Subject : ______

Date : / /

① a)

$$\begin{bmatrix} 1 & 2 & 3 & 2 & 4 \\ 4 & -9 & -5 & 8 & -1 \\ -2 & -2 & -4 & -4 & C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ -18 \\ D \end{bmatrix}$$

b)

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 4 & 4 \\ 4 & -9 & -5 & 8 & -1 & -18 \\ -2 & -2 & -4 & -4 & C & D \end{array}\right] \underset{\substack{R_2-4R_1\to R_2 \\ R_3+2R_1\to R_3}}{\sim} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 4 & 4 \\ 0 & -17 & -17 & 0 & -17 & -34 \\ 0 & 2 & 2 & 0 & C+8 & D+8 \end{array}\right]$$

$$\frac{R_2}{-17} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 4 & 4 \\ 0 & 1 & 1 & 0 & 1 & 2 \\ 0 & 2 & 2 & 0 & C+8 & D+8 \end{array}\right] \underset{\sim}{R_3-2R_2\to R_3} \left[\begin{array}{ccccc|c} \boxed{1} & 2 & 3 & 2 & 4 & 4 \\ 0 & \boxed{1} & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & C+6 & D+4 \end{array}\right]$$

For $\underline{A}$ to have rank 2 $\quad C+6=0$

$C=-6$

c) If $C=-6$ then $D+4=0$ to make the system consistent.

ie $D=-4$, $C=-6$

$$\left[\begin{array}{ccccc|c} \boxed{1} & 2 & 3 & 2 & 4 & 4 \\ 0 & \boxed{1} & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \quad x_1+2x_2+3x_3+2x_4+4x_5=4$$

↑ ↑ ↑

$x_2 = 2-x_3-x_5$

x_3, x_4, x_5 are Free variables

$x_1 = 4-2(2-x_3-x_5)-3x_3-2x_4-4x_5$

$x_1 = 4-4+2x_3+2x_5-3x_3-2x_4-4x_5$

$x_1 = -x_3-2x_4-2x_5$

$$\underline{X} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_4 \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_5 \begin{bmatrix} -2 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

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d) To make matrix $\underline{A}$ rank 3, $c+6 \neq 0$

$c \neq -6$

ie $c = \mathbb{R} - \{-6\}$

If matrix $\underline{A}$ has rank=3, its echelon form contains pivot in every row. Therefore no zero row exists in echelon form of A the $\underline{A}\underline{x} = \underline{b}$ has never had no solution. $r = m$ and $m < n$ ie. it will always consistent.

2)
$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 4 & 1 & 1 \\ 2 & -1 & k \end{array}\right] \underset{\substack{R_2-4R_1 \\ R_3-2R_1}}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -11 \\ 0 & -1 & k-6 \end{array}\right] \underset{R_3+R_2}{} \left[\begin{array}{cc|c} \boxed{1} & 0 & 3 \\ 0 & \boxed{1} & -11 \\ 0 & 0 & k-17 \end{array}\right]$$

a) $k-17 = 0 \therefore k = 17$.

unique solution $x_1 = 3$, $x_2 = -11$

$$\underline{x} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$$

b) $k - 17 \neq 0$ ie $k \neq +17$

$k = \mathbb{R} - \{+17\}$

c) No since the rank of matrix is 2 or number of pivots $r = 2$ and #column $= 2$ ie $r = n$ the matrix $\underline{A}$ contains pivot in every column with 1 zero row in its echelon form $\therefore$ the system $\underline{A}\underline{x} = \underline{b}$ can either have one unique solution or no solution.

Subject : ______ Date :

d) To make matrix $\underline{A}$ rank 3 , $C+6 \neq 0$

$C \neq -6$

ie $C = \mathbb{R} - \{-6\}$

If Matrix $\underline{A}$ has rank=3 , its echelon form contains pivot in every row. Therefore no zero row exists in echelon form of A the $\underline{A}\underline{x}=\underline{b}$ has never had no solution. $r=m$ and $m<n$ ie. it will always consistent.

