# Spatial Regression

EE464: Urban Economics
EE562: Selected Topics in Development Economics 2
Semester 1 / 2020
Faculty of Economics, Thammasat University

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(1) The latest textbook on spatial analysis

(2) Criteria for selecting model's specification

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# (1) The latest textbook on spatial analysis

# Spatial Analysis Methods and Practice Describe – Explore – Explain through GIS

*by George Grekousis*

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# SPATIAL ANALYSIS METHODS AND PRACTICE

DESCRIBE – EXPLORE – EXPLAIN THROUGH GIS

GEORGE GREKOUSIS

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One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

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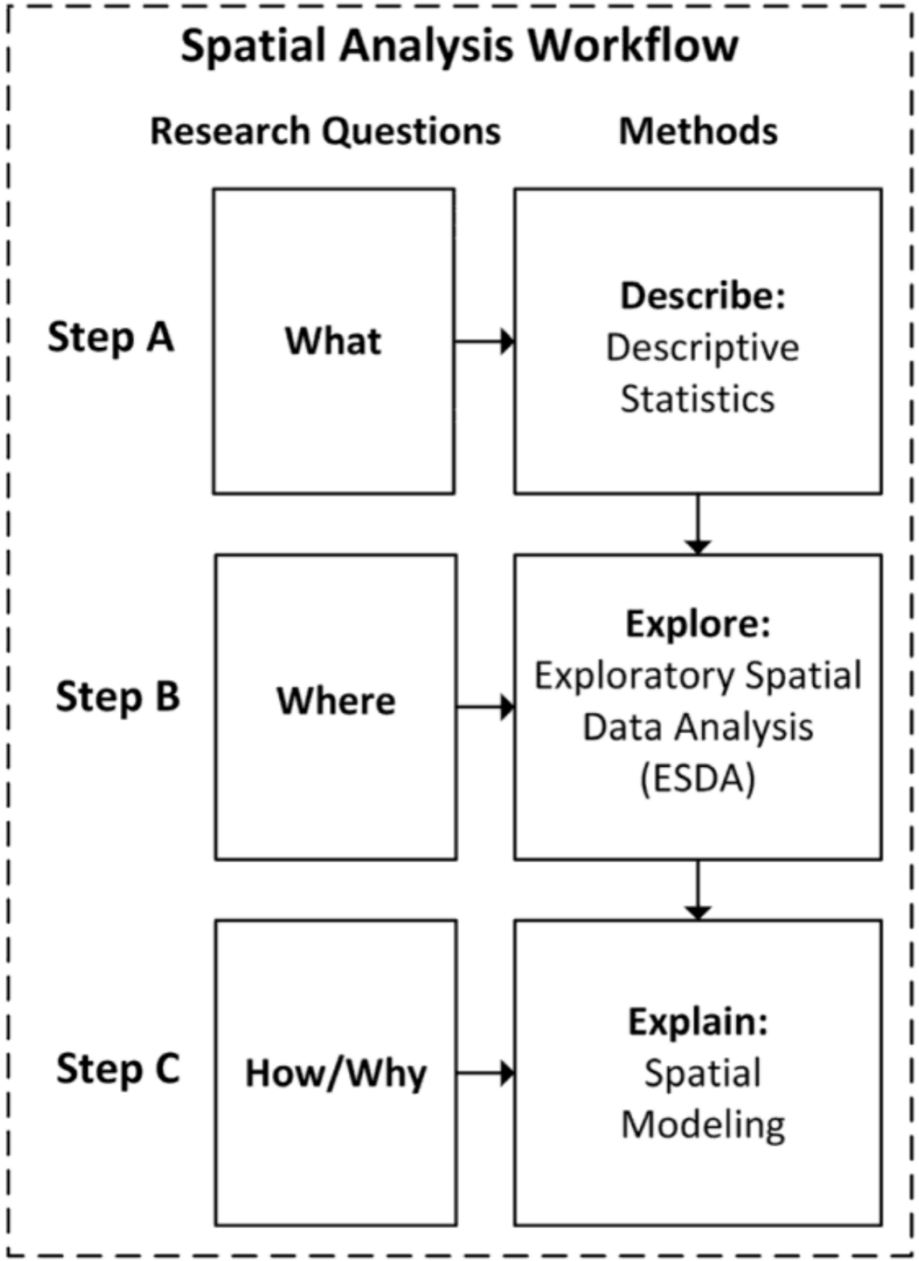



