Answer

Practice ☺

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{\mu}_i$$

$$\hat{\beta}_2 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

X_i	Y_i	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$(Y_i - \bar{Y})$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	2	-0.5	0.25	0	0
-1	0	-2.5	6.25	-2	5
2	3	0.5	0.25	1	0.5
1	2	-0.5	0.25	0	0
4	1	2.5	6.25	-1	-2.5
2	4	0.5	0.25	2	1
$\sum X_i = 9$	$\sum Y_i = 12$	$\sum (X_i - \bar{X}) = 0$	$\sum (X_i - \bar{X})^2 = 13.5$	$\sum (Y_i - \bar{Y}) = 0$	$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 4$

$$\bar{X} = 1.5$$

$$\bar{Y} = 2$$

Practice ☺

$$\hat{\sigma}^2 = \frac{\sum \hat{\mu}_i^2}{n-2}$$

$$\text{var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum (X_i - \bar{X})^2}$$

$$\text{var}(\hat{\beta}_1) = \frac{\sum X_i^2}{n\sum (X_i - \bar{X})^2}\hat{\sigma}^2$$

$$COV(\hat{\beta}_1, \hat{\beta}_2) = -\bar{X}\ \text{var}(\hat{\beta}_2)$$

$$\hat{\beta}_2 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} = \frac{4}{13.5} \approx 0.296$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 2 - \left(\frac{4}{13.5}\right)1.5 = 1.556$$

X_i	Y_i	$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$	$\hat{\mu}_i$	$\hat{\mu}_i^2$	$(X_i - \bar{X})^2$	X_i^2
1	2	1.852	0.148	0.021904	0.25	1
-1	0	1.26	-1.26	1.5876	6.25	1
2	3	2.148	0.852	0.725904	0.25	4
1	2	1.852	0.148	0.021904	0.25	1
4	1	2.74	-1.74	3.0276	6.25	16
2	4	2.148	1.852	3.429904	0.25	4
				$\sum \hat{\mu}_i^2 = 8.815$	$\sum (X_i - \bar{X})^2 = 13.5$	$\sum X_i^2 = 27$

The Coefficient of Determination

$$\bar{X} = 1.5$$

$$\bar{Y} = 2$$

$$r^2 = \frac{\sum (\hat{Y}_i - \bar{Y})^2}{\sum (Y_i - \bar{Y})^2}$$

X_i	Y_i	$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$	$\hat{\mu}_i$	$\hat{\mu}_i^2$	$(\hat{Y}_i - \bar{Y})^2$	$(Y_i - \bar{Y})^2$
1	2	1.852	0.148	0.021904	0.021904	0
-1	0	1.26	-1.26	1.5876	0.5476	4
2	3	2.148	0.852	0.725904	0.021904	1
1	2	1.852	0.148	0.021904	0.021904	0
4	1	2.74	-1.74	3.0276	0.5476	1
2	4	2.148	1.852	3.429904	0.021904	4
				$\sum \hat{\mu}_i^2 = 8.815$	$\sum (Y_i - \bar{Y})^2 = 1.182816$	$\sum (Y_i - \bar{Y})^2 = 10$