(3) Output is homogenous and the demand curve is

$$P = 448 - Q$$

There are two firms with identical costs given by $C = q_i^2$ where q_i is the production of firm i.
The marginal cost of firm i is $MC_i(q_i) = 2q_i$
(a) Find the Cournot eq. from outputs.

Sol $Q = (Q_1 + Q_2)$, $P = 448 - (Q_1 + Q_2)$

$TR_1 = (448 - Q_1 - Q_2)Q_1$

$= 448Q_1 - Q_1^2 - Q_1Q_2$

FOC: $MR_1 = \frac{dTR_1}{dQ_1} = 448 - 2Q_1 - Q_2$

$MR_1 = MC_1$

$448 - 2Q_1 - Q_2 = 2Q_1$

$448 - 4Q_1 = Q_2$

$Q_1 = \frac{Q_2 - 448}{-4}$

$= 112 - \frac{1}{4}Q^2$ — ①

$TR_2 = (448 - Q_1 - Q_2)Q_2$

$= 448Q_2 - Q_1Q_2 - Q_2^2$

FOC: $MR_2 = \frac{dTR_2}{dQ_2} = 448 - Q_1 - 2Q_2$

$MR_2 = MC_2$

$448 - Q_1 - 2Q_2 = 2Q_2$

$448 - 4Q_2 = Q_1$

$Q_2 = \frac{Q_1 - 448}{-4}$

$= 112 - \frac{1}{4}Q_1$ — ②

plug ② in ①

$Q_1 = 112 - \frac{1}{4}(112 - \frac{1}{4}Q_1)$

$Q_1 = 112 - 28 + \frac{1}{16}Q_1$

$Q_1 = 84 + \frac{1}{16}Q_1$

$\frac{15}{16}Q_1 = 84$

$Q_1 = 84 \times \frac{16}{15}$ (84 → 80, 15 → 5)

$Q_1 = 89.6 = Q_2$ #