**Figure 1.1** Spatial analysis workflow.

**Source:** Grekousis (2020)

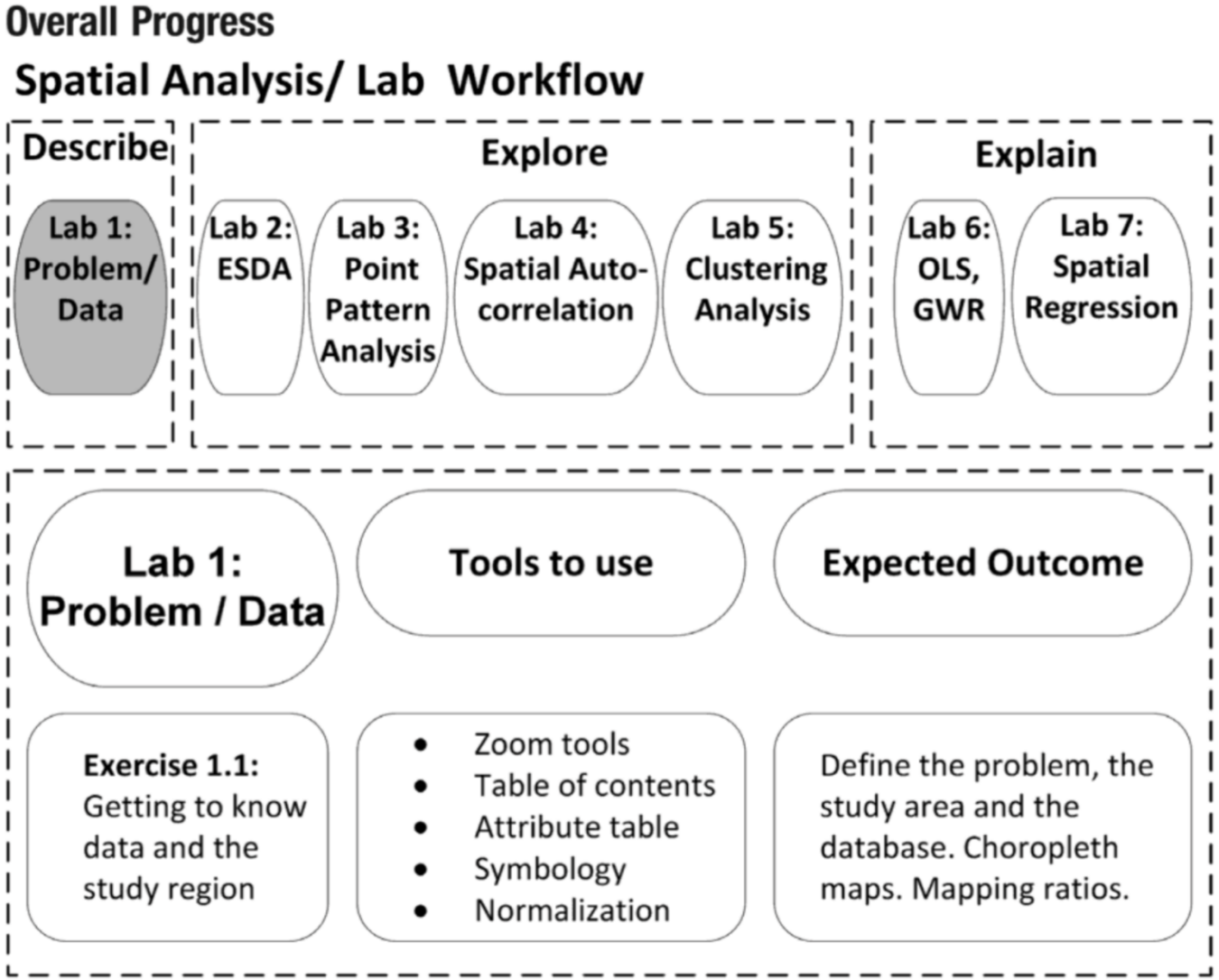
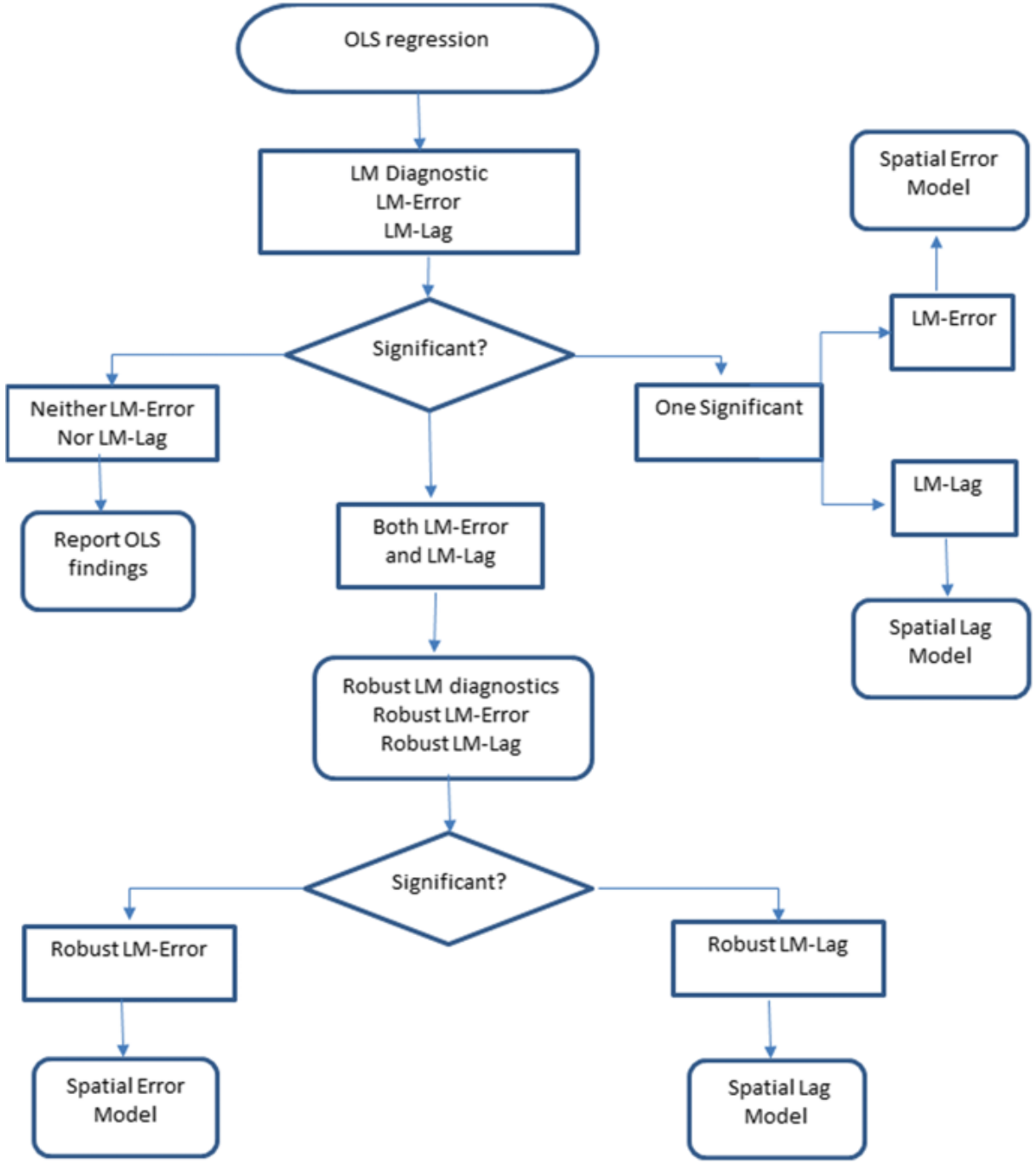



