EE415/418 Game Theory

Lecture 10-11. Repeated Games, continued.

- 2.3.C Collusion between Cournot Duopolists
- 2.3.D Efficiency Wages

2.3.C Repeated Cournot (1/20)

- Consider the game in which two firms play the infinitely repeated game in which Cournot game is the stage game and both firms have discount factor δ.
- Recall the static Cournot game
- The inverse demand function is $P(Q) = a - Q$, where $Q = q_1 + q_2$.
- Firms have identical marginal cost c and no fixed cost.

Repeated Cournot (2/20)

- In Nash equilibrium, both firms produces "Cournot" quantity $q_c = (a - c)/3$.
- The aggregate Cournot quantity $2q_c = 2(a - c)/3$ exceeds the monopolist quantity $q_m = (a - c)/2$.
- Thus, both firms will be better off if they produce half of the monopolist quantity, $q_i = q_m/2 = (a - c)/4$.

Repeated Cournot (3/20)

- Consider the following trigger strategy:

  Produce half the monopoly quantity, $q_m/2$, in the first period. In the t^{th} period, produce $q_m/2$ if both firms have produced $q_m/2$ in each of the $t-1$ periods; otherwise, produce the Cournot quantity.

- Note the similarity between this model and the repeated Prisoners' Dilemma.

Repeated Cournot (4/20)

- If both firms produce Cournot quantity, then both firms gain $\pi_c = (a - c)^2/9$.
- If both firms produce half monopolist quantity, then they gain $\pi_m/2 = (a - c)^2/8$.
- If firm i is going to produce q_{m} in current period, then the quantity q_{j} that firm j maximizes current period profit is

$$\max_{q_j}\left(a - q_j - \frac{1}{2}q_m - c\right)q_j.$$

Repeated Cournot (5/20)

- The solution is given by $q_j = 3(a-c)/8$, associates with profit $\pi_d = 9(a-c)^2/64$.
- Analogous to the Prisoners' Dilemma, the condition for which the trigger strategy is the Nash equilibrium is given by

$$\frac{1}{1-\delta}\frac{1}{2}\pi_m \geq \pi_d + \frac{\delta}{1-\delta}\pi_c \Rightarrow \delta \geq \frac{9}{17}.$$

Repeated Cournot (6/20)

- What if $\delta < 9/17$?
- We can explore this issue using the two approach proposed in previous section.
- First, for fixed value of $\delta < 9/17$, one knows that the trigger strategy cannot support the quantity as low as half of monopoly profit.
- Also, for any value of δ, it is always a subgame-perfect Nash equilibrium to repeat Cournot quantity forever.

Repeated Cournot (7/20)

- Thus, the most-profitable quantity that trigger strategies can support is between $q_m/2$ and q_c.
- Consider the following trigger strategy

  Produce q^* in the first period. In the t^{th} period, produce q^* if both firms have produce q^* in each of $t-1$ previous periods; otherwise, produce the Cournot quantity, q_c.

Repeated Cournot (8/20)

- If both firms play q^*, then each firm gets $\pi^* = (a - 2q^* - c)q^*$.
- If firm i is going the produce q^*current period, then q_j, which maximizes firm j's current period profit solves

$$\max_{q_j}(a - q_j - q^* - c)q_j.$$

Repeated Cournot (9/20)

- The solution is given by $q_i = (a - q^* - c)/2$, with associated profit $(a - q^* - c)^2/4$.
- It is a Nash equilibrium for both firms to play the trigger strategy if and only if

$$\frac{1}{1-\delta}\pi^* \geq \pi_d + \frac{\delta}{1-\delta}\pi_c .$$

- Using the equation for π^*, π_c, and π_d into above equation results in quadratic in q^*.

Repeated Cournot (10/20)

- Solving for q^* shows that the lowest value of for which the trigger strategy in subgame-perfect Nash equilibrium is

$$q^* = \frac{9-5\delta}{3(9-\delta)}(a-c).$$

- q^* is monotonically decreasing in δ, approaching $q_m/2$ as δ approaches $9/17$ and approaching q_c as δ approaches 0.

Repeated Cournot (11/20)

- Now, consider the second approach proposed by Abreu (1986). We show that we can support collusion with a lower discount factor.
- Consider the “two-phase” (or carrot-and-stick) strategy:

  Produce half the monopoly quantity, $q_m/2$, in the first period. In the t^{th} period, produce $q_m/2$ if both firms produced $q_m/2$ in period $t-1$, produce $q_m/2$ if both firms produced x in period $t-1$, and otherwise produce x.

