BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X$

Table 1

$\hat{Y}$

$\hat{u}_i$

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$2.8 - \hat{Y}_i$

$\hat{Y}_i$

$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X + \hat{u}$

$Y_i - \hat{Y}_i = \hat{u}_i$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$ OLS

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

1.

Student	Y_i	X_i	x_i: $X - \bar{X}$	y_i: $Y - \bar{Y}$	x_iy_i: $(X-\bar{X})(Y-\bar{Y})$	x_i^2: $(X-\bar{X})^2$	X_i^2	$\hat{u}_i^2$
1	2.8	63	-14.625	-0.4125	6.0328	213.8906	3969	0.0074
2	3.4	72	-5.625	0.1875	-1.0547	31.6406	5184	0.1439
3	3.0	78	0.375	-0.2125	-0.0797	0.1406	6084	0.0508
4	3.5	81	3.375	0.2875	0.9703	11.3906	6561	0.0297
5	3.6	87	9.375	0.3875	3.6328	87.8906	7569	0.0046
6	3.0	75	-2.625	-0.2125	0.5578	6.8906	5625	0.0151
7	2.7	75	-2.625	-0.5125	1.3453	6.8906	5625	0.1789
8	3.7	90	12.375	0.4875	6.0328	153.1406	8100	0.0043

$\bar{Y}_i = 3.2125$ $\quad \sum x_iy_i = 17.4375$ $\quad \sum \hat{u}_i^2 = 0.434703$

$\bar{X}_i = 77.625$ $\quad \sum x_i^2 = 511.8748$ $\quad \sum X_i^2 = 48{,}717$

$$\hat{\beta}_2 = \frac{\sum x_iy_i}{\sum x_i^2} = \frac{17.4375}{511.8748} = 0.0341$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X} = 3.2125 - (0.0341)(77.625) = 0.5655$$

Interpretation

$\therefore$ As $Y_i = 0.5655 + 0.0341X_i + u_i$, for any given level of exam point (X_i), the slope of estimated GPA is 0.0341 with estimated intercept on Y-axis is 0.5655 and the error (u_i) is nearly zero.

- $\hat{\beta}_1 = 0.5655$ $\Rightarrow$ As if the student has got no point at all, the maximum GPA they can get is 0.5655.
- $\hat{\beta}_2 = 0.0341$ $\Rightarrow$ As one point earned, the GPA is raised up by 0.0341.

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 x$$

$$\hat{Y}_1 = 0.5655 + 0.0341(63) = 2.7138$$
$$\hat{Y}_2 = 0.5655 + 0.0341(72) = 3.0207$$
$$\hat{Y}_3 = 0.5655 + 0.0341(78) = 3.2253$$
$$\hat{Y}_4 = 0.5655 + 0.0341(81) = 3.3276$$
$$\hat{Y}_5 = 0.5655 + 0.0341(87) = 3.5322$$
$$\hat{Y}_6 = 0.5655 + 0.0341(75) = 3.1230$$
$$\hat{Y}_7 = 0.5655 + 0.0341(75) = 3.1230$$
$$\hat{Y}_8 = 0.5655 + 0.0341(90) = 3.6345$$

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 x + \hat{u}_i$$
$$Y_i = \hat{Y}_i + \hat{u}_i$$
$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$\hat{u}_1 = 2.8 - 2.7138 = 0.0862$$
$$\hat{u}_2 = 3.4 - 3.0207 = 0.3793$$
$$\hat{u}_3 = 3.0 - 3.2253 = -0.2253$$
$$\hat{u}_4 = 3.5 - 3.3276 = 0.1724$$
$$\hat{u}_5 = 3.6 - 3.5322 = 0.0678$$
$$\hat{u}_6 = 3.0 - 3.1230 = -0.123$$
$$\hat{u}_7 = 2.7 - 3.1230 = -0.423$$
$$\hat{u}_8 = 3.7 - 3.6345 = 0.0655$$

$$\sum \hat{u}_i = \hat{u}_1 + \hat{u}_2 + \hat{u}_3 + \hat{u}_4 + \hat{u}_5 + \hat{u}_6 + \hat{u}_7 + \hat{u}_8$$
$$= -0.0001$$
$$\approx 0$$

$$Var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{0.4347}{6} = 0.0725$$

$$Var(\hat{\beta}_1) = \frac{\Sigma X_i^2 \sigma^2}{n \Sigma (x_i - \bar{x})^2} = \frac{48,717 \, (0.0725)}{8 \, (511.8748)} = \frac{3531.982}{4095} = 0.8625$$

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma (x_i - \bar{x})^2} = \frac{0.0725}{511.8748} = 0.000141$$

2)

X_i	Y_i	x_i: $x - \bar{x}$	y_i: $Y - \bar{Y}$	$x_i y_i$: $(x-\bar{x})(Y-\bar{Y})$	x_i^2: $(x-\bar{x})^2$	X_i^2	$\hat{u}_i^2$
10	0	−10	−9.1	91	100	100	0.0210
12	2	−8	−7.1	56.8	64	144	0.0041
14	5	−6	−4.1	24.6	36	196	1.6205
16	6	−4	−3.1	12.4	16	256	0.2323
18	7	−2	−2.1	4.2	4	324	0.0955
22	10	2	0.9	1.8	4	484	0.7939
24	10	4	0.9	3.6	16	576	7.1931
26	15	6	5.9	35.4	36	676	0.2777
28	16	8	6.9	55.2	64	784	0.0697
30	20	10	10.9	109	100	900	3.7830