**Figure 1.7** Lab 1 workflow and overall progress.

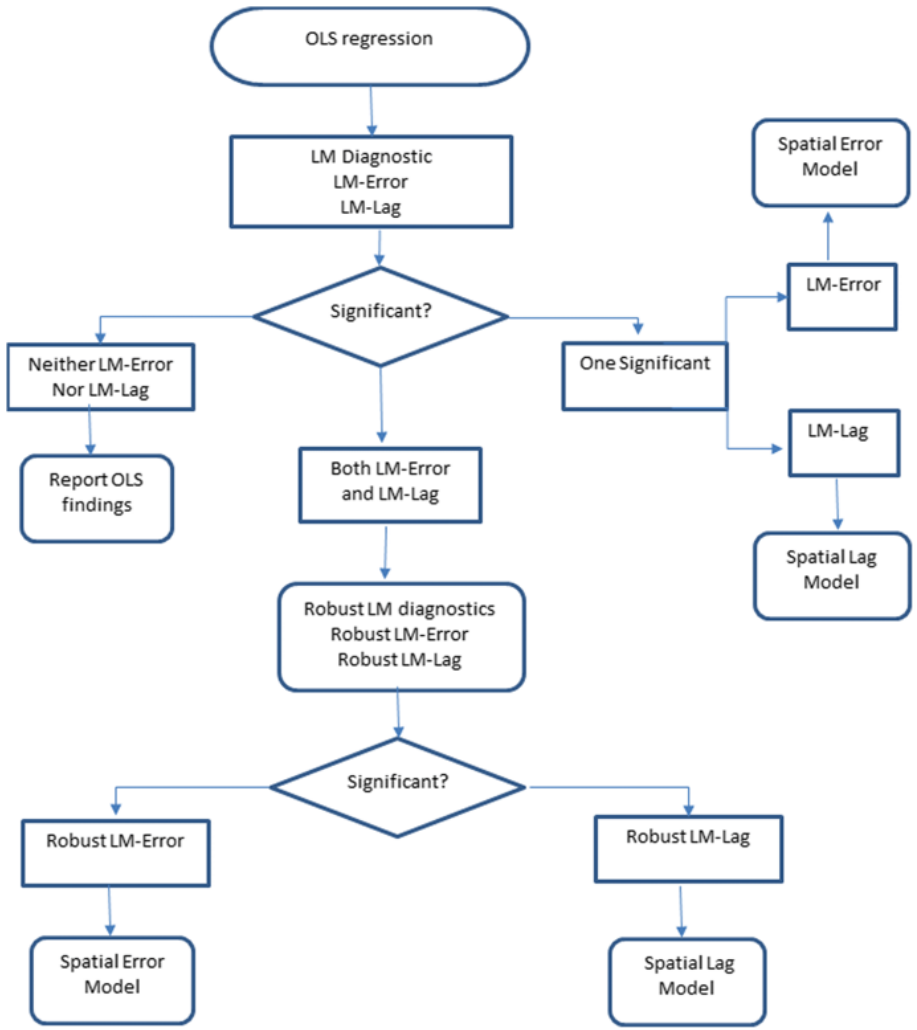
**Source:** Grekousis (2020)

