

C6. Use the data in WAGE2 for this exercise.

i. Consider the standard wage equation

$$\log(\text{wage}) = \beta_0 + \beta_1 \text{educ} + \beta_2 \text{exper} + \beta_3 \text{tenure} + u.$$

State the null hypothesis that another year of **general workforce experience** has the same effect on $\log(\text{wage})$ as another year of tenure with the current employer.

ii. Test the null hypothesis in part (i) against a two-sided alternative, at the 5% significance level, by constructing a 95% confidence interval. What do you conclude?

Source	SS	df	MS	Number of obs	=	935
Model	25.6953242	3	8.56510806	F(3, 931)	=	56.97
Residual	139.960959	931	.150334005	Prob > F	=	0.0000
				R-squared	=	0.1551
				Adj R-squared	=	0.1524
				Root MSE	=	.38773

lwage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
educ	.0748638	.0065124	11.50	0.000	.062083 .0876446
exper	.0153285	.0033696	4.55	0.000	.0087156 .0219413
tenure	.0133748	.0025872	5.17	0.000	.0082974 .0184522
_cons	5.496696	.1105282	49.73	0.000	5.279782 5.713609

i. $H_0: \beta_2 = \beta_3, \beta_2 - \beta_3 = 0$

$H_a: \beta_2 \neq \beta_3, \beta_2 - \beta_3 \neq 0$

ii. $H_0: \beta_2 = \beta_3, \beta_2 - \beta_3 = 0$

$H_a: \beta_2 \neq \beta_3, \beta_2 - \beta_3 \neq 0$

two-tailed test $\rightarrow t = \frac{(\hat{\beta}_2 - \hat{\beta}_3) - 0}{\text{se}(\hat{\beta}_2 - \hat{\beta}_3)}$

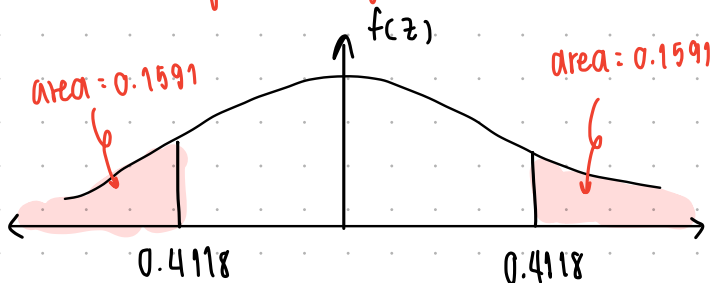
let $\hat{\theta}_1 = \hat{\beta}_2 - \hat{\beta}_3$; $H_0: \theta_1 = 0, t = \frac{\hat{\theta}_1 - 0}{\text{se}(\hat{\theta}_1)}$
 $H_a: \theta_1 \neq 0$

if rearrange $\hat{\theta}_1 = \hat{\beta}_2 - \hat{\beta}_3$, we have $\hat{\beta}_2 = \hat{\theta}_1 + \hat{\beta}_3$ or $\beta_2 = \theta_1 + \beta_3$

$$\begin{aligned} \log(\text{wage}) &= \beta_0 + \beta_1(\text{educ}) + \beta_2(\text{exper}) + \beta_3(\text{tenure}) + u \\ &= \beta_0 + \beta_1(\text{educ}) + (\theta_1 + \beta_3)(\text{exper}) + \beta_3(\text{tenure}) + u \\ &= \beta_0 + \beta_1(\text{educ}) + \theta_1(\text{exper}) + \beta_3(\text{exper}) + \beta_3(\text{tenure}) + u \\ &= \beta_0 + \beta_1(\text{educ}) + \theta_1(\text{exper}) + \beta_3(\text{exper} + \text{tenure}) + u \end{aligned}$$

new V

do the regression again



$$t = \frac{0.0019537 - 0}{0.0047434} = 0.4118 \approx 0.41$$

at 5% significant, p value is 0.1591 > 0.025 (significant level)

So, we do not reject H_0 because p value (0.1591) is greater than significant level (0.025)

Therefore, another year of general workforce experience has the same effect on $\log(\text{wage})$ as another year of tenure with the current employer.

