1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

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Table 1

Student	Y_i	X_i	Y_iX_i	X_i^2	$X_i - \bar{X}$	$(X_i-\bar{X})^2$
1	2.8	63	176.4	3969	-14.625	213.8906
2	3.4	72	244.8	5184	-5.625	31.6406
3	3.0	78	234.0	6084	0.375	0.1406
4	3.5	81	283.5	6561	3.375	11.3906
5	3.6	87	313.2	7569	9.375	87.8906
6	3.0	75	225	5625	-2.625	6.8906
7	2.7	75	202.5	5625	-2.625	6.8906
8	3.7	90	333.0	8100	12.375	153.1406
sum	25.7	621	2012.4	48,717	0	511.8748
Mean	3.2125	77.625	251.55	6089.63		

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

$$\hat{\beta}_2 = \frac{n\sum X_iY_i - \sum X_i \sum Y_i}{n\sum X_i^2 - (\sum X_i)^2} = \frac{8(2012.4)-(621)(25.7)}{8(48717)-(621)^2}$$

$= 0.0341$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$= 3.2125 - (0.0341)(77.625)$

$= 0.5655$

$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

$\hat{Y}_i = 0.5655 + 0.0341X_i$

- $\hat{\beta}_1 = 0.5655$, GPA of student is 0.5655 when total exam score is zero.
- $\hat{\beta}_2 = 0.0341$, total score change by 1 point, on average, student's GPA will change by 0.0341.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$ $\quad \hat{Y}_i = 0.5655 + 0.0341X_i$

Student	Y_i	X_i	$\hat{Y}_i$	$\hat{u}_i = Y_i - \hat{Y}_i$	$\hat{u}_i^2$
1	2.8	63	2.7138	0.0862	0.0074
2	3.4	72	3.0207	0.38	0.1444
3	3.0	78	3.2253	-0.23	0.0529
4	3.5	81	3.3276	0.17	0.0289
5	3.6	87	3.5322	0.07	0.0049
6	3.0	75	3.123	-0.12	0.0144
7	2.7	75	3.123	-0.42	0.1764
8	3.7	90	3.6345	0.07	0.0049
				$\sum \hat{u}_i = 0.01$ $\sum \hat{u}_i \approx 0$	$\sum \hat{u}_i^2 =$ 0.4342

$\therefore \sum \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.4342}{8-2} = 0.0724$$

$$var(\hat{\beta}_1) = \frac{\hat{\sigma}^2 \sum X_i^2}{n\sum(X_i - \bar{X})^2} = \frac{0.0724(48717)}{8(511.8748)} = 0.8613$$

$$var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum(X_i-\bar{X})^2} = \frac{0.0725}{511.875} = 0.000142$$

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2. Data is listed in the table

$\bar{X} = \frac{\sum X_i}{n} = \frac{200}{10} = 20$

$\bar{Y} = \frac{\sum Y_i}{n} = \frac{91}{10} = 9.1$

X_i	Y_i	$X_i - \bar{X}$	$(X_i-\bar{X})^2$	$Y_i - \bar{Y}$	$(X_i-\bar{X})(Y_i-\bar{Y})$
10	0	−10	100	−9.1	91
12	2	−8	64	−7.1	56.8
14	5	−6	36	−4.1	24.6
16	6	−4	16	−3.1	12.4
18	7	−2	4	−2.1	4.2
22	10	2	4	0.9	1.8
24	10	4	16	0.9	3.6
26	15	6	36	5.9	35.4
28	16	8	64	6.9	55.2
30	20	10	100	10.9	109
$\sum X_i = 200$	$\sum Y_i = 91$	$\sum(X_i-\bar{X}) = 0$	$\sum(X_i-\bar{X})^2 = 440$	$\sum(Y_i-\bar{Y}) = 0$	$\sum(X_i-\bar{X})(Y_i-\bar{Y}) = 394$

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

$\hat{\beta}_2 = \frac{\sum(X_i-\bar{X})(Y_i-\bar{Y})}{\sum(X_i-\bar{X})^2} = \frac{394}{440} = 0.8955$

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X} = 9.1 - (0.8955)(20) = -8.81$

$\hat{Y}_i = -8.81 + 0.8955X_i$

$\therefore \hat{\beta}_2 = 0.8955$ implies that if X change by 1 unit on average, Y will change by 0.8955 unit in the same direction.