# (2) Criteria for selecting model's specification



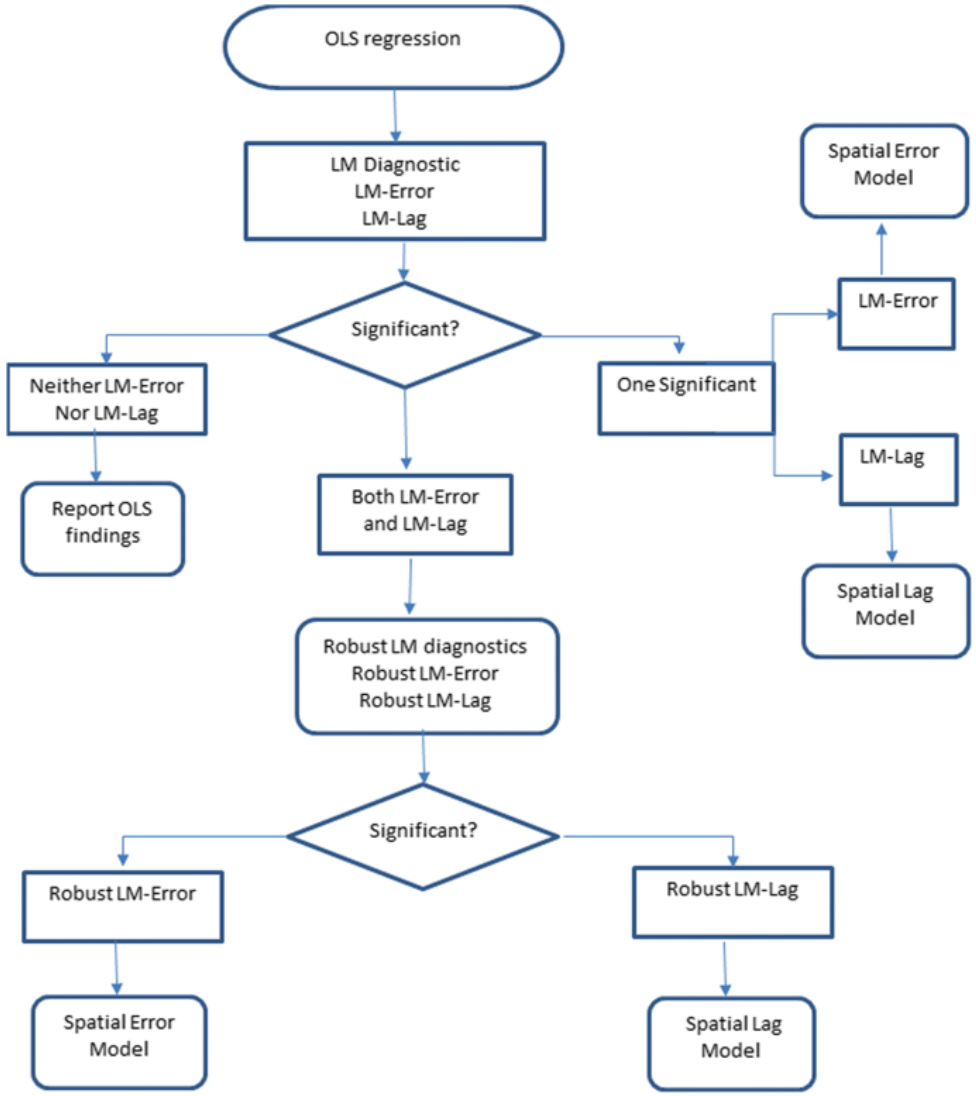

**Source:** Nilima et al. (2018)

# Example 1




```
>>01/12/20 11:43:01
REGRESSION
----------
SUMMARY OF OUTPUT: ORDINARY LEAST SQUARES ESTIMATION
Data set            :  TRF_Thailand data 2008
Dependent Variable  :      ln_gpp  Number of Observations:   75
Mean dependent var  :     10.6273  Number of Variables   :    6
S.D. dependent var  :     1.10065  Degrees of Freedom    :   69

R-squared           :    0.475942  F-statistic           :     12.5329
Adjusted R-squared  :    0.437966  Prob(F-statistic)     :1.15138e-008
Sum squared residual:     47.6149  Log likelihood        :    -89.3826
Sigma-square        :    0.690071  Akaike info criterion :     190.765
S.E. of regression  :    0.830705  Schwarz criterion     :      204.67
Sigma-square ML     :    0.634866
S.E of regression ML:    0.796785

------------------------------------------------------------------------------
       Variable      Coefficient      Std.Error    t-Statistic    Probability
------------------------------------------------------------------------------
          CONSTANT      -11.4482        12.2164      -0.937113      0.35197
          Ln_Pop_D     -0.240969        0.22103       -1.09021      0.27941
            Ln_NTL       1.24261       0.221595         5.6076      0.00000
         Ln_NDVI_M      -1.09656        1.27192      -0.862127      0.39160
         Ln_Rain_M      0.957134        0.46944        2.03889      0.04529
         Ln_LSTD_M        4.2676        3.44137        1.24009      0.21914
------------------------------------------------------------------------------

REGRESSION DIAGNOSTICS
MULTICOLLINEARITY CONDITION NUMBER   423.656107
TEST ON NORMALITY OF ERRORS
TEST                     DF          VALUE              PROB
Jarque-Bera               2            4.8372           0.08905

DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS
TEST                     DF          VALUE              PROB
Breusch-Pagan test        5           16.8776           0.00474
Koenker-Bassett test      5           10.5731           0.06053

DIAGNOSTICS FOR SPATIAL DEPENDENCE
FOR WEIGHT MATRIX : TRF_Thailand data 2008
   (row-standardized weights)
TEST                             MI/DF          VALUE              PROB
Moran's I (error)                0.0280          0.8742           0.38199
Lagrange Multiplier (lag)             1          0.0050           0.94362
Robust LM (lag)                       1          0.3243           0.56904
Lagrange Multiplier (error)           1          0.0867           0.76838
Robust LM (error)                     1          0.4060           0.52400
Lagrange Multiplier (SARMA)           2          0.4110           0.81423
================================ END OF REPORT ================================
```

