Serial Correlation and Heteroskedasticity in Time Series Regressions

1 Properties of OLS with Serially Correlated Errors

2 Unbiasedness and Consistency

3 Efficiency and Inference

With serial correlation, $\hat{\beta}_{OLS}$ would not be BLUE ($var(\hat{\beta}_{OLS})$ would not be minimized). Consider

$$u_t = \rho u_{t-1} + e_t \ ; \ t = 1, 2, ..., n \quad \text{and} \quad |\rho| < 1$$

where u_t is from a regression model

$$y_t = \beta_0 + \beta_1 x_t + u_t.$$

4 Testing for Serial Correlation

Given the model

$$y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + ... + \beta_k x_{tk} + u_t$$

4.1 A "t-test" for AR(1) serial correlation with strictly exogeneous regressors

The most common type of serial correlation or autocorrelation is the AR(1) type:

To perform the test:

1. Estimate $y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + ... + \beta_k x_{tk} + u_t$
2. Obtain $\hat{u}_t$,$\hat{u}_{t-1}$; $\forall\ t = 1, 2, ..., n$
3. Estimate $\hat{u}_t = \rho\ \hat{u}_{t-1} + error$

4. Perform the $t-test$ for

4.2 The Durbin-Watson Test (DW test)

This implies

$$\begin{aligned} \hat{\rho} &= 0 \quad \Rightarrow \quad DW = 2 \\ \hat{\rho} &> 0 \quad \Rightarrow \quad DW < 2 \\ \hat{\rho} &< 0 \quad \Rightarrow \quad DW > 2 \end{aligned}$$

H_o : no positive autocorrelation, serial-correlation
H_a : no negative serial correlation

To perform the test:

1. Estimate $y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + ... + \beta_k x_{tk} + u_t$
2. Obtain $\hat{u}_t$,$\hat{u}_{t-1}$; $\forall\ t = 1, 2, ..., n$
3. Calculate DW from eq.(2)
4. Find the critical d_L and d_u values (say, at the 5% level of significance) for the given sample size and # of regressors.
5. Follow the decision rule in the picture.

Example:
Suppose the calculated value of $DW = 0.80$, $n = 45$, $k = 4$.
From this, we get $d_L =$ _____ and $d_u =$ _____

4.3 Testing for AR(1) serial correlation "without" strictly exogenous regressors

4.4 Testing for AR(q) serial correlation "without" strictly exogenous regressors

5 Correcting for serial correlation

5.1 Passive way

Use the type of standard error that is robust to the serial correlation, autocorrelation problem

5.2 Active way –