Source	SS	df	MS	Number of obs	=	935
Model	25.6953242	3	8.56510806	F(3, 931)	=	56.97
Residual	139.960959	931	.150334005	Prob > F	=	0.0000
				R-squared	=	0.1551
				Adj R-squared	=	0.1524
				Root MSE	=	.38773

lwage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
educ	.0748638	.0065124	11.50	0.000	.062083 .0876446
exper	.0019537	.0047434	0.41	0.681	-.0073554 .0112627
newV	.0133748	.0025872	5.17	0.000	.0082974 .0184522
_cons	5.496696	.1105282	49.73	0.000	5.279782 5.713609

C8. The data set 401KSUBS contains information on net financial wealth (*nettfa*), age of the survey respondent (*age*), annual family income (*inc*), family size (*fsize*), and participation in certain pension plans for people in the United States. The wealth and income variables are both recorded in thousands of dollars. For this question, use only the data for single-person households (so *fsize* = 1).

i. How many single-person households are there in the data set?

ii. Use OLS to estimate the model

$$nettfa = \beta_0 + \beta_1 inc + \beta_2 age + u$$

and report the results using the usual format. Be sure to use only the single-person households in the sample. Interpret the slope coefficients. Are there any surprises in the slope estimates?

iii. Does the intercept from the regression in part (ii) have an interesting meaning? Explain.

iv. Find the p -value for the test $H_0: \beta_2 = 1$ against $H_1: \beta_2 < 1$. Do you reject H_0 at the 1% significance level? $\alpha = 1\%$.

v. If you do a simple regression of *nettfa* on *inc*, is the estimated coefficient on *inc* much different from the estimate in part (ii)? Why or why not?

iii. The interception point is -43.03981, it means that if both *inc* and *Age* are zero the net financial wealth would be about negative \$43,039.81. Therefore, when the age of the survey respondent and annual family income are zero, the saving of 43,039.81 will occur.

. regress nettfa inc age if fsize == 1

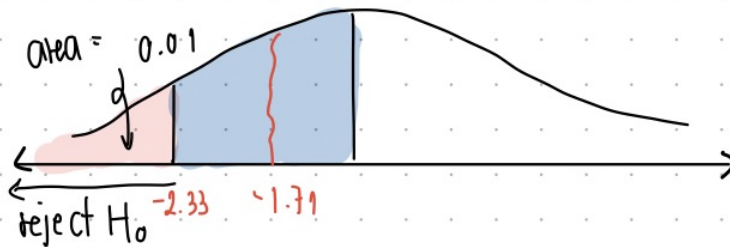
Source	SS	df	MS	Number of obs	=	2,017
Model	544916.989	2	272458.495	F(2, 2014)	=	136.46
Residual	4021048.06	2,014	1996.54819	Prob > F	=	0.0000
				R-squared	=	0.1193
				Adj R-squared	=	0.1185
Total	4565965.05	2,016	2264.86361	Root MSE	=	44.683

	nettfa	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
$\hat{\beta}_1$	inc	.7993167	.0597307	13.38	0.000	.6821762 .9164572
$\hat{\beta}_2$	age	.8426563	.0920169	9.16	0.000	.6621982 1.023115
Intercept	_cons	-43.03981	4.080393	-10.55	0.000	-51.04204 -35.03758

i. N = 2017

ii. We can see that if annual family income increase by \$1000, on average, net financial wealth will increase by \$799.3. For every additional age of survivor respondent, the average net financial wealth go up by about \$842.7. There is no surprise for the increase of *nettfa* that come from each additional *inc* and *age*, the more year of working can mean the more year of saving too.

iv. $H_0: \beta_2 = 1$
 $H_a: \beta_2 < 1$



area left = 0.049
 $z = -2.33$

$$z = \frac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} = \frac{0.8426 - 1}{0.0920} = -1.71$$

$-1.71 > -2.33$, the computed z value did not fall into the rejection region, therefore, we do not reject H_0

$$inc = \beta_0 + \beta_1 nettfa + u$$

$$inc = 28.07666 + 0.100737 nettfa + u$$

we can see from the slope of this equation that if net financial wealth increase by \$1,000, on average, the annual family income will increase by \$100.737. And it shows the different at the interception point, as the orientation one result is negative but in this equation when the net financial wealth is zero, the remaining amount is positive \$28,076.66.

. regress inc nettfa if fsize == 1

Source	SS	df	MS	Number of obs	=	2,017
Model	46335.1731	1	46335.1731	F(1, 2015)	=	181.60
Residual	514127.962	2,015	255.150354	Prob > F	=	0.0000
				R-squared	=	0.0827
				Adj R-squared	=	0.0822
Total	560463.135	2,016	278.007508	Root MSE	=	15.973

	inc	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
	nettfa	.100737	.0074754	13.48	0.000	.0860768 .1153973
	_cons	28.07666	.3699027	75.90	0.000	27.35123 28.80209

To summarise is the effect of (*inc* on *nettfa*) has a higher potential than effect of (*nettfa* on *inc*) due to the significance change of the slope from \$799.3 to \$100.737 when we change to regress *inc* on *nettfa*.