# Example 2




```
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SUMMARY OF OUTPUT: ORDINARY LEAST SQUARES ESTIMATION
Data set            :  TRF_Thailand data 2016
Dependent Variable  :     A11_Avg  Number of Observations:   77
Mean dependent var  :     8.73412  Number of Variables   :    6
S.D. dependent var  :    0.236003  Degrees of Freedom    :   71

R-squared           :    0.687733  F-statistic           :      31.274
Adjusted R-squared  :    0.665743  Prob(F-statistic)     :1.10685e-016
Sum squared residual:     1.33922  Log likelihood        :     46.7328
Sigma-square        :   0.0188623  Akaike info criterion :    -81.4656
S.E. of regression  :     0.13734  Schwarz criterion     :    -67.4028
Sigma-square ML     :   0.0173925
S.E of regression ML:    0.131881

------------------------------------------------------------------------------
       Variable      Coefficient      Std.Error    t-Statistic    Probabili
------------------------------------------------------------------------------
          CONSTANT       6.76934        1.95872          3.456      0.00093
          Ln_Pop_D    -0.0259939       0.0344682       -0.75414      0.45326
            Ln_NTL      0.184405       0.0192027        9.60305      0.00000
         Ln_NDVI_M     0.0697841        0.234638       0.297412      0.76702
         Ln_Rain_M      0.176842       0.0857219        2.06297      0.04277
         Ln_LSTD_M      0.321049        0.529664       0.606137      0.54636
------------------------------------------------------------------------------

REGRESSION DIAGNOSTICS
MULTICOLLINEARITY CONDITION NUMBER   379.980969
TEST ON NORMALITY OF ERRORS
TEST                     DF           VALUE             PROB
Jarque-Bera               2             0.6733          0.71416

DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS
TEST                     DF           VALUE             PROB
Breusch-Pagan test        5             6.5858          0.25331
Koenker-Bassett test      5             7.8637          0.16391

DIAGNOSTICS FOR SPATIAL DEPENDENCE
FOR WEIGHT MATRIX : TRF_Thailand data 2016
   (row-standardized weights)
TEST                          MI/DF          VALUE           PROB
Moran's I (error)            0.0964         1.6592         0.09707
Lagrange Multiplier (lag)         1         3.3019         0.06920
Robust LM (lag)                   1         2.2860         0.13054
Lagrange Multiplier (error)       1         1.0895         0.29659
Robust LM (error)                 1         0.0736         0.78622
Lagrange Multiplier (SARMA)       2         3.3755         0.18494
================================ END OF REPORT ================================
```
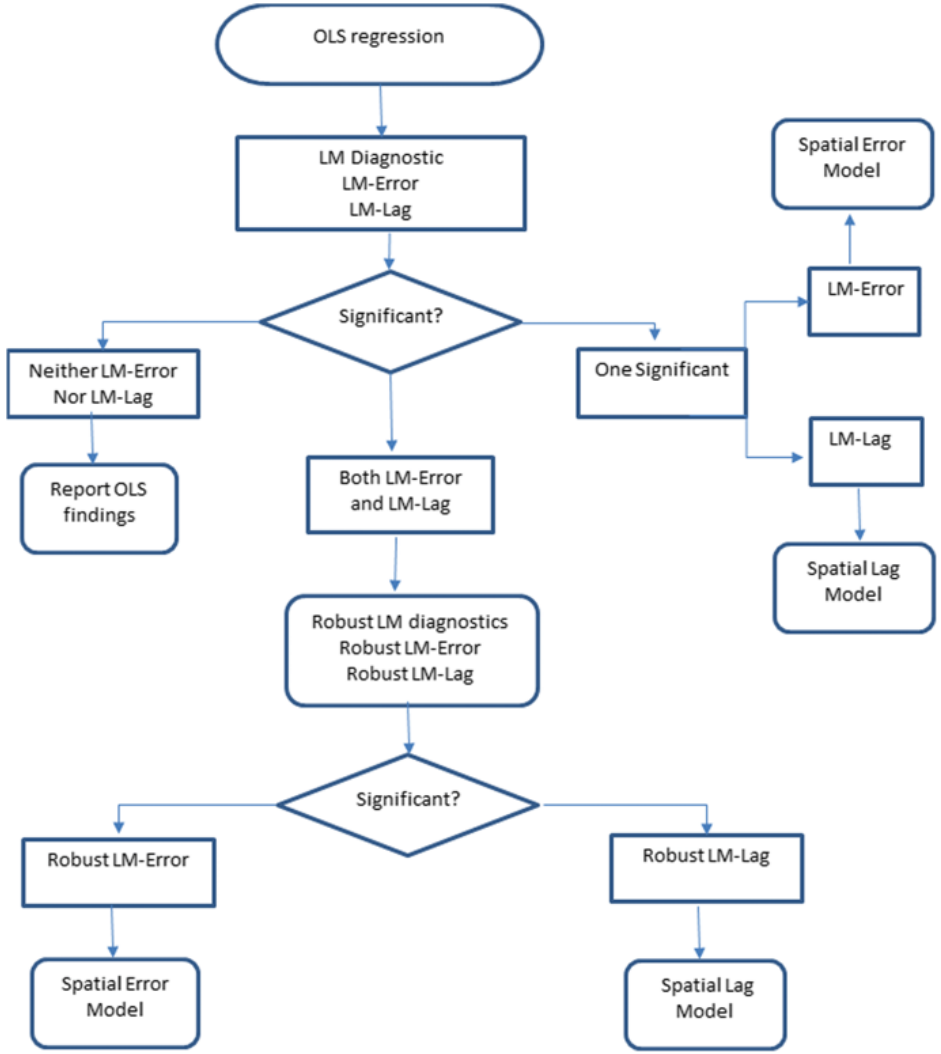
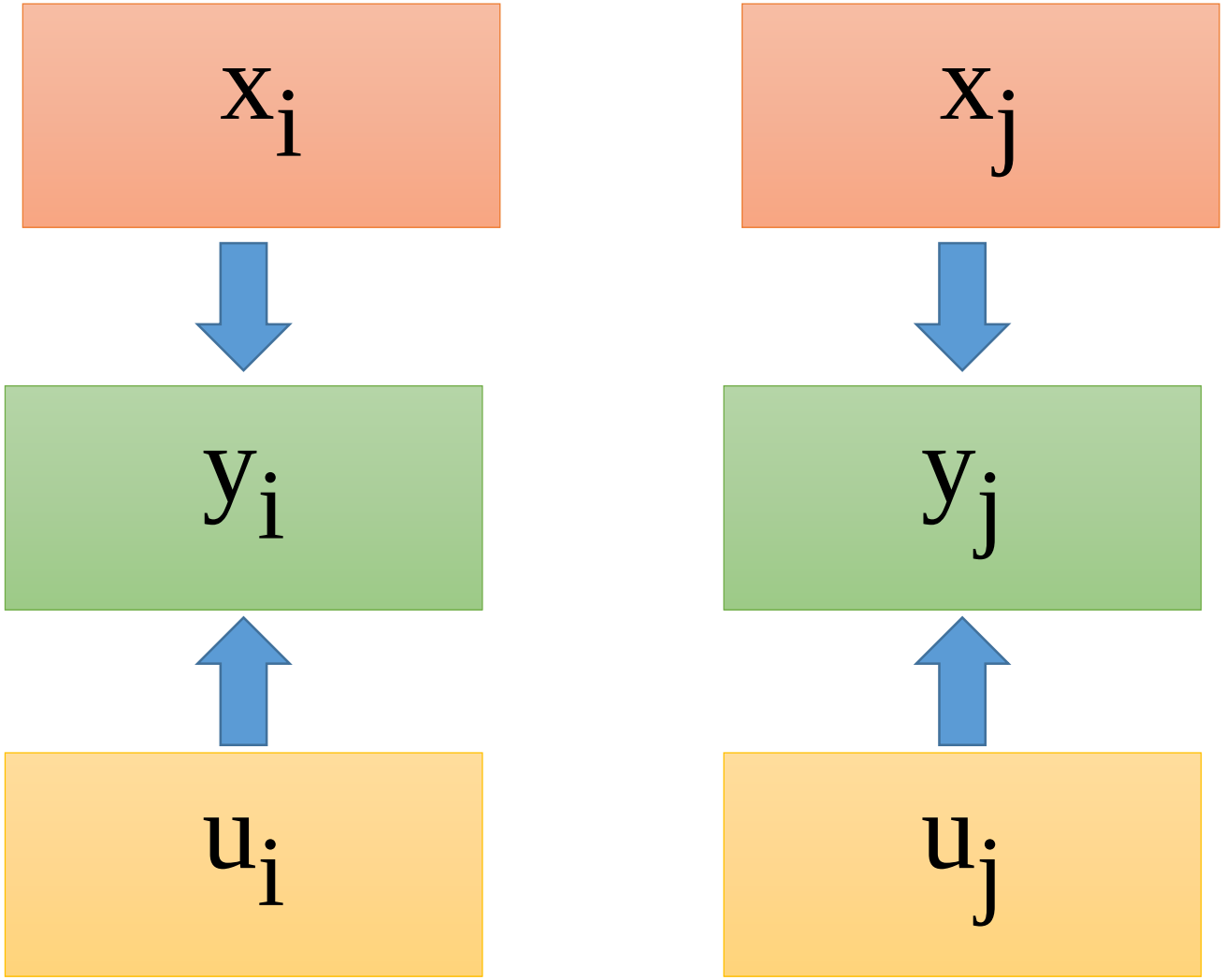

