Tentative Solution for the Quiz.

Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score.....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c. **Please read and answer the following questions carefully and completely.**

Score.....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

Based on the model:

we know that

$$\frac{1}{R_f} = \delta E\left[\frac{U'(C_1)}{U'(C_0)}\right]$$

Since

$$U'(C_1) = \frac{1}{C_1}$$

$$U'(C_0) = \frac{1}{C_0}$$

$$\frac{1}{R_f} \uparrow = \delta E\left[\frac{C_0}{C_1}\right] \uparrow$$

so that, when the interest is high, so will be the expected growth in the consumption.

Score......................

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

we know that the elasticity of intertemporal substitution can be defined as:

$$\varepsilon = \frac{\partial \ln(c_1/c_0)}{\partial \ln(R_f)}$$

Therefore, from question ①, if we take logs both sides.

$$\ln(R_f) = -\ln\delta + \ln\left(\frac{c_1}{c_0}\right)$$

$$\therefore \ \varepsilon = 1$$

$\Rightarrow$ a change in the interest rate has no effect on savings.

Score....................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

For question 1.3. We know that the pricing kernel can be explained as:

$$P_i = E\left[\delta \frac{U'(C_1)}{U'(C_0)} X_i\right]$$

$$P_i = E\left[\delta \left[\frac{C_0}{C_1}\right] X_i\right]$$

the marginal rate of substitution b/w initial and the end-of period consumption

in the state that C_1 ↑↑ → marginal utility ↓↓↓ c why? → Law of diminishing marginal utility) then the asset's payoffs in this state is not highly valued. → ∴ P_i ↓↓

Score.....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

we know that if C_1/C_0 is assumed to be lognormally distributed, with μ_c and σ_c

then

$$\left|\frac{E[R_i]-R_f}{\sigma_{R_i}}\right| \leq \frac{\sigma_{m01}}{E[m_{01}]}$$

$$\xrightarrow{} m_{01} = \delta\left(\frac{C_1}{C_0}\right) = \delta e^{\ln(C_1/C_0)}$$

$$\leq \frac{\sqrt{var[e^{\ln(C_1/C_0)}]}}{E[e^{\ln(C_1/C_0)}]}$$

$\rightarrow$ from formula: $var[X] = E[X^2] - E[X]^2$

$$\leq \frac{\sqrt{E[e^{2\ln(C_1/C_0)}] - E[e^{\ln(C_1/C_0)}]^2}}{E[e^{\ln(C_1/C_0)}]}$$

$$\leq \sqrt{E\left[e^{2\ln(c_1/c_0)}\right] / E\left[e^{\ln(c_1/c_0)}\right]^2 - 1}$$

$$\leq \sqrt{e^{\cancel{2\mu_c} + 2\sigma_c^2} / e^{\cancel{2\mu_c} + \sigma_c^2} - 1}$$

$$\leq \sqrt{e^{\sigma_c^2} - 1} \approx \sigma_c$$

therefore,

$$\left| \frac{E[R_i] - R_f}{\sigma_{R_i}} \right| \leq \sigma_c$$

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the sharpe ratio cannot exceed the stand deviation of consumption growth.

↳ when we match is HB withe the real-data, then we figure out that the sharpe ratio is

... contradict to the

greater than σ_c which contradict to the condition! $\Rightarrow$ we then called it as "Equity premium Puzzle"

Ellen S.

April 19, 2022.