

From the given data set (assign5-1.dta):

- Estimate the interest rate structure models applying all 9 models using GMM. Also perform Overidentification Test. (Hint: command for generating $\Delta r_t = r_{t+1} - r_t$ is `gen dr=f.r-r`)
- Determine the most appropriated model using Wald Test.

```
1 . use "C:\Users\user\Documents\BE TU\Year2\EE426\HW5\assign5-1.dta"
2 . do "C:\Users\user\AppData\Local\Temp\STD03000000.tmp"
```

```
3 . gen dr=f.r-r
   (1 missing value generated)
```

a) **Estimate Unrestricted model & perform overidentification test ↴**

```
4 . gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
> {sigma2}*r^(2*{gamma})) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r^(2*{gamma}))*r)
> winitial(identity)nolog
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values
numerical derivatives are approximate
flat or discontinuous region encountered
```

Final GMM criterion Q(b) = **4.10e-18**

note: model is exactly identified

GMM estimation

Number of parameters = **4**
 Number of moments = **4**
 Initial weight matrix: **Identity** Number of obs = **1,334**
 GMM weight matrix: **Robust**

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/alpha	-.0023725	.0011574	-2.05	0.040	-.0046409	-.0001041
/beta	.0004291	.0002873	1.49	0.135	-.000134	.0009922
/sigma2	.0005043	.000324	1.56	0.120	-.0001307	.0011393
/gamma	.0985177	.1823933	0.54	0.589	-.2589666	.456002

Instruments for equation **1: _cons**
 Instruments for equation **2: _cons**
 Instruments for equation **3: _cons**
 Instruments for equation **4: _cons**

```
6 . estat overid
```

Test of overidentifying restriction:

Hansen's J chi2(0) = **5.5e-15** (p = .)

Note: test cannot be performed because there are no overidentifying restrictions.

```
7 . est store Unrestricted
```

Estimate Merton model & perform overidentification test ↴

```
8 . gmm (dr-{alpha}) ((dr-{alpha})*r) ((dr-{alpha})^2-{sigma2}) (((dr-{alpha})^2-
> {sigma2})*r) winitial(identity)nolog
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values
```

Final GMM criterion Q(b) = **.0055237**

GMM estimation

Number of parameters = 2
 Number of moments = 4
 Initial weight matrix: Identity Number of obs = 1,334
 GMM weight matrix: Robust

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/alpha	-.0008137	.0006821	-1.19	0.233	-.0021507	.0005232
/sigma2	.0004277	.0002902	1.47	0.141	-.0001412	.0009965

Instruments for equation 1: cons
 Instruments for equation 2: cons
 Instruments for equation 3: cons
 Instruments for equation 4: cons

10 . estat overid

Test of overidentifying restriction:

Hansen's J chi2(2) = 7.36859 (p = 0.0251)

11 . est store Merton

12 . *Estimate Vasicek model & perform overidentification test ↴*

13 . gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-
 > {sigma2}) (((dr-{alpha}-{beta}*r)^2-{sigma2})*r) winitial(identity)nolog
 note: 1 missing value returned for equation 1 at initial values
 note: 1 missing value returned for equation 2 at initial values
 note: 1 missing value returned for equation 3 at initial values
 note: 1 missing value returned for equation 4 at initial values

Final GMM criterion Q(b) = .0002049

GMM estimation

Number of parameters = 3
 Number of moments = 4
 Initial weight matrix: Identity Number of obs = 1,334
 GMM weight matrix: Robust

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/alpha	-.0026994	.0009734	-2.77	0.006	-.0046072	-.0007915
/beta	.0005368	.0001999	2.69	0.007	.000145	.0009286
/sigma2	.0005887	.0002977	1.98	0.048	5.20e-06	.0011722

Instruments for equation 1: cons
 Instruments for equation 2: cons
 Instruments for equation 3: cons
 Instruments for equation 4: cons

14 . estat overid

Test of overidentifying restriction:

Hansen's J chi2(1) = .273315 (p = 0.6011)