# Example 3




```
>>03/13/20 21:54:22
REGRESSION
----------
SUMMARY OF OUTPUT: ORDINARY LEAST SQUARES ESTIMATION
Data set            :  TH_JETRO_IndusSpillOvr
Dependent Variable  :     TH_lnVA  Number of Observations:    76
Mean dependent var  :     23.4965  Number of Variables   :     6
S.D. dependent var  :      1.7812  Degrees of Freedom    :    70

R-squared           :    0.882644  F-statistic           :     105.295
Adjusted R-squared  :    0.874261  Prob(F-statistic)     :3.70775e-031
Sum squared residual:     28.2975  Log likelihood        :    -70.2968
Sigma-square        :    0.404249  Akaike info criterion :     152.594
S.E. of regression  :    0.635806  Schwarz criterion     :     166.578
Sigma-square ML     :    0.372335
S.E of regression ML:    0.610193

------------------------------------------------------------------------------
       Variable      Coefficient      Std.Error    t-Statistic    Probability
------------------------------------------------------------------------------
       CONSTANT         6.87966        1.61102        4.27038        0.00006
         TH_lnL        0.344717       0.128305         2.6867        0.00901
         TH_lnK        0.421677       0.107584         3.9195        0.00020
          lnNTL         0.68795       0.192754        3.56906        0.00065
        TH_lnIM      -0.0176677      0.0197637      -0.893948        0.37441
        TH_lnEX        0.164259      0.0335603        4.89446        0.00001
------------------------------------------------------------------------------

REGRESSION DIAGNOSTICS
MULTICOLLINEARITY CONDITION NUMBER   93.190925
TEST ON NORMALITY OF ERRORS
TEST                     DF          VALUE              PROB
Jarque-Bera               2            1.7363           0.41974

DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS
TEST                     DF          VALUE              PROB
Breusch-Pagan test        5           11.3263           0.04528
Koenker-Bassett test      5            9.5991           0.08743

DIAGNOSTICS FOR SPATIAL DEPENDENCE
FOR WEIGHT MATRIX : TH_JETRO_IndusSpillOvr
     (row-standardized weights)
TEST                           MI/DF        VALUE           PROB
Moran's I (error)              0.2793        3.5698         0.00036
Lagrange Multiplier (lag)         1         14.7833         0.00012
Robust LM (lag)                   1          7.3401         0.00674
Lagrange Multiplier (error)       1          9.1910         0.00243
Robust LM (error)                 1          1.7478         0.18615
Lagrange Multiplier (SARMA)       2         16.5311         0.00026
================================ END OF REPORT ================================
```

