q_2 + q_3))q_1 - c_1$

$= aq_1 - q_1^2 b - q_1 q_2 b - q_1 q_3 b - c_1$

$\frac{d\Pi}{dq_1} = a - 2q_1 b - q_2 b - q_3 b$

$2q_1 b = a - q_2 b - q_3 b$

$q_1 = (a - q_2 b - q_3 b)/2b$

$q_1 = a - \left(\frac{a - q_3 b}{3b}\right) b - q_3 b$

$q_1 = (3a - a + q_3 b - 3q_3 b)/6b$

$= \frac{2a - 2q_3 b}{6b} = \frac{a - q_3 b}{3b} = a - \left(\frac{a}{4b}\right) b / 3b$

$= \frac{4a - a}{12b} = \frac{a}{4b}$ #

Firm 2 : $\Pi = PQ_2 - c_2$

$= (a - b(q_1 + q_2 + q_3))q_2 - c_2 \quad = aq_2 - q_1 q_2 b - q_2^2 b - q_2 q_3 b - c_2$

$\frac{d\Pi}{dq_2} : a - q_1 b - 2q_2 b - q_3 b$

$0 = a - q_1 b - 2q_2 b - q_3 b$

$2q_2 b = a - q_1 b - q_3 b$

$2q_2 b = a - \left[\frac{a - q_2 b - q_3 b}{2b}\right] b - q_3 b$

$2q_2 b = (2a - a + q_2 b + q_3 b - 2q_3 b)/2$

$4q_2 b = a + q_2 b - q_3 b$

$3q_2 b = a - q_3 b$

$q_2 = (a - q_3 b)/3b$

$= a - \left(\frac{a}{4b}\right) b$

$= \frac{4a - a}{12b} = \frac{a}{4b}$ #

Firm 3 : $\Pi = PQ_3 - C_3$

$$= (a - b(q_1 + q_2 + q_3))q_3 - C_3$$

$$= aq_3 - bq_1q_3 - bq_2q_3 - bq_3^2 - C_3$$

$$\frac{d\Pi}{dq_3} = a - bq_1 - bq_2 - 2bq_3$$

$$2bq_3 = a - bq_1 - bq_2$$

$$q_3 = (a - bq_1 - bq_2)/2b$$

$$= \left[a - b\left(\frac{a - q_3b}{3b}\right) - b\left(\frac{a - q_3b}{3b}\right)\right]/2b$$

$$= 3a - a + q_3b - a + q_3b / 6b$$

$$q_3 = (a + 2q_3b)/6b$$

$$6bq_3 = a + 2q_3b$$

$$4bq_3 = a$$

$$q_3 = \frac{a}{4b} \ \#$$

Equilibrium price : $P = a - ba$

$$= a - b(q_1 + q_2 + q_3)$$

$$= a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$$

$$= a - \cancel{b}\left(\frac{3a}{4\cancel{b}}\right) = \frac{4a - 3a}{4} = \frac{a}{4}$$

$$\Pi_1 = PQ - C_1$$

$$= \left(\frac{a}{4}\right)\left(\frac{a}{4b}\right) - C_1 = \frac{a^2}{16b} - C_1$$

$$\Pi_2 = PQ - C_2$$

$$= \left(\frac{a}{4}\right)\left(\frac{a}{4b}\right) - C_1 = \frac{a^2}{16b} - C_2$$

$$\Pi_3 = PQ - C_3$$

$$= \left(\frac{a}{4}\right)\left(\frac{a}{4b}\right) - C_3 = \frac{a^2}{16b} - C_3$$

2.) If there are N firm

$$q_i^+ = f(N) \ , \ P = (N) \ , \ \pi_i = f(N)$$

$$P = a - bq \ ; \ q = q_n$$

$$c_N = c_N = c_N = c$$

Assume that $q_1 + q_2 + \dots + q_n = A$

$$q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1 + q_2 + \dots + q_n)$$

$$0.5q_1 = \frac{a}{2b} - 0.5A$$

$$q_1 = \frac{a}{b} - A$$

$$q_n = \frac{a}{b} - A$$

$$P = a - b(q_1 + q_2 + \dots + q_n)$$

$$P = a - bq_1 - bq_2 - \dots - bq_n$$

$$\pi_i = (a - bq_1 - bq_2 - \dots - bq_n)q_i - c_i$$

$$\vdots$$

$$\pi_n = (a - bq_1 - bq_2 - \dots - bq_n)q_n - c_n$$

$$\frac{\partial \pi_n}{q_n} = a - 2bq_1 - bq_2 - \dots - bq_n = 0$$

$$\frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_n) = q_i$$

$$\frac{A}{2b} - 0.5(q_1 + q_2 + \dots + q_n) = q_n$$

Let $q_1 + q_2 + \dots + q_n \rightarrow A = n\left(\frac{a}{b} - A\right) = nq$

$$= n\frac{a}{b} - nA$$

$$(n+1)A = n\frac{a}{b}$$

$$* \ A = nq \longrightarrow A = \frac{na}{(n+1)b}$$

$$q_i = \frac{a}{(n+1)b} \ \#$$

$$P = a - b(A) = a - b\left(\frac{na}{(n+1)b}\right)$$

$$P = a - \frac{n}{n+1}(a) = \frac{a(n+1) - na}{n+1} = \frac{na + a - na}{n+1}$$

$$P = \frac{a}{n+1}$$

$$\Pi_i = P \cdot q_i - c_i = \left(\frac{a}{n+1}\right)\left(\frac{a}{(n+1)b}\right) - c_i$$

$$= \frac{a^2}{(n+1)^2 b} - c_i$$

3.) from q2, what happen if $n \to \infty$

if $n \to \infty$

$$q_i = \frac{a}{(n+1)b} \to 0$$

$$A = nq_i \to \infty$$

$$P = \frac{a}{n+1} \to 0$$

$$\Pi = \frac{a^2}{(n+1)^2 b} - c_i \to -c_i$$

if $n \to 1$

$$q_i = \frac{a}{2b} \to q = \frac{a}{2b} < a = \frac{na}{(n+1)b}$$

$A = nq_i = Q$, $n = 1$ $\therefore$ Monopoly

$$P = \frac{a}{n+1} = \frac{a}{2} \to P_m = \frac{a}{2} > P = \frac{a}{n+1}$$

$$\Pi = \frac{a^2}{(n+1)^2 b} - c_i = \frac{a^2}{4b} - c_i \therefore \Pi_m > \Pi_i = \frac{a^2}{(n+1)^2 b} - c_i$$