Repeated Cournot (12/20)

- This strategy involves a (one-period) punishment phase in which the firm produces x and a (potentially infinite) collusive phase in which the firm produces $q_m/2$.
- It either firm deviates from the collusive phase, then the punishment phase begins.
- If either firm deviates from the punishment phase, then the punishment phase begins again.
- If neither firm deviates from the punishment phase, (x, x) then the collusive phase begins again.

Repeated Cournot (13/20)

- If both firms produce x, then each gets
$$\pi(x) = (a - 2x - c)x.$$
- Let $V(x)$ denotes the present value of getting $\pi(x)$ this period and half the monopoly profit forever.
$$V(x) = \pi(x) + \frac{\delta}{1-\delta}\frac{1}{2}\pi_m.$$
- This is the present-value payoff being in punishment phase. Notice that we can use this instead of punishing with Cournot forever.

Repeated Cournot (14/20)

- Next, we check payoff for deviation from punishment, π_{dp} .
- If firm i is going to produce x this period, but firm j deviates, then the quantity that maximizes firm j's profit solves

$$\max_{q_j}(a - q_j - x - c)q_j.$$

- The solution is $q_j = (a - x - c)/2$, with associated profit $\pi_{dp}(x) = (a - x - c)^2/4$.

Repeated Cournot (15/20)

- If both firms play the two-phase strategy above, then the subgames in the infinitely repeated game can be grouped into two classes:
- (i) collusive subgames, in which the outcome of the previous period was either $(q_m/2, q_m/2)$ or (x, x); and
- (ii) punishment subgames, in which the outcome of the previous period was neither $(q_m/2, q_m/2)$, nor (x, x).

Repeated Cournot (16/20)

- For it to be a subgame-perfect Nash equilibrium for both firms to play the two-phase strategy, it must be a Nash equilibrium to obey the strategy in each class of subgames.

Repeated Cournot (17/20)

- In the collusive subgames, each firm must prefer to receive half the monopoly profit forever than to receive π_d this period and the punishment present value $V(x)$ next period:

$$\frac{1}{1-\delta}\frac{1}{2}\pi_m \geq \pi_d + \delta V(x).$$

Repeated Cournot (18/20)

- In the punishment subgames, each firm must prefer to administer the punishment than to receive π_{dp} this period and begin the punishment again next period:

$$V(x) \geq \pi_{dp}(x) + \delta V(x). \qquad \ldots(*)$$

- Use both equations to eliminate $V(x)$, one gets

$$\delta\left(\frac{1}{2}\pi_m - \pi(x)\right) \geq \pi_d - \frac{1}{2}\pi_m. \qquad \ldots(**)$$

Repeated Cournot (19/20)

- That is, the gain this period from deviating must not exceed the discounted value of the loss next period from the punishment.
- Also, one can express (*) as

$$\delta\left(\frac{1}{2}\pi_m - \pi(x)\right) \geq \pi_{dp} - \pi(x),$$

- with analogous interpretation: gain from deviation from punishment period must not exceed the pv of the loss from punishment next period.

Repeated Cournot (20/20)

- For $\delta = 1/2$, (*) is satisfied provided $x/(a - c)$ is not between $1/8$ and $3/8$, and $(**)$ is satisfied if $x/(a - c)$ is between $3/10$ and $1/2$.
- Thus, for $\delta = 1/2$, the two-phase strategy achieves the monopoly outcome as a subgame-perfect Nash equilibrium provided that $3/8 < x/(a - c) < 1/2$.

2.3.D Efficiency Wage (1/15)

- Shapiro and Stiglitz (1984) developed a model in which firms induce workers to work hard by paying high wage, and threatening to fire workers caught shirking.
- As a consequence of high wages, the demand for labor is reduced so that some workers are employed at high wage and some left unemployed.
- Large pool of unemployed makes a fired worker take longer time to find a new job.

Efficiency Wage (2/15)

- Consider the following stage game.
  1. Firm offers worker wage, w.
  2. The worker accept or reject firm's offer.
  3. If the worker reject the offer, then he is self employed and get w_0.
  4. If the worker accept the offer, then he can choose to supply effort, e, which entail disutility (work hard) or to shirk.

Efficiency Wage (3/15)

5. Worker's effort cannot be observed by the firm but the firm can observed the output, which can either be high, $y > 0$, or low, 0.
6. If the worker works hard, then the output is high for certain. If the worker shirks, the output can be high with probability p, and low with probability $1 - p$.

Efficiency Wage (4/15)

- If the worker works hard and output is high for sure, then the payoff for the firm is $y - w$ and $w - e$ for the worker.
- If the worker shirks (e=0), then the expected payoff for the firm is (a) if output is low, then y=0; or (b) if output is high y > 0, then $py - w$ and w- 0 for the worker.
- Assume the $y - e > w_0 > py$, so that it is efficient for the worker to be employed and work hard and also it is better for the worker to be self employed than to be employed and shirk.