$\bar{X} = 20$ $\Sigma x_i y_i = 394$ $\Sigma u_i^2 = 14.0908$

$\bar{Y} = 9.1$ $\Sigma x_i^2 = 440$ $\Sigma X_i^2 = 4,440$

2.1)

$$\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{394}{440} = 0.8955$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - (0.8913)(20)$$

$$= 9.1 - 17.91$$

$$= -8.81$$

Interpretation

- $\hat{\beta}_1 = -8.81$ ⟹ the y-intercept when $x = 0$ is at $(0, -8.81)$
- $\hat{\beta}_2 = 0.8955$ ⟹ As x is increased by 1 unit, y increases by 0.8955

2.2) $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 x$

$\hat{Y}_1 = -8.81 + 0.8955(10) = 0.145$

$\hat{Y}_2 = -8.81 + 0.8955(12) = 1.936$

$\hat{Y}_3 = -8.81 + 0.8955(14) = 3.727$

$\hat{Y}_4 = -8.81 + 0.8955(16) = 5.518$

$\hat{Y}_5 = -8.81 + 0.8955(18) = 7.309$

$\hat{Y}_6 = -8.81 + 0.8955(22) = 10.891$

$\hat{Y}_7 = -8.81 + 0.8955(24) = 12.682$

$\hat{Y}_8 = -8.81 + 0.8955(26) = 14.473$

$\hat{Y}_9 = -8.81 + 0.8955(28) = 16.964$

$\hat{Y}_{10} = -8.81 + 0.8955(30) = 18.055$

$Y_i = \hat{\beta}_1 + \hat{\beta}_2 x + \hat{u}_i$

$Y_i = \hat{Y}_i + \hat{u}_i \Rightarrow \hat{u}_i = Y_i - \hat{Y}_i$

$\hat{u}_1 = 0 - 0.145 = -0.145$

$\hat{u}_2 = 2 - 1.936 = 0.064$

$\hat{u}_3 = 5 - 3.727 = 1.273$

$\hat{u}_4 = 6 - 5.518 = 0.482$

$\hat{u}_5 = 7 - 7.308 = -0.309$

$\hat{u}_6 = 10 - 10.891 = -0.891$

$\hat{u}_7 = 10 - 12.682 = -2.682$

$\hat{u}_8 = 15 - 14.473 = 0.527$

$\hat{u}_9 = 16 - 16.964 = -0.964$

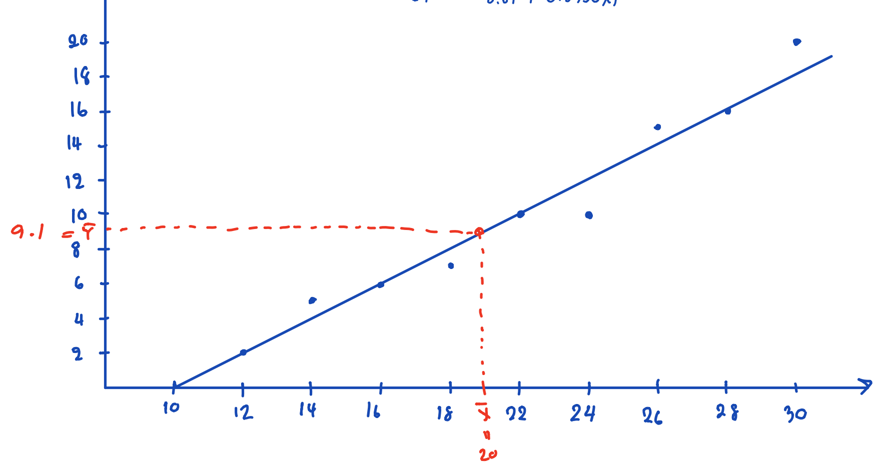
$\hat{u}_{10} = 20 - 18.055 = 1.945$

$\sum \hat{u}_i = 0$

2.3)

Regression line : $\hat{y}_i = \beta_1 + \beta_2 X_i$

$\hat{y}_i = -8.81 + 0.8955 X_i$



$\therefore$ The Regression line passes the $(\bar{X}, \bar{Y}) = (20, 9.1)$

2.4) If $X_1 = 18$, $\hat{y}_1 = ?$

$$\hat{y}_i = \beta_1 + \beta_2 X_i$$

$$= -8.81 + \left(\frac{394}{440}\right)(18)$$

$$= 7.3081$$

2.5) $$Var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.0908}{8} = 1.7614$$

$$Var(\hat{\beta}_1) = \frac{\sum X_i^2 \sigma^2}{n \sum (x_i - \bar{x})^2} = \frac{4440(1.7614)}{10(440)} = 1.7614$$

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} = \frac{(1.7614)^2}{440} = 0.0071$$

3.

$$\hat{\beta}_1 = \frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n \sum X_i^2 - (\sum X_i)^2}$$

$$= \bar{Y} - \hat{\beta}_2 \bar{X}$$

Proof that this is an unbiased estimator

$$E(\hat{\beta}_1) = E(\bar{Y} - \hat{\beta}_2 \bar{X})$$

$$= E(\bar{Y}) - \hat{\beta}_2 E(\bar{X})$$

$$= \beta_1 + \cancel{\beta_2 \bar{X}} - \cancel{\bar{X} \beta_2}$$

$$\therefore E(\hat{\beta}_1) = \beta_1$$

$\therefore$ Assumption 1, 7, 8