# (3) Spatial regression techniques

# OLS

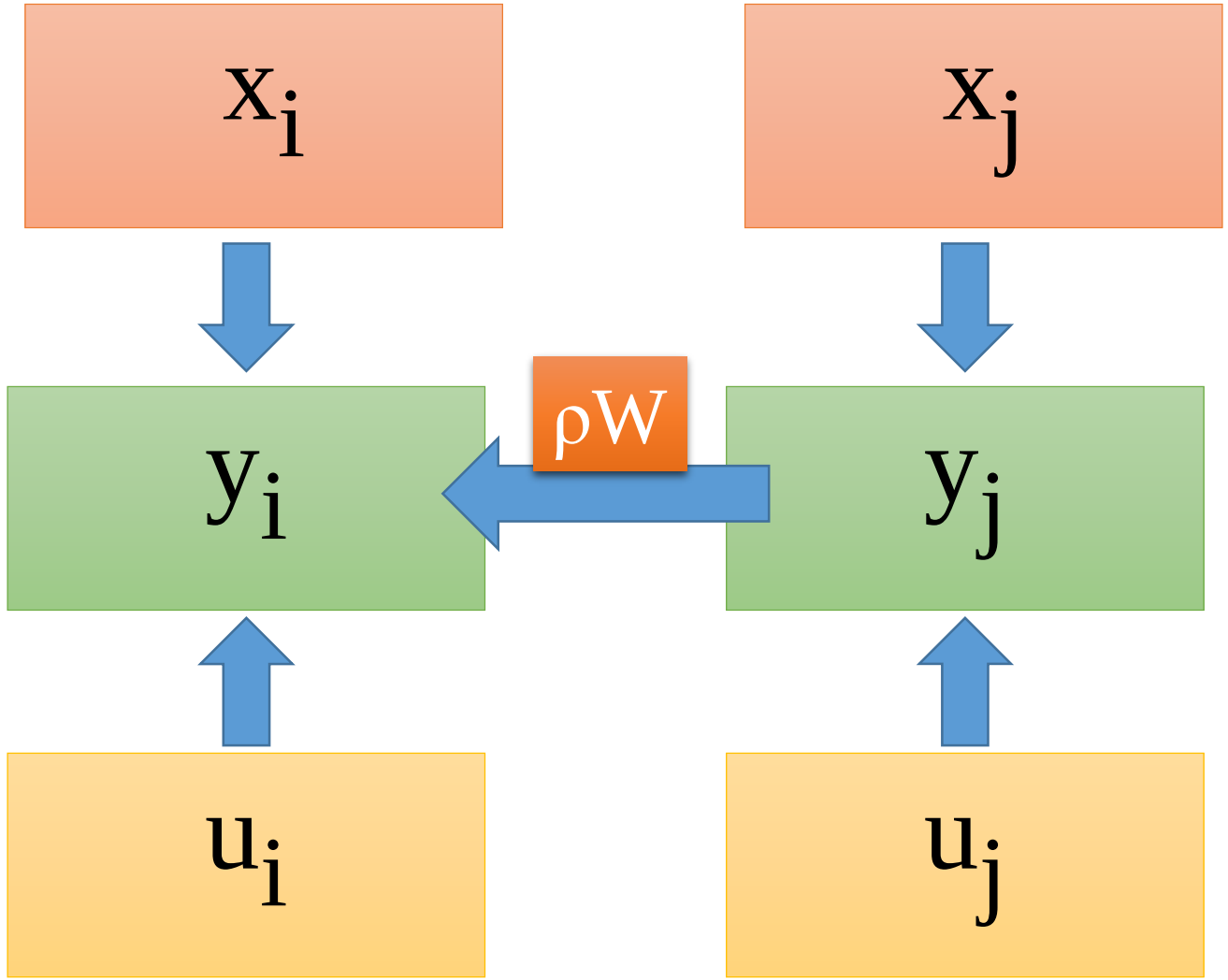



$$Y_i = a + \beta X_i + u_i \qquad Y_j = a + \beta X_j + u_j$$

# SLM

Spatial Lag Model

$$Y_i = \rho W_{ij} Y_j + a + \beta X_i + u_i$$




$$Y_i = a + \beta X_i + u_i$$

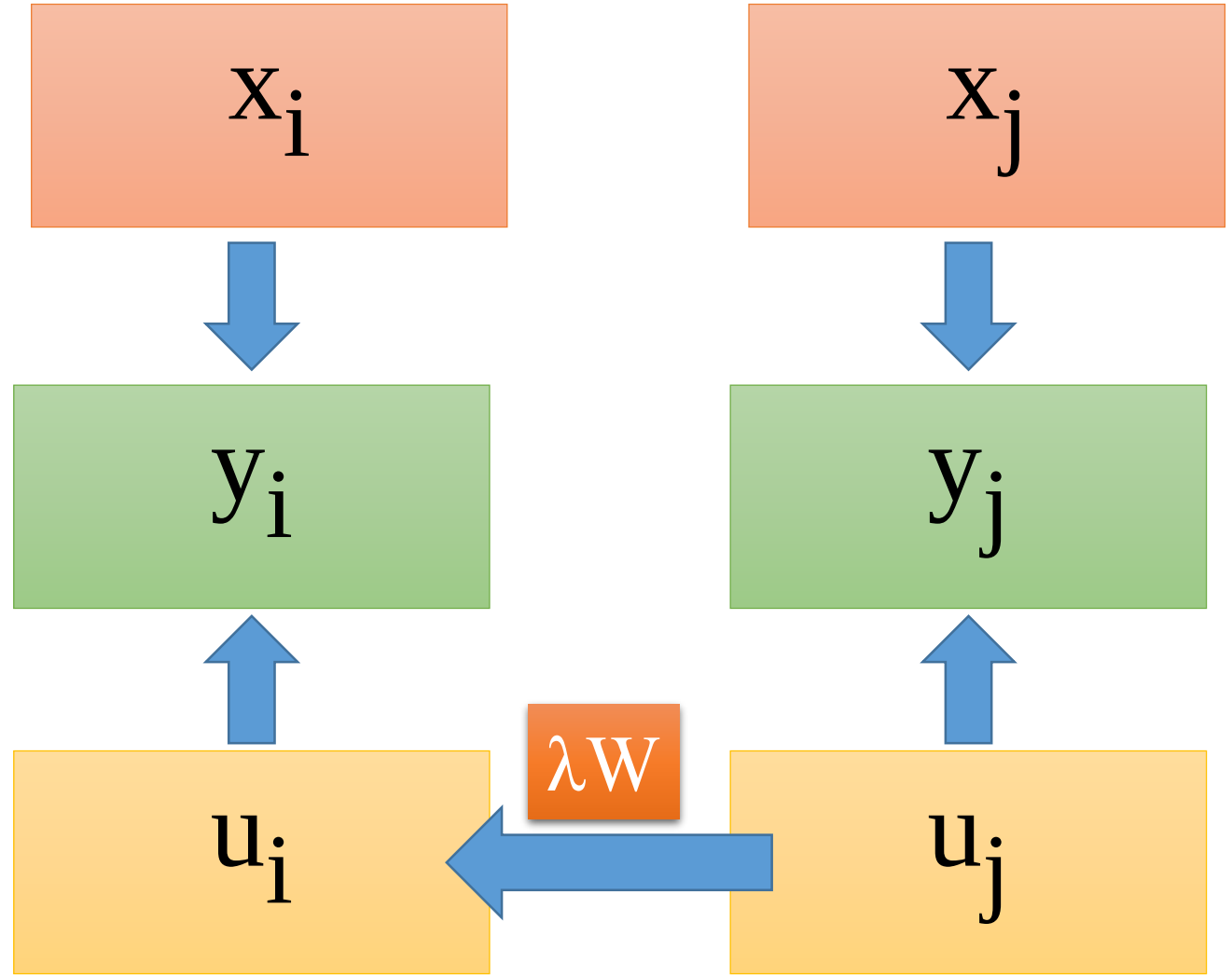
$$Y_j = a + \beta X_j + u_j$$

# SEM

Spatial Error Model