(2)

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 4 & 1 & 1 \\ 2 & -1 & k \end{array}\right] \underset{\substack{R_2-4R_1 \\ R_3-2R_1}}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -11 \\ 0 & -1 & k-6 \end{array}\right] \underset{R_3+R_2}{} \left[\begin{array}{cc|c} \boxed{1} & 0 & 3 \\ 0 & \boxed{1} & -11 \\ 0 & 0 & k-17 \end{array}\right]$$

a) $k-17 = 0$. $k = 17$.

unique solution $x_1 = 3$, $x_2 = -11$

$$\underline{x} = \begin{bmatrix} 3 \\ -11 \end{bmatrix}$$

b) $k-17 \neq 0$ ie $k \neq +17$

$k = \mathbb{R} - \{+17\}$

c) No since the rank of matrix is 2 or number of pivot $r=2$ and #column $=2$ ie $r=n$ the matrix $\underline{A}$ contains pivot in every column with 1 zero row in its echelon form ∴ the system $\underline{A}\underline{x}=\underline{b}$ can either have one unique solution or no solution.

Subject: ______ Date: / /

③ a)

$$\begin{bmatrix} 10 & 20 & 30 \\ 10 & 30 & 50 \\ 10 & 50 & 60 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 150 \\ 180 \\ 210 \end{bmatrix}$$

b)

$$\left[\begin{array}{ccc|ccc} 10 & 20 & 30 & 1 & 0 & 0 \\ 10 & 30 & 50 & 0 & 1 & 0 \\ 10 & 50 & 60 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 10 & 20 & 30 & 1 & 0 & 0 \\ 0 & 10 & 20 & -1 & 1 & 0 \\ 0 & 30 & 30 & -1 & 0 & 1 \end{array}\right]$$

$R_3 - 3R_2 \to R_3$

$$\sim \left[\begin{array}{ccc|ccc} 10 & 20 & 30 & 1 & 0 & 0 \\ 0 & 10 & 20 & -1 & 1 & 0 \\ 0 & 0 & -30 & 2 & -3 & 1 \end{array}\right]$$

$R_1 + R_3 \to R_1$

$R_2 + \frac{2}{3}R_3 \to R_2$

$$\sim \left[\begin{array}{ccc|ccc} 10 & 20 & 0 & 3 & -3 & 1 \\ 0 & 10 & 0 & 1/3 & -1 & 2/3 \\ 0 & 0 & -30 & 2 & -3 & 1 \end{array}\right]$$

$R_1 - 2R_2 \to R_1$

$$\left[\begin{array}{ccc|ccc} 10 & 0 & 0 & 7/3 & -1 & -1/3 \\ 0 & 10 & 0 & 1/3 & -1 & 2/3 \\ 0 & 0 & -30 & 2 & -3 & 1 \end{array}\right]$$

$R_1/10$

$R_2/10$

$R_3/-30$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/30 & -1/10 & -1/30 \\ 0 & 1 & 0 & 1/30 & -1/10 & 2/30 \\ 0 & 0 & 1 & \frac{2}{-30} & \frac{-3}{-30} & \frac{1}{-30} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0.233 & -0.1 & -0.033 \\ 0.033 & -0.1 & -0.066 \\ -0.066 & +0.1 & -0.033 \end{bmatrix}$$ ✓

c)

$$Ax = b$$

$$x = A^{-1} b$$

$$x = \begin{bmatrix} 0.233 & -0.1 & -0.033 \\ 0.033 & -0.1 & -0.066 \\ -0.066 & 0.1 & -0.033 \end{bmatrix} \begin{bmatrix} 150 \\ 180 \\ 210 \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \\ 1 \end{bmatrix}$$

Subject :

(4) $2x + 3y - z = 5$
$x + 2y + z = 3$
? $ax + by + cz = d$

ie $\left[\begin{array}{ccc|c} 2 & 3 & -1 & 5 \\ 1 & 2 & 1 & 3 \\ a & b & c & d \end{array}\right]$

For the system to have infinitely many solutions the last row can be

$2x + 3y - z = 5$ or multiple of 1^{st} or 2^{nd} row

or $4x + 6y - 2z = 10$

or $2x + 4y + 2z = 6$

or $-2x - 4y - 2z = -6$

to make last row zero after 1 row replacement.