15 . est store Vasicek

16 . **Estimate CIR SR model & perform overidentification test ↴**

```
17 . gmm (dr-(alpha)-(beta)*r) ((dr-(alpha)-(beta)*r)*r) ((dr-(alpha)-(beta)*r)^2-
> {sigma2}*r) (((dr-(alpha)-(beta)*r)^2-{sigma2}*r)*r) winitial(identity)nolog
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values

Final GMM criterion Q(b) = .001642

GMM estimation

Number of parameters = 3
Number of moments = 4
Initial weight matrix: Identity Number of obs = 1,334
GMM weight matrix: Robust
```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/alpha	-.0010252	.0007112	-1.44	0.149	-.0024191	.0003686
/beta	.0002288	.0002532	0.90	0.366	-.0002675	.000725
/sigma2	.0000917	.0000646	1.42	0.156	-.000035	.0002183

```
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons
```

```
18 . estat overid
```

Test of overidentifying restriction:

Hansen's J chi2(1) = 2.19038 (p = 0.1389)

```
19 . est store CIR_SR
```

20 . **Estimate Dothan model & perform overidentification test ↴**

```
21 . gmm (dr) (dr*r) (dr^2-{sigma2}*r^2) ((dr^2-{sigma2}*r^2)*r) winitial(identity)no
> log
note: no parameters in equation 1
note: no parameters in equation 2
note: 1 missing value returned for equation 1 at initial values
note: 1 missing value returned for equation 2 at initial values
note: 1 missing value returned for equation 3 at initial values
note: 1 missing value returned for equation 4 at initial values

Final GMM criterion Q(b) = .003444

GMM estimation

Number of parameters = 1
Number of moments = 4
Initial weight matrix: Identity Number of obs = 1,334
GMM weight matrix: Robust
```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/sigma2	.0000162	7.20e-06	2.25	0.025	2.08e-06	.0000303

```
Instruments for equation 1: _cons
Instruments for equation 2: _cons
Instruments for equation 3: _cons
Instruments for equation 4: _cons
```

22 . estat overid

Test of overidentifying restriction:

Hansen's J chi2(3) = 4.59432 (p = 0.2040)

23 . est store Dothan

24 . *Estimate ̢BM model & perform overidentification test ↴*

25 . gmm (dr-{beta}*r) ((dr-{beta}*r)*r) ((dr-{beta}*r)^2-{sigma2}*r^2) (((dr-{beta}*r)^2-{sigma2}*r^2)*r) winitial(identity)nolog
 note: 1 missing value returned for equation 1 at initial values
 note: 1 missing value returned for equation 2 at initial values
 note: 1 missing value returned for equation 3 at initial values
 note: 1 missing value returned for equation 4 at initial values

Final GMM criterion Q(b) = .0034254

GMM estimation

Number of parameters = 2
 Number of moments = 4
 Initial weight matrix: Identity Number of obs = 1,334
 GMM weight matrix: Robust

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/beta	.0000248	.0001265	0.20	0.844	-.000223	.0002727
/sigma2	.0000154	8.33e-06	1.84	0.065	-9.60e-07	.0000317

Instruments for equation 1: _cons
 Instruments for equation 2: _cons
 Instruments for equation 3: _cons
 Instruments for equation 4: _cons

26 . estat overid

Test of overidentifying restriction:

Hansen's J chi2(2) = 4.56951 (p = 0.1018)

27 . est store GBM

28 . *Estimate Brennan & Schwartz model & perform overidentification test ↴*

29 . gmm (dr-{alpha}-{beta}*r) ((dr-{alpha}-{beta}*r)*r) ((dr-{alpha}-{beta}*r)^2-{sigma2}*r^2) (((dr-{alpha}-{beta}*r)^2-{sigma2}*r^2)*r) winitial(identity)no
 > log
 note: 1 missing value returned for equation 1 at initial values
 note: 1 missing value returned for equation 2 at initial values
 note: 1 missing value returned for equation 3 at initial values
 note: 1 missing value returned for equation 4 at initial values

Final GMM criterion Q(b) = .0025923

GMM estimation

Number of parameters = 3
 Number of moments = 4
 Initial weight matrix: Identity Number of obs = 1,334
 GMM weight matrix: Robust

