Instruction:

Exam time: 30 minutes.

You may use a calculator, turn off cell phones. Phone communication are strictly prohibited during the exam.

For each question, write your answer in the blank space provided.

Manage your time carefully and answer as many questions as you can.

Solution:

Quiz 2/2020

Seat No..............................
ID.No................................

Question1 (40 points)

Your score.....................

Suppose the daily log return r_t of Stock A follows the model:

$$r_t = 0.002 + a_t$$

$$a_t = \sigma_t \epsilon_t$$

where ϵ_t is an independent and identically distributed (iid) sequence of standardized Student-t distribution with 5 degrees of freedom. In addition,

$$\sigma_t^2 = 0.01 + 0.1 a_{t-1}^2$$

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Question1.1 (10 points)

Your score.....................

From the above model, Find out the unconditional Expectation of a_t : $E(a_t)$ and the unconditional expectation of r_t : $E(r_t)$

From $a_t = \sigma_t \varepsilon_t$

Take $E(\cdot)$ to both sides, then

$$E[a_t] = E[\sigma_t \varepsilon_t]$$

Using Law of iterated expectation:

$$E[a_t] = E[E[\sigma_t \varepsilon_t | F_{t-1}]]$$ where F_{t-1} is the information set

Since σ_t is a function of past information as shown:

$$\sigma_t^2 = 0.01 + 0.1 a_{t-1}^2$$

Thus, Given F_{t-1} we have already known σ_t

$\therefore$ $E[a_t] = E[\sigma_t E[\varepsilon_t | F_{t-1}]]$

$= 0$ since $\varepsilon_t \overset{iid}{\sim} t(0,1)$

$\therefore$ $E[a_t] = 0$ #

For $E(r_t) = 0.002 \rightarrow$ # $E(r_t) = E[0.002 + a_t]$
$= 0.002 + E(a_t)$ ($E(a_t)=0$)

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Question1.2 (10 points) Your score.....................

Find out the unconditional variance of a_t :$Var(a_t)$ and the conditional variance of a_t :$Var(a_t|F_{t-1})$

For $Var(a_t|F_{t-1}) = E[(\sigma_t \varepsilon_t)^2|F_{t-1}] = \sigma_t^2 E[\varepsilon_t^2|F_{t-1}] = 0.01+0.1a_{t-1}^2$ ($E[\varepsilon_t^2|F_{t-1}]=1$)

$Var(a_t) = E[(a_t - E(a_t))^2] = E[a_t^2] = E[E[a_t^2|F_{t-1}]]$

$E[a_t^2] = E[0.01+0.1a_{t-1}^2] = 0.01+0.1E[a_{t-1}^2]$

since $E[a_t^2] = E[a_{t-1}^2]$

Question1.3 (10 points) $\therefore E[a_t^2] = \frac{0.01}{0.9} = 0.0111$ # Your score.....................

Let h = 100 be the forecast origin with $a_h = 0.015$ and $\sigma_h = 0.2$. Calculate the 1-step ahead prediction $r_h(1)$ and 1-step ahead volatility forecast.

$$r_t = 0.002 + a_t$$

$$a_t = \sigma_t \epsilon_t$$

tically distributed (iid) sequence of st

$$\sigma_t^2 = 0.01 + 0.1a_{t-1}^2$$

Given $t=h$

for 1-step ahead forecasting

$\sigma_{h+1}^2 = 0.01+0.1a_h^2$

$\therefore E[\sigma_{h+1}^2|F_h] = \sigma_h^2(1) = 0.01+0.1a_h^2$ #

$= 0.01+0.1(0.015)^2$

$= 0.01+0.0000225 = 0.0100225$ #

$r_h(1) = 0.002$ # since $r_t = 0.002+a_t$

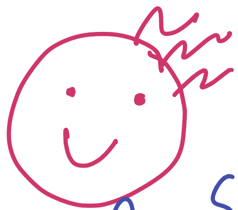
Question1.4 (10 points)

Your score......................

Calculate the ∞-step ahead prediction $r_h(\infty)$ and the ∞-step ahead volatility forecast at the forecast origin h.

For $r_h(\infty) = 0.002$ as well since $E(r_t) = 0.002$ #

for $\sigma_h^2(\infty) = Var(a_t) = 0.011$ #



S.

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May 21, 2021

Hint: for kurtosis of a_t

if $\sigma_t^2 = \omega + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2$

you should get

$$E[a_t^4] = 3E\left[\omega^2 + \alpha^2 a_{t-1}^4 + \beta^2 \sigma_{t-1}^4 + 2\omega\alpha a_{t-1}^2 + 2\omega\beta\sigma_{t-1}^2 + 2\alpha\beta a_{t-1}^2 \sigma_{t-1}^2\right]$$

(1)

then you should find:

$E[a_{t-1}^4] = E[a_t^4]$

$E[\sigma_{t-1}^2] = ?$

$E[\sigma_{t-1}^4] = \overset{?}{\ldots\ldots}\ E[a_{t-1}^4]$

$E[a_{t-1}^2 \cdot \sigma_{t-1}^2]$

$= \dfrac{E[a_{t-1}^4]}{3}$

$E[a_{t-1}^2] = \overset{?}{\ldots}$

Once you subsitute in (1) and rearrange,

$$\frac{E[a_t^4]}{E[a_t^2]^2} = \frac{3(1-(\alpha+\beta)^2)}{1-3\alpha^2-\beta^2-2\alpha\beta} \#$$

Try it!