EE431/438 Economics of Financial Markets and Institutions

Exercise 3: Mean-Variance Analysis

1. Suppose that there are two securities in the market. The risk free rate is equal to 10%

	Expected Return	Standard Deviation
Security A	20%	30%
Security B	12%	15%

The correlation between the return of security A and security B is (-0.10).

(a) Calculate the Sharpe ratio for each of the two assets.

The Sharpe ratio for asset i is defined by $s_i = \frac{E(R_i) - R_f}{\sigma_i}$. In words, it is the excess of rate of return above the risk-free rate divided by the standard deviation of return for each asset. The Sharpe ratio can be interpreted for each asset as its excess rate of returns per unit of standard deviation. Sharpe ratio is useful as it makes all assets comparable with a unit of standard deviation.

The Sharpe ratio for asset A : $s_A = \frac{20 - 5}{30} = 0.50.$

Alternatively, $s_A = \frac{0.20 - 0.50}{0.30} = 0.50.$

The Sharpe ratio for asset B : $s_B = \frac{12 - 5}{15} = 0.467.$

Alternatively, $s_A = \frac{0.12 - 0.10}{0.15} = 0.467.$

(b) Find the portfolio which has the minimum variance.

The optimal percentage to invest in security A in order to obtain the minimum variance portfolio is:

$$a^* = \frac{\sigma_B^2 - r_{AB}\sigma_A\sigma_B}{\sigma_A^2 + \sigma_B^2 - 2r_{AB}\sigma_A\sigma_B} = \frac{(15)^2 - (-0.1)(30)(15)}{(30)^2 + (15)^2 - 2(-0.1)(30)(15)} = 0.22.$$

Alternatively, $a^* = \frac{(0.15)^2 - (-0.1)(0.30)(0.15)}{(0.30)^2 + (0.15)^2 - 2(-0.1)(0.30)(0.15)} = 0.22.$

(c) Let portfolio C consisting of f 50% in security A and 50% in security B. Calculate expected rate of return and variance of the portfolio C.

$E(R_p) = aE(R_A) + (1-a)E(R_B) = 0.5 * 20 + 0.5 * 12 = 16\%$

$VAR(R_p) = a^2 VAR(R_A) + (1-a)^2 VAR(R_B) + 2a(1-a)r_{AB}\sigma_A\sigma_B = (0.5)^2(30)^2 + (0.5)^2(15)^2 + 2(0.5)(0.5)(-0.1)(30)(15) = 191.25$

$\sigma_p = \sqrt{191.25} = 13.83\%$

Alternatively, $E(R_p) = aE(R_A) + (1-a)E(R_B) = 0.5 * 0.20 + 0.5 * 0.12 = 0.16 = 16\%$

$VAR(R_p) = a^2 VAR(R_A) + (1-a)^2 VAR(R_B) + 2a(1-a)r_{AB}\sigma_A\sigma_B = (0.5)^2(0.30)^2 + (0.5)^2(0.15)^2 + 2(0.5)(0.5)(-0.1)(0.30)(0.15) = 0.019125$

$\sigma_p = \sqrt{0.019125} = 0.1383 = 13.83\%$

(d) Suppose that Mr.Smith choose to invest 70% in portfolio of risky asset C and 30% in the risk-free asset. What is the expected rate of return and standard deviation of the rate of return on his portfolio?

$E(R_p) = aE(R_C) + (1-a)R_f = 0.7 * 16 + 0.5 * 5 = 13.7\%$
$VAR(R_p) = a^2 VAR(R_C) = (0.7)^2(191.25) = 93.125$
$\sigma_p = \sqrt{93.125} = 9.68\%$

Alternatively, $E(R_p) = aE(R_C) + (1-a)R_f = 0.7 * 0.16 + 0.5 * 0.05 = 0.137 = 13.7\%$
$VAR(R_p) = a^2 VAR(R_C) = (0.7)^2(0.019125) = 0.0093125$
$\sigma_p = \sqrt{0.0093125}0.0968 = 9.68\%$

(e) Suppose that Mrs.Smith wants to invest a proportion of her total investment budget in risky portfolio C. If she wish her investmet portfolio to have an expected rate of return of 17%, what proportion should she invest in the risky portfolio, C ? and what proportion in the risk-free asset?

$E(R_p) = aE(R_C) + (1-a)R_f = a * 16 + (1-a) * 5 = 17: \ a = \frac{12}{11}, 1-a = -\frac{1}{11}$

Alternatively, $E(R_p) = aE(R_C)+(1-a)R_f = a*0.16+(1-a)*0.05 = 0.17 : a = \frac{0.12}{0.11} = \frac{12}{11}$

Therefore, she should invest $-\frac{1}{11}$ of her fund in risk-free asset. In other words, she should borrow $-\frac{1}{11}$ of her funds from the financial market at the risk-free rate. Then, she would have $\frac{12}{11}$ of her initial funds to invest in the risky portfolio C.