ID ....SOLUTION...............

Quiz#3
EE320 Introductory Mathematical Economics
Section 046402

Instructor: Panit Wattanakoon
Semester 1, 2014

December 6, 2014

Question 1 (100 points) Given the following utility function,

$$u(x_1, x_2) = \sqrt{x}_1 + \sqrt{x}_2 \tag{1}$$

Suppose that p_1, p_2, and I are price of goods x_1, x_2 and income. Answer the following questions

1.1) **(8 points)** This consumer maximizes utility subject to budget constraint. Write down the maximization problem

Ans

Max. $u(x_1, x_2) = \sqrt{x}_1 + \sqrt{x}_2$

subject to $p_1x_1 + p_2x_2 = I$

1.2) **(8 points)** Write down the Lagrange function.

Ans

$$L = u(x_1, x_2) = \sqrt{x}_1 + \sqrt{x}_2 + \lambda(I - p_1x_1 - p_2x_2) \tag{2}$$

or

$$L = u(x_1, x_2) = \sqrt{x}_1 + \sqrt{x}_2 - \lambda(p_1x_1 + p_2x_2 - I)$$

1.3) **(12 points)** Find first order conditions with respect to x_1, x_2, and Lagrange multiplier

Ans From (2),

$\frac{\partial L}{\partial x_1} = \frac{x_1^{\frac{-1}{2}}}{2} - \lambda p_1 = 0 \Rightarrow$

$$\frac{x_1^{\frac{-1}{2}}}{2} = \lambda p_1 \tag{3}$$

$\frac{\partial L}{\partial x_2} = \frac{x_2^{\frac{-1}{2}}}{2} - \lambda p_2 = 0 \Rightarrow$

$$\frac{x_2^{\frac{-1}{2}}}{2} = \lambda p_2 \tag{4}$$

$\frac{\partial L}{\partial \lambda} = I - p_1 x_1 - p_2 x_2 = 0 \Rightarrow$

$$p_1 x_1 + p_2 x_2 = I \tag{5}$$

1.4) **(24 points)** Solve for x_1 and x_2 in term of p_1, p_2, and I

Ans

From $\frac{(3)}{(4)}$:

$\frac{\sqrt{x_2}}{\sqrt{x_1}} = \frac{p_1}{p_2} \Rightarrow x_2 = \frac{p_1^2}{p_2^2} x_1$, substitute in (5), then we get:

$I = p_1 x_1 + p_2 \frac{p_1^2}{p_2^2} x_1 \Rightarrow I = p_1 x_1 + \frac{p_1^2}{p_2} x_1 \Rightarrow I = x_1 (p_1 + \frac{p_1^2}{p_2}) \Rightarrow$

$x_1^* = \frac{I}{p_1 + \frac{p_1^2}{p_2}} = (\frac{I}{p_1+p_2}) \frac{p_2}{p_1}$. So the answer is in term of $p_1, p_2,$ and I.

Then solve for x_2^*. Rearrange (3), (4) and substitute in (5), we should get:,

$x_2^* = (\frac{I}{p_1+p_2}) \frac{p_1}{p_2}$

1.5) **(16 points)** Confirm that the solution found in the previous question is the maximum by using bordered Hessian matrix.

Ans

Consider n-k = 2-1 = 1 leading principal minor, which is $|\bar{H}_{k+1}| = |\bar{H}_{1+1}| = |\bar{H}_2| > 0$ for max.

$$\det \bar{H} = \begin{vmatrix} 0 & p_1 & p_2 \\ p_1 & L_{11} & L_{12} \\ p_2 & L_{21} & L_{22} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & p_1 & p_2 \\ p_1 & -1/4(x_1^*)^{-3/2} & 0 \\ p_2 & 0 & -1/4(x_2^*)^{-3/2} \end{vmatrix}$$

$$= -p_2^2(-1/4(x_1^*)^{-3/2}) - p_1^2(-1/4(x_2^*)^{-3/2})$$

$$= p_2^2(1/4(x_1^*)^{-3/2}) + p_1^2(1/4(x_2^*)^{-3/2})$$

Since x_1^* and x_2^* are greater than zero, so $|\bar{H}| > 0$

1.6) **(12 points)** Compute $\frac{\partial x_1^*}{\partial I}$ and explain whether x_1 is normal goods.

Ans

From $x_1^* = (\frac{I}{p_1+p_2})\frac{p_2}{p_1}$,

$\frac{\partial x_1^*}{\partial I} = (\frac{1}{p_1+p_2})\frac{p_2}{p_1} > 0$

As income increases, people consume more of x_1^*. We conclude that goods 1 is normal.

1.7) **(20 points)** Suppose that there are n numbers of goods consumer can purchase. Write out the new bordered Hessian matrix. What will be the dimension of zero matrix in bordered Hessian matrix and the bordered Hessian matrix itself?

Ans

$$\det \bar{H} = \begin{vmatrix} 0 & p_1 & p_2 & \dots & p_n \\ p_1 & L_{11} & L_{12} & \dots & L_{1n} \\ p_2 & L_{21} & L_{22} & \dots & L_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_n & L_{n1} & L_{n2} & \dots & L_{nn} \end{vmatrix}_{(n+1)\times(n+1)}$$

Dimension of zero matrix in bordered Hessian matrix = k × k = 1 × 1

Dimension of bordered Hessian matrix = (n+k)×(n+k) = (n+1)×(n+1)