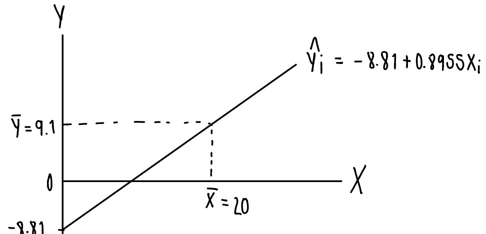
2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

$\hat{Y}_i = -8.81 + 0.8955X_i$

X_i	Y_i	$\hat{Y}_i$	$\hat{u}_i = Y_i - \hat{Y}_i$	$\hat{u}_i^2$	X_i^2
10	0	0.145	−0.145	0.0210	100
12	2	1.936	0.064	0.0041	144
14	5	3.727	1.273	1.6205	196
16	6	5.518	0.482	0.2323	256
18	7	7.309	−0.309	0.0955	324
22	10	10.891	−0.891	0.7939	484
24	10	12.682	−2.682	7.1931	576
26	15	14.473	0.527	0.2777	676
28	16	16.264	−0.264	0.0697	784
30	20	18.055	1.945	3.7830	900
			$\sum \hat{u}_i \approx 0$	$\sum \hat{u}_i^2 = 14.0908$	$\sum X_i^2 = 4440$

$\therefore \sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



$\hat{Y}_i = -8.81 + 0.8955X_i$

$\bar{Y} = -8.81 + 0.8955(\bar{X})$

$\bar{Y} = -8.81 + 0.8955(20)$

$\bar{Y} = 9.1$

$\therefore$ The regession line passes through the sample means of x and y.

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2.4 If $X_i = 18$, what is the predicted Y?

$$\hat{Y}_i = -8.81 + 0.8955X_i$$
$$\hat{Y}_i = -8.81 + 0.8955(18)$$
$$\hat{Y}_i = 7.309$$

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.0908}{10-2} = 1.7614$$

$$var(\hat{\beta}_1) = \frac{\hat{\sigma}^2 \sum X_i^2}{n\sum(X_i - \bar{X})^2} = \frac{(1.7614)(4440)}{10(440)} = 1.7774$$

$$var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum(X_i - \bar{X})^2} = \frac{1.7614}{440} = 0.004$$

3. Consider the below regression function: consider the two-variable model

$$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator.
Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

$$\bar{Y} = \hat{\beta}_1 + \hat{\beta}_2\bar{X}, \quad \hat{\beta}_2 = \sum k_i Y_i$$
$$\hat{\beta}_1 = \bar{Y} - (\sum k_i Y_i)\bar{X}$$
$$\hat{\beta}_1 = \frac{\sum Y_i}{n} - \sum \bar{X} k_i Y_i$$
$$\hat{\beta}_1 = \sum\left(\frac{Y_i}{n} - \bar{X} k_i Y_i\right)$$
$$\hat{\beta}_1 = \sum\left(\frac{1}{n} - \bar{X} k_i\right)Y_i$$

$\therefore \hat{\beta}_1$ is linear.

$$Y_i = \beta_1 + \beta_2 X_i + \mu_i \quad \text{—} ①$$
$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X} \quad \text{—} ②$$
$$\bar{Y} = \beta_1 + \beta_2\bar{X} + \bar{\mu} \quad \text{—} ③$$

② into ③; $\hat{\beta}_1 = \beta_1 + \beta_2\bar{X} + \bar{\mu} - \hat{\beta}_2\bar{X}$

$$\hat{\beta}_1 = \beta_1 + (\beta_2 - \hat{\beta}_2)\bar{X} + \bar{\mu}$$
$$E(\hat{\beta}_1) = E(\beta_1) + \bar{X}E(\beta_2 - \hat{\beta}_2) + E(\bar{\mu})$$
$$E(\hat{\beta}_1) = \beta_1$$

$\therefore \hat{\beta}_1$ is unbiased estimator.