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/alpha	- .0008979	.0008413	-1.07	0.286	-.0025469	.000751
/beta	.0002882	.0002774	1.04	0.299	-.0002555	.0008319
/sigma2	8.91e-06	.0000103	0.87	0.387	-.0000113	.0000291

Instruments for equation 1: **_cons**
 Instruments for equation 2: **_cons**
 Instruments for equation 3: **_cons**
 Instruments for equation 4: **_cons**

30 . estat overid

Test of overidentifying restriction:

Hansen's J chi2(1) = **3.45819** (p = **0.0629**)

31 . est store Brennan_Schwartz

32 . **Estimate CIR VR model & perform overidentification test ↴**

33 . gmm (dr)(dr*r)(dr^2-{sigma2}*r^3) ((dr^2-{sigma2}*r^3)*r) winitial(identity)n
 > olog

note: no parameters in equation 1
 note: no parameters in equation 2
 note: 1 missing value returned for equation 1 at initial values
 note: 1 missing value returned for equation 2 at initial values
 note: 1 missing value returned for equation 3 at initial values
 note: 1 missing value returned for equation 4 at initial values

Final GMM criterion Q(b) = **.0038055**

GMM estimation

Number of parameters = **1**
 Number of moments = **4**
 Initial weight matrix: **Identity** Number of obs = **1,334**
 GMM weight matrix: **Robust**

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/sigma2	2.70e-06	1.26e-06	2.15	0.032	2.35e-07	5.16e-06

Instruments for equation 1: **_cons**
 Instruments for equation 2: **_cons**
 Instruments for equation 3: **_cons**
 Instruments for equation 4: **_cons**

34 . estat overid

Test of overidentifying restriction:

Hansen's J chi2(3) = **5.07654** (p = **0.1663**)

35 . est store CIR_VR

36 . **Estimate CEV model & perform overidentification test ↴**

37 . gmm (dr-{beta}*r) ((dr-{beta}*r)*r) ((dr-{beta}*r)^2-{sigma2}*r^(2*{gamma}))
 > (((dr-{beta}*r)^2-{sigma2}*r^(2*{gamma}))*r) winitial(identity)nolog

note: 1 missing value returned for equation 1 at initial values
 note: 1 missing value returned for equation 2 at initial values
 note: 1 missing value returned for equation 3 at initial values
 note: 1 missing value returned for equation 4 at initial values
 numerical derivatives are approximate
 flat or discontinuous region encountered

Final GMM criterion Q(b) = **.0031388**

GMM estimation

Number of parameters = 3
 Number of moments = 4
 Initial weight matrix: **Identity** Number of obs = 1,334
 GMM weight matrix: **Robust**

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
/beta	-.0000453	.0001702	-0.27	0.790	-.0003789	.0002883
/sigma2	.0000881	.0001436	0.61	0.539	-.0001933	.0003696
/gamma	.5717551	.3668112	1.56	0.119	-.1471817	1.290692

Instruments for equation 1: **_cons**
 Instruments for equation 2: **_cons**
 Instruments for equation 3: **_cons**
 Instruments for equation 4: **_cons**

38 . estat overid

Test of overidentifying restriction:

Hansen's J chi2(1) = 4.18715 (p = 0.0407)

39 . est store CEV

40 .
 end of do-file

b) 41 . *Wald Test

42 . est restore Unrestricted
 (results Unrestricted are active now)

43 . *Test Unrestricted vs Merton

44 . test (_b[/beta]=0) (_b[/gamma]=0)

(1) [beta]_cons = 0
 (2) [gamma]_cons = 0

 chi2(2) = 7.92
 Prob > chi2 = 0.0191

45 . *Test Unrestricted vs Vasicek

46 . test (_b[/gamma]=0)

(1) [gamma]_cons = 0

 chi2(1) = 0.29
 Prob > chi2 = 0.5891

47 . *Test Unrestricted vs CIR_SR

48 . test (_b[/gamma]=0.5)