C1. The following model can be used to study whether campaign expenditures affect election outcomes:

$$\text{voteA} = \beta_0 + \beta_1 \log(\text{expendA}) + \beta_2 \log(\text{expendB}) + \beta_3 \text{prtystrA} + u$$

where voteA is the percentage of the vote received by Candidate A, expendA and expendB are campaign expenditures by Candidates A and B, and prtystrA is a measure of party strength for Candidate A (the percentage of the most recent presidential vote that went to A's party).

- What is the interpretation of β_1 ?
- In terms of the parameters, state the null hypothesis that a 1% increase in A's expenditures is offset by a 1% increase in B's expenditures.
- Estimate the given model using the data in VOTE1 and report the results in usual form. Do A's expenditures affect the outcome? What about B's expenditures? Can you use these results to test the hypothesis in part (ii)?
- Estimate a model that directly gives the t statistic for testing the hypothesis in part (ii). What do you conclude? (Use a two-sided alternative.)

i. This equation shows that if campaign expenditure by Candidate A increases by 1, on average, the percentage of the vote received by candidate A will increase by β_1

ii. $H_0: \beta_2 = -\beta_1$, $H_0: \beta_2 + \beta_1 = 0$
 $H_a: \beta_2 \neq -\beta_1$, $H_a: \beta_2 + \beta_1 \neq 0$

regress voteA lexpendA lexpendB prtystrA					
Source	SS	df	MS	Number of obs	= 173
Model	38405.1096	3	12801.7032	F(3, 169)	= 215.23
Residual	10052.1389	169	59.480112	Prob > F	= 0.0000
				R-squared	= 0.7926
				Adj R-squared	= 0.7889
				Root MSE	= 7.7123

voteA	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
lexpendA	6.083316	.38215	15.92	0.000	5.328914 6.837719
lexpendB	-6.615417	.3788203	-17.46	0.000	-7.363246 -5.867588
prtystrA	.1519574	.0620181	2.45	0.015	.0295274 .2743873
_cons	45.07893	3.926305	11.48	0.000	37.32801 52.82985

iii. $\text{VoteA} = 45.07893 + 6.083316 \log(\text{expendA}) - 6.615417 \log(\text{expendB}) + 0.1519574 \text{prtystrA} + u$

Both variables have effects on voteA but we can not analyse how different effect from each variable. Therefore, we can't use these results to test the hypothesis in part ii

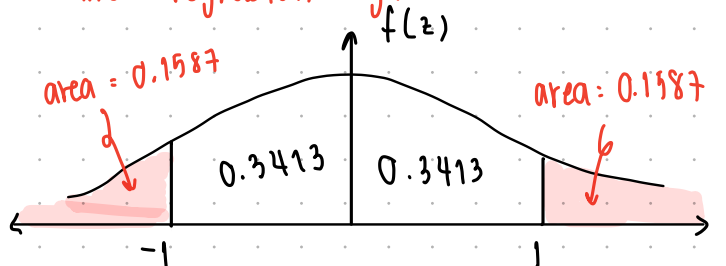
iv. $H_0: \beta_2 = -\beta_1$, $H_0: \beta_2 + \beta_1 = 0 \rightarrow t = \frac{(\hat{\beta}_2 + \hat{\beta}_1) - 0}{\text{se}(\hat{\beta}_2 + \hat{\beta}_1)}$
 $H_a: \beta_2 \neq -\beta_1$, $H_a: \beta_2 + \beta_1 \neq 0$

let $\hat{\theta}_1 = \hat{\beta}_2 + \hat{\beta}_1 \rightarrow H_0: \theta_1 = 0$, $t = \frac{\hat{\theta}_1 - 0}{\text{se}(\hat{\theta}_1)}$
 $H_a: \theta_1 \neq 0$

if rearrange $\hat{\theta}_1 = \hat{\beta}_2 + \hat{\beta}_1$, we have $\hat{\beta}_1 = \hat{\theta}_1 - \hat{\beta}_2$ or $\beta_1 = \theta_1 - \beta_2$

$$\begin{aligned} \text{voteA} &= \beta_0 + \beta_1 \log(\text{expendA}) + \beta_2 \log(\text{expendB}) + \beta_3 \text{prtystrA} + u \\ &= \beta_0 + \theta_1 - \beta_2 \log(\text{expendA}) + \beta_2 \log(\text{expendB}) + \beta_3 \text{prtystrA} + u \\ &= \beta_0 + \theta_1 \log(\text{expendA}) - \beta_2 \log(\text{expendA}) + \beta_2 \log(\text{expendB}) + \beta_3 \text{prtystrA} + u \\ &= \beta_0 + \theta_1 \log(\text{expendA}) - \beta_2 (\log(\text{expendB}) - \log(\text{expendA})) + \beta_3 \text{prtystrA} + u \end{aligned}$$

