EE325 HW1 Answer

1.

a. $\sum_{i=1}^{8} a + b\sum_{i=1}^{8} x_i$

b.

$\sum_{y=1}^{7} f(x-y) = f(x-0) + f(x-1) + f(x-2) + f(x-3) + f(x-4) + f(x-5) + f(x-6) + f(x-7)$

c. $1^3 + 2^3 + 3^3 + 4^3 + 5^3$

d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(3x+y) = 28$

2

a. $C = \frac{1}{5}$

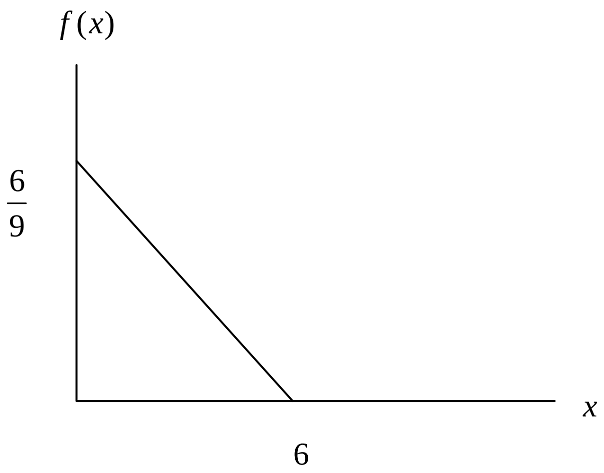
b. $P(X \le 2) = 0.5$

c. $P(-2 \le X \le 3) = 0.9$

d. $P(X \ge 1) = 0.7$

3

a. $f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$



b. $P(2 \le x \le 3) = \frac{7}{18}$

c. $P(X \ge 1) = \frac{8}{9}$

d. $E(X) = 2$

4

a.

		X					
		1	2	3	4	5	6
Y	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

b. & c.

$$f(X=1)=\frac{2}{12}$$

$$f(X=2)=\frac{2}{12}$$

$$f(X=3)=\frac{2}{12}$$

$$f(X=4)=\frac{2}{12}$$

$$f(X=5)=\frac{2}{12}$$

$$f(X=6)=\frac{2}{12}$$

$$f(Y=0)=\frac{6}{12}$$

$$f(Y=1)=\frac{6}{12}$$

d.

$$f(X=1\,|\,Y=1)=\frac{1}{6}$$

$\vdots$

$$f(X=6\,|\,Y=1)=\frac{1}{6}$$

e.

E(X|Y=1) = $\sum_{x} xf(x\,|\,Y=1)=\frac{21}{6}$

f.

$$Var(X\,|\,Y=1)=\frac{35}{12}$$

5.

$$\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$$

$$E(\bar{X}) = E\left(\frac{1}{3}\sum_{i=1}^{3} X_i\right)$$

$$= \frac{1}{3}3E(X_i) = \mu$$

$$Var(\bar{X}) = \frac{1}{9}Var(X_1 + X_2 + X_3)$$

$$= \frac{1}{9}(Var(X_1) + Var(X_2) + Var(X_3) + 2Cov(X_1, X_2) + 2Cov(X_1, X_3) + 2Cov(X_2, X_3))$$

$$= \frac{1}{9}(\sigma^2 + \sigma^2 + \sigma^2 + 2(\frac{1}{4}\sigma^2) + 2(\frac{1}{4}\sigma^2) + 2(\frac{1}{4}\sigma^2))$$

$$= \frac{1}{9}(3\sigma^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\sigma^2 + \frac{1}{2}\sigma^2)$$

$$= \frac{1}{9}(3\sigma^2 + \frac{3}{2}\sigma^2) = \frac{\sigma^2}{2}$$

6a.

$$E(\bar{X}) = E\left(\frac{1}{5}\sum_{i=1}^{5} X_i\right)$$

$$= \frac{1}{3}5E(X_i) = \mu$$

$$Var(\bar{X}) = \frac{1}{25}Var(X_1 + X_2 + X_3 + X_4 + X_5)$$

$$= \frac{1}{25}(Var(X_1) + Var(X_2) + Var(X_3) + Var(X_4) + Var(X_5))$$

$$= \frac{1}{25}(\sigma^2 + \sigma^2 + \sigma^2 + \sigma^2 + \sigma^2)$$

$$= \frac{1}{5}\sigma^2$$

b.

$$\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{4}X_4 + \frac{1}{4}X_5$$

$$= \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{4}E(X_4) + \frac{1}{4}E(X_5)$$

$$= \frac{1}{8}\mu + \frac{1}{4}\mu + \frac{1}{8}\mu + \frac{1}{4}\mu + \frac{1}{4}\mu = \mu$$

$\tilde{X}$ is unbiased estimator of μ

c.

$$Var(\tilde{X}) = Var(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{4}X_4 + \frac{1}{4}X_5)$$

$$= Var(\frac{X_1 + 2X_2 + X_3 + 2X_4 + 2X_5}{8})$$

$$= \frac{14\sigma^2}{64}$$

$$\therefore Var(\overline{X}) < Var(\tilde{X})$$

$\overline{X}$ is better estimator for μ