Efficiency Wage (5/15)

- The SPNE for this stage game is obvious.
- Since the must pay w in advance, thus the worker has no incentive to work so he shirks.
- Therefore, the firm would want to pay $w = 0$ (or any $w \leq w_0$) since $py < w_0$ by assumption; and the worker chooses self-employment.

Efficiency Wage (6/15)

- In the infinitely repeated game, however, the firm can induce effort by paying a wage w in excess of w_0 and threatening to fire the worker if output is ever low.
- We will show that for some parameter firms can pay wage premium w^* to induce effort.
- We will say that the history of play is *high-wage, high-output* if all previous offers have been w^*, all previous offers have been accepted, and all previous outputs have been high.

Efficiency Wage (7/15)

- Consider the strategies
  - The firm's strategy is to offer $w = w^*$ in the first period, and in each subsequent period to offer $w = w^*$ provided that the history of play is high-wage, high-output, but to offer $w = 0$ otherwise.
  - The worker's strategy is to accept the firm's offer if $w \geq w_0$ (choosing self-employment otherwise) and to supply effort if the history of play, including the current offer, is high-wage, high-output (shirking otherwise).

Efficiency Wage (8/15)

- The firm's strategy is analogous to the trigger strategy discussed in the collusion model.
- The worker strategy is slightly different since the worker observes firm's action before he chooses to act. Thus, if the worker sees $w \neq w^*$ but $w \geq w_0$, then the worker accepts the offer but shirks.
- To show these strategies are SPNE, we shows 2 steps: conditions which strategies are NE; and they are subgame-perfect.

Efficiency Wage (9/15)

- Suppose the firm offers w^* in the first period. Given the firm's strategy, it is optimal for the worker to accept w^* .
- If putting effort (e) is optimal, the present value of worker's payoffs is

$$V_e = (w^* - e) + \delta V_e.$$

- If it is optimal for the worker to shirk (s), then the present value of worker's payoffs is

$$V_s = w^* + \delta\left\{ pV_s + (1-p)\frac{w_0}{1-\delta} \right\}.$$

Efficiency Wage (10/15)

- The worker's strategy is sustainable if $V_e > V_s$, or
$$w^* \geq w_0 + \frac{1-p\delta}{\delta(1-p)}e = w_0 + \left(1 + \frac{1-\delta}{\delta(1-p)}\right)e. \quad \ldots(*)$$
- Thus, to induce effort, the firm must pay not only $w_0 + e$ to compensate the worker for the foregone opportunity of self-employment and for the disutility of effort, but also the wage premium $(1-\delta)e/\delta(1-p)$.

Efficiency Wage (11/15)

- If p is near one (shirking is rarely detected), then the wage premium is very high.
- In limiting case, if p is zero (so that worker can never shirk without being fired), the condition (*) for supplying efforts becomes

$$\frac{1}{1-\delta}(w^* - e) \geq w^* + \frac{\delta}{1-\delta} w_0,$$

- which is similar to the infinitely repeated Prisoners' dilemma game.

Efficiency Wage (12/15)

- Even if (*) holds, it may not optimal for the firm to pay efficiency wage. We also needs condition for firm to pay wage premium.
- Paying wage premium is better than paying w=0 and get zero payoff iff $y - w^* \geq 0$.
- Using this and (*), the trigger strategy is NE for firm if

$$y - w_0 - \left(1 + \frac{1-\delta}{\delta(1-p)}\right)e \geq 0$$

$$\Rightarrow y - e \geq w_0 + \frac{1-\delta}{\delta(1-p)}e. \qquad \ldots(**)$$

Efficiency Wage (13/15)

- (**) is a familiar condition for cooperation
- Next, we show that they are subgame-perfect.
- The subgame can be grouped into two classes, (i) those beginning after high-wage, high-output history, and
- (ii) those beginning after all other histories (no premium no effort).

Efficiency Wage (14/15)

- We just show that the strategies are NE for the first class of subgames.
- As for the second class, the worker will never supply effort, and firm is best to induce self-employment. Since the firm will offer $w = 0$ in next stage and forever after, then the worker will not supply effort in this stage and will accept the current offer only if $w \geq w_0$.

Efficiency Wage (15/15)

- In this equilibrium, self-employment, if occurs, is permanent. That is, if the worker is ever caught shirking, then the firm offers w = 0 forever; if firm does not pay w*, then worker never works hard again.
- But it is better if worker is employed since the worker will be better paid and the firm makes profit.
- Both agents want to return to high-wage, high-output equilibrium than to play subgame-perfect outcome of the stage game forever.
- Therefore, we have a "renegotiation" problem.