do the regression again



$$t = \frac{\hat{\theta}_1 - 0}{\text{se}(\hat{\theta}_1)} = \frac{-0.532101 - 0}{0.5330858} = -0.99851 \approx -1$$

regress voteA lexpendA newV prtystrA					
Source	SS	df	MS	Number of obs	= 173
Model	38405.1097	3	12801.7032	F(3, 169)	= 215.23
Residual	10052.1388	169	59.480115	Prob > F	= 0.0000
				R-squared	= 0.7926
				Adj R-squared	= 0.7889
				Root MSE	= 7.7123

voteA	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
θ lexpendA	-.532101	.5330858	-1.00	0.320	-1.584466 .5202638
newV	-6.615417	.3788203	-17.46	0.000	-7.363246 -5.867588
prtystrA	.1519574	.0620181	2.45	0.015	.0295274 .2743873
_cons	45.07893	3.926305	11.48	0.000	37.32801 52.82985

at 1% significant, p value is 0.1587 > 0.005 (significant level)

- ii. In terms of the parameters, state the null hypothesis that a 1% increase in A's expenditures is offset by a 1% increase in B's expenditures.

So, we do not reject H_0 because p value (0.1587) is greater than significant level (0.005). Therefore, a 1% increase in A's expenditures is offset by a 1% increase in B's expenditure.

1. Which of the following can cause the usual OLS t statistics to be invalid (that is, not to have t distributions under H_0)?

- i. Heteroskedasticity.
- ii. A sample correlation coefficient of .95 between two independent variables that are in the model.
- iii. Omitting an important explanatory variable.

i. Due to assumption 5 on MLR refer to homoskedasticity therefore the OLS t statistics will be invalid.

iii. Omitting an important explanatory variable will cause the OLS to be invalid as the equation would not find the accurate outcome from the incomplete equation. The coefficient of regression will fluctuate and will eventually cause the equation to be invalid.

2. Consider an equation to explain salaries of CEOs in terms of annual firm sales, return on equity (roe , in percentage form), and return on the firm's stock (ros , in percentage form):

$$\log(\text{salary}) = \beta_0 + \beta_1 \log(\text{sales}) + \beta_2 roe + \beta_3 ros + u.$$

i. In terms of the model parameters, state the null hypothesis that, after controlling for $sales$ and roe , ros has no effect on CEO salary. State the alternative that better stock market performance increases a CEO's salary.

ii. Using the data in CEOSAL1, the following equation was obtained by OLS:

$$\widehat{\log(\text{salary})} = 4.32 + .280 \log(\text{sales}) + .0174 roe + .00024 ros$$

(.32) (.035) (.0041) (.00054)

$n = 209, R^2 = .283.$

By what percentage is $salary$ predicted to increase if ros increases by 50 points? Does ros have a practically large effect on $salary$?

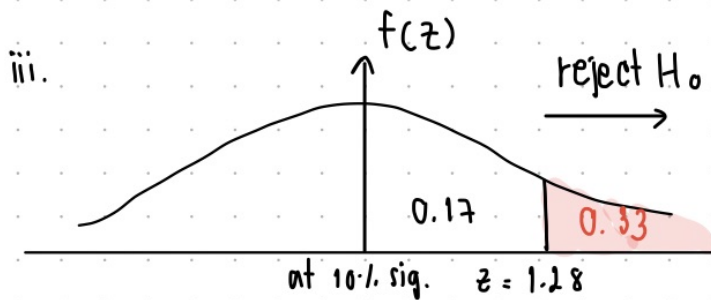
iii. Test the null hypothesis that ros has no effect on $salary$ against the alternative that ros has a positive effect. Carry out the test at the 10% significance level.

iv. Would you include ros in a final model explaining CEO compensation in terms of firm performance? Explain.

i. $H_0: \beta_3 = 0 \rightarrow$ stock performance has no effect on CEO's salary.

$H_a: \beta_3 > 0 \rightarrow$ better stock market performance increase a CEO's salary

ii. if ros increase by 50 points, salary will increase by 0.012 or percentage increase equal to 1.2%. ros does not have a particular large effect on salary. This can be seen when compared to $\log(\text{sales})$ and roe . For example 50 point increase in $\log(\text{sales})$ and roe will result in salary to increase by 14 and 0.87 respectively.



$$z = \frac{\hat{\beta}_3 - 0}{se \hat{\beta}_3} = \frac{0.00024 - 0}{0.00054} = 0.44$$

at 10% significant critical value is 1.28

that is greater than 0.44 ($0.44 < 1.28$) so we do not reject H_0 at 10% significant level. the computed z value did not fall into the rejection region. Therefore, ros has no effect on CEO's salary.

iv. I would not include ros into the equation as ros has no effect on CEO's salary so I have no reason to put it in the final model to calculate for $\widehat{\log(\text{salary})}$.