No solution

$2x + 4y + 2z = 7$

or $4x + 6y - 2z = -5$

or $-2x - 6y - 2z = 8$

to make last row of echelon form of coefficient matrix zero but the right hand side is nonzero.

~~(5). $|A| = \begin{vmatrix} x^2 & y^2 & z^2 & 1 \\ 3 & 9 & 8 & 1 \\ 3 & 4 & 5 & 1 \\ 3 & 7 & 5 & 1 \end{vmatrix}$ $R_3 - R_2$, $R_4 - R_2$, $R_5 - R_2$ $\begin{vmatrix} x^2 & y^2 & z^2 & 1 \\ 3 & 9 & 8 & 1 \\ 0 & -5 & -3 & 0 \\ 0 & -2 & -3 & 0 \end{vmatrix}$ $R_2 - R_1 \to R_2$ $\begin{vmatrix} x^2 & y^2 & z \\ 3-x^2 & 9-y^2 & 8 \\ 0 & -5 & - \\ 0 & -2 & - \end{vmatrix}$~~

~~$\therefore |A| = (1)(-1)^{1+4} \begin{vmatrix} 3-x^2 & 9-y^2 & 8-z^2 \\ 0 & -5 & -3 \\ 0 & -2 & -3 \end{vmatrix} = (-1)(3-x^2)(15-6) = 9(x^2-3)$~~

~~$|B| = (-1)(-1) \begin{vmatrix} 4 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -13 \\ 0 & 0 & 0 & 3 \end{vmatrix} = (1)(4)(3)(1)(3) = 36.$; $|C| = 1 \cdot \frac{1}{2} \cdot (-4)(-1/4)$, $|C| = \frac{1}{2}$~~

(1) 2012-13

ⓐ
$$|A| = \begin{vmatrix} x^2 & y^2 & z^2 & 1 \\ 3 & 9 & 8 & 1 \\ 3 & 4 & 5 & 1 \\ 3 & 7 & 5 & 1 \end{vmatrix} \begin{matrix} \\ R_3 - R_2 \\ R_4 - R_2 \\ R_5 - R_2 \end{matrix} \begin{vmatrix} x^2 & y^2 & z^2 & 1 \\ 3 & 9 & 8 & 1 \\ 0 & -5 & -3 & 0 \\ 0 & -2 & -3 & 0 \end{vmatrix} R_2 - R_1 \to R_2 \begin{vmatrix} x^2 & y^2 & z^2 & 1 \\ 3-x^2 & 9-y^2 & 8-z^2 & 0 \\ 0 & -5 & -3 & 0 \\ 0 & -2 & -3 & 0 \end{vmatrix}$$

$$|A| = (1)(-1)^{1+4} \begin{vmatrix} 3-x^2 & 9-y^2 & 8-z^2 \\ 0 & -5 & -3 \\ 0 & -2 & -3 \end{vmatrix} = (-1)(3-x^2)(15-6) = 9(x^2-3)$$

$$|B| = (-1)(-1) \begin{vmatrix} 4 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & -13 \\ 0 & 0 & 0 & 3 \end{vmatrix} = (1)(4)(3)(1)(3) = 36$$

$$|C| = 1 \cdot \frac{1}{2}(-6)\left(\frac{-1}{6}\right) = \frac{1}{2}.$$

$|A| = |BC^2|$

$|A| = |B||C|^2$

$9(x^2-3) = 36 \cdot \frac{1}{4}$

$x^2 - 3 = 1$

$x^2 - 4 = 0$

$\boxed{x = \pm 2}$

ⓑ If $x = 3$, $y = 1$, $z = 2$, $|A| = ?$

$|A| = 9(x^2-3) = 9(9-3) = 54.$

$|-2A^T| = (-2)^4 (54) = 864$

Find D.