(1) [gamma]_cons = .5

 chi2(1) = 4.85
 Prob > chi2 = 0.0277

conducting Wald test,
 I found that Vasicek is
 the most appropriated model as its
 P-value is 0.5891, which
 can be interpreted that
 H₀ can't be rejected at
 1%, 5% and 10% level. #

```

49 . *Test Unrestricted vs Dothan
50 . test (_b[/alpha]=0)(_b[/beta]=0)(_b[/gamma]=1)
    ( 1) [alpha]_cons = 0
    ( 2) [beta]_cons = 0
    ( 3) [gamma]_cons = 1
           chi2( 3) =    34.04
           Prob > chi2 =    0.0000
51 . *Test Unrestricted vs GBM
52 . test (_b[/alpha]=0)(_b[/gamma]=1)
    ( 1) [alpha]_cons = 0
    ( 2) [gamma]_cons = 1
           chi2( 2) =    27.48
           Prob > chi2 =    0.0000
53 . *Test Unrestricted vs Brennan_Schwartz
54 . test (_b[/gamma]=1)
    ( 1) [gamma]_cons = 1
           chi2( 1) =    24.43
           Prob > chi2 =    0.0000
55 . *Test Unrestricted vs CIR_VR
56 . test (_b[/alpha]=0)(_b[/beta]=0)(_b[/gamma]=1.5)
    ( 1) [alpha]_cons = 0
    ( 2) [beta]_cons = 0
    ( 3) [gamma]_cons = 1.5
           chi2( 3) =    95.71
           Prob > chi2 =    0.0000
57 . *Test Unrestricted vs CEV
58 . test (_b[/alpha]=0)
    ( 1) [alpha]_cons = 0
           chi2( 1) =     4.20
           Prob > chi2 =    0.0404
59 . est table Unrestricted Merton Vasicek CIR_SR Dothan GBM Brennan_Schwartz CIR_
    > VR CEV, star(0.1 0.05 0.01)

```

Variable	Unrestricted
alpha _cons	-.00237253**
beta _cons	.00042912
sigma2 _cons	.00050427
gamma _cons	.09851773

legend: * p<.1; ** p<.05; *** p<.01

Variable	Merton
alpha _cons	-.00081372
beta _cons	
sigma2 _cons	.00042766
gamma _cons	

legend: * p<.1; ** p<.05; *** p<.01

Variable	Vasicek
alpha _cons	-.00269937***
beta _cons	.00053681***
sigma2 _cons	.00058872**
gamma _cons	

legend: * p<.1; ** p<.05; *** p<.01

Variable	CIR_SR
alpha _cons	-.00102525
beta _cons	.00022877
sigma2 _cons	.00009168
gamma _cons	

legend: * p<.1; ** p<.05; *** p<.01

Variable	Dothan
alpha _cons	
beta _cons	
sigma2 _cons	.00001618**
gamma _cons	

legend: * p<.1; ** p<.05; *** p<.01

Variable	GBM
alpha _cons	
beta _cons	.00002483
sigma2 _cons	.00001536*
gamma _cons	

legend: * p<.1; ** p<.05; *** p<.01

Variable	Brennan_Sch~z
alpha _cons	-.00089794
beta _cons	.0002882
sigma2 _cons	8.909e-06
gamma _cons	

legend: * p<.1; ** p<.05; *** p<.01

Variable	CIR_VR
alpha _cons	
beta _cons	
sigma2 _cons	2.698e-06**
gamma _cons	


legend: * p<.1; ** p<.05; *** p<.01

Variable	CEV
alpha _cons	
beta _cons	-.00004534
sigma2 _cons	.00008815
gamma _cons	.57175507

legend: * p<.1; ** p<.05; *** p<.01

60 . log close
 name: <unnamed>
 log: C:\Users\user\Documents\BE TU\Year2\EE426\HW5\5.9.smcl
 log type: smcl
 closed on: 23 Feb 2021, 12:37:27

(R)



Statistics/Data Analysis

MP - Parallel Edition

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2. From the model:

$$y_i = \alpha + \beta x_i + u_i$$

where: y_i is dependent variable

x_i is explanatory variable

u_i is stochastic error term

$E(u_i) = 0$ but $E(x_i u_i) \neq 0$.