$$Y_i = a + \beta X_i + u_i$$

$$u_i = \lambda W_{ij} u_j + \varepsilon_i$$

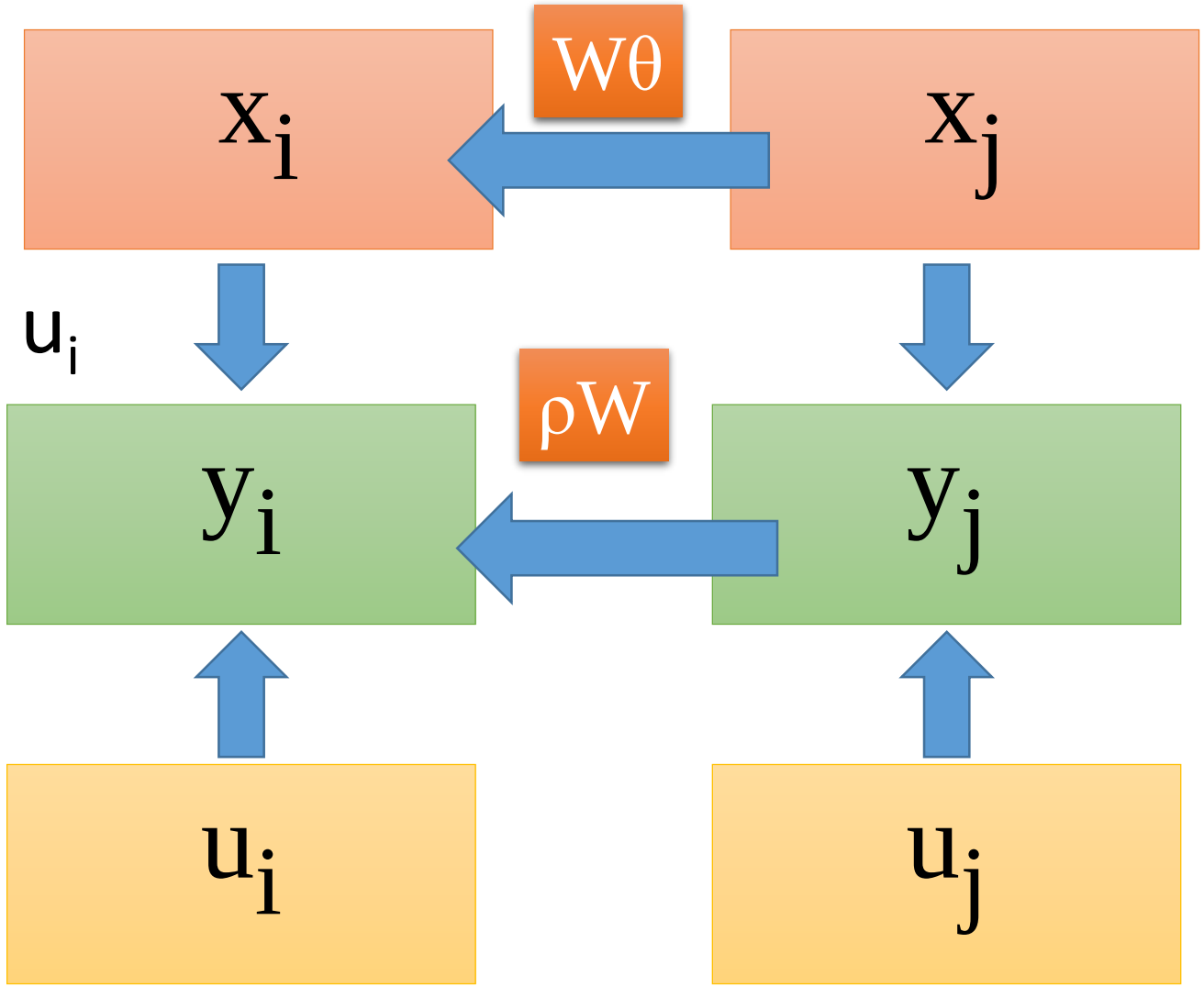



$$Y_i = a + \beta X_i + u_i \qquad Y_j = a + \beta X_j + u_j$$

# SDM

Spatial Durbin Model

$$Y_i = \rho W_{ij} Y_j + a + \beta X_i + W_{ij} X_j \theta + u_i$$




$$Y_i = a + \beta X_i + u_i \qquad Y_j = a + \beta X_j + u_j$$