$$B = 9A D^3 C^{-1}$$

$$|B| = 9^4 |A| |D|^3 |C^{-1}|$$

$$36 = 9^4 (54)(|D|^3)(2)$$

$$\frac{2}{9^3 \cdot 54} = |D|^3$$

$$|D|^3 = \frac{1}{9^3} \cdot \frac{1}{3^3}$$

$$|D| = \frac{1}{27}.$$

c) $\begin{bmatrix} 9 & 1 & 4 & 1 \\ 3 & 9 & 8 & 1 \\ 3 & 4 & 5 & 1 \\ 3 & 7 & 5 & 1 \end{bmatrix}$; $x=3, y=1, z=2$ $\therefore |A| = 54.$

$$|A_{b1}| = \begin{vmatrix} 0 & 1 & 4 & 1 \\ 2 & 9 & 8 & 1 \\ 0 & 4 & 5 & 1 \\ 0 & 7 & 5 & 1 \end{vmatrix} = (-2)\begin{vmatrix} 1 & 4 & 1 \\ 4 & 5 & 1 \\ 7 & 5 & 1 \end{vmatrix} = (-2)\begin{vmatrix} 1 & 4 & 1 \\ 4 & 5 & 1 \\ 3 & 0 & 0 \end{vmatrix} = (-2)(3)(4-5)$$

$$= 6.$$

$$\therefore x_1 = \frac{6}{54} = \frac{3}{27} = \frac{1}{9}.$$

Subject : Date : /

$$|Ab_2| = \begin{vmatrix} 9 & 0 & 4 & 1 \\ 3 & 2 & 8 & 1 \\ 3 & 0 & 5 & 1 \\ 3 & 0 & 5 & 1 \end{vmatrix} = (2)\begin{vmatrix} 9 & 4 & 1 \\ 3 & 5 & 1 \\ 3 & 5 & 1 \end{vmatrix} = (2)\begin{vmatrix} 9 & 4 & 1 \\ 3 & 5 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

(1) $\therefore x_2 = \frac{0}{56} = 0$

$$|Ab_3| = \begin{vmatrix} 9 & 1 & 0 & 1 \\ 3 & 9 & 2 & 1 \\ 3 & 4 & 0 & 1 \\ 3 & 7 & 0 & 1 \end{vmatrix} = (-2)\begin{vmatrix} 9 & 1 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 1 \end{vmatrix} = (-2)\begin{vmatrix} 9 & 1 & 1 \\ 3 & 4 & 1 \\ 0 & 3 & 0 \end{vmatrix} = (-2)(-3)\begin{vmatrix} 9 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= (6)(9-3) = 36$$

(1) $\therefore x_3 = \frac{36}{54} = \frac{4}{6} = \frac{2}{3}$

$$|Ab_4| = \begin{vmatrix} 9 & 1 & 4 & 0 \\ 3 & 9 & 8 & 2 \\ 3 & 4 & 5 & 0 \\ 3 & 7 & 5 & 0 \end{vmatrix} = (2)\begin{vmatrix} 9 & 1 & 4 \\ 3 & 4 & 5 \\ 3 & 7 & 5 \end{vmatrix} = 2\begin{vmatrix} 9 & 1 & 4 \\ 3 & 4 & 5 \\ 0 & 3 & 0 \end{vmatrix} = (2)(-3)\begin{vmatrix} 9 & \\ 3 & \end{vmatrix}$$

$$= (-6)(45-12) = -198$$

(1) $\therefore x_4 = \frac{-198}{54} = \frac{-22}{6} = -\frac{11}{3}$

$$\underline{x} = \begin{bmatrix} 1/9 \\ 0 \\ 2/3 \\ -\frac{11}{3} \end{bmatrix}$$ ✓