From the given data set (assign5-2.dta):

- a. Estimate model (3) using OLS.
- b. Based on z_1, z_2, z_3, z_4 , determine the best set of instrumental variables, then, estimate model (3) using GMM.

a) `. reg y x` → estimating the model using OLS

Source	SS	df	MS	Number of obs	=	500
Model	132481.702	1	132481.702	F(1, 498)	=	449.66
Residual	146722.774	498	294.624043	Prob > F	=	0.0000
Total	279204.475	499	559.528007	R-squared	=	0.4745
				Adj R-squared	=	0.4734
				Root MSE	=	17.165

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x	5.431333	.2561312	21.21	0.000	4.928102	5.934564
_cons	-33.31333	2.673868	-12.46	0.000	-38.56678	-28.05988

b) `. ivregress gmm y (x = z1 z2)`

Instrumental variables (GMM) regression

Number of obs = 500
 Wald chi2(1) = 36.13
 Prob > chi2 = 0.0000
 R-squared = 0.3136
 Root MSE = 19.577

GMM weight matrix: Robust

y	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
x	2.269419	.3775527	6.01	0.000	1.529429	3.009409
_cons	-1.848121	3.763999	-0.49	0.623	-9.225423	5.529182

Instrumented: x
 Instruments: z1 z2

The best set of instrumental variables is z_1 and z_4 because both of them are highly correlate with x (rank 1st and 2nd).
 Moreover, when using the J-test, H_0 can't be rejected.
 Thus, the over-identified moment are satisfied.

10 . ivregress gmm y (x = z3 z4)

```
Instrumental variables (GMM) regression      Number of obs   =          500
Wald chi2( 1)      =          457.33
Prob > chi2        =          0.0000
R-squared          =          0.4615
GMM weight matrix: Robust                 Root MSE        =          17.341
```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
y						
x	6.237532	.2916741	21.39	0.000	5.665861	6.809202
_cons	-40.18052	3.069851	-13.09	0.000	-46.19731	-34.16372

```
Instrumented:  x
Instruments:  z3 z4
```

11 . corr
(obs=500)

	t	y	x	z1	z2	z3	z4
t	1.0000						
y	0.0681	1.0000					
x	0.0517	0.6888	1.0000				
z1	0.0661	0.2002	0.7215	1.0000			
z2	0.0062	0.1203	0.2352	0.1937	1.0000		
z3	0.0387	0.9214	0.5487	0.0918	0.0933	1.0000	
z4	0.0143	0.1887	0.6954	0.4738	0.1607	0.0828	1.0000

12 . qui ivregress gmm y (x = z1 z2)

13 . estat overid

```
Test of overidentifying restriction:
Hansen's J chi2( 1) = 2.55533 (p = 0.1099)
```

14 . qui ivregress gmm y (x = z1 z3)

15 . estat overid

```
Test of overidentifying restriction:
Hansen's J chi2( 1) = 159.838 (p = 0.0000)
```

16 . qui ivregress gmm y (x = z1 z4)

17 . estat overid

```
Test of overidentifying restriction:
Hansen's J chi2( 1) = .012951 (p = 0.9094)
```

18 . qui ivregress gmm y (x = z2 z3)

19 . estat overid

```
Test of overidentifying restriction:
Hansen's J chi2( 1) = 24.2483 (p = 0.0000)
```

```

20 . qui ivregress gmm y ( x = z2 z4 )
21 . estat overid
    Test of overidentifying restriction:
    Hansen's J chi2( 1) = 2.47701 (p = 0.1155)
22 . qui ivregress gmm y ( x = z3 z4 )
23 . estat overid
    Test of overidentifying restriction:
    Hansen's J chi2( 1) = 148.756 (p = 0.0000)
24 . log close
    name: <unnamed>
    log: C:\Users\user\Documents\BE TU\Year2\EE426\HW5\5.93.smcl
    log type: smcl
    closed on: 23 Feb 2021, 20:17:48

```

- c. Determine whether OLS estimated results in (a) or GMM estimated results in (b) is more appropriate using the Hansen's J chi2 statistic test.

c) Through J-test of the model using z_1 and z_4 as instrumental variables, I found that the over-identified moment conditions are true as mentioned the details in (b).

It can be concluded that the GMM estimated results in (b) is more appropriate than the OLS estimated results in (a) because X in the OLS model has an endogeneity $E(x_i u_i) \neq 0$.