Name:...ID...........................

Quiz 1

1. Consider the following function:

$$f(x) = \begin{cases} \frac{2x^3+|x|}{2-|x|^3}, & \text{for} \quad x \le -1 \\ \\ (x+1)^2 \sin(\frac{1}{3x+3}) & \text{for} \quad x > -1. \end{cases}$$

   Find $\lim\limits_{x \to -1} f(x)$.

Solution:

Note that $|x| = \begin{cases} -x, & \text{for} \quad x < 0 \\ \\ x, & \text{for} \quad x > 0 \end{cases}$ and since we consider x approaching negative number -1 ($x \to -1$) from both left and right, we use $|x| = -x$. So the left-handed limit can be computed as shown below.

$$\begin{aligned} \lim_{x \to -1^-} f(x) &= \lim_{x \to -1^-} \frac{2x^3+|x|}{2-|x|^3} \\ &= \lim_{x \to -1^-} \frac{2x^3-x}{2-(-x)^3} \\ &= \frac{2(-1)^3-(-1)}{2-(-(-1))^3} = \frac{-1}{1} = -1. \end{aligned}$$

Note that this can be computed directly using

$$\lim_{x \to -1^-} \frac{2x^3+|x|}{2-|x|^3} = \frac{2(-1)^3+|-1|}{2-|-1|^3} = \frac{-1}{1} = -1.$$

The right-hand limit $\lim\limits_{x \to -1^+} f(x)$ can be computed using the Squeeze Theorem.

$$\begin{array}{rcl} -1 \le & \sin(\frac{1}{3x+3}) & \le 1 \\ -(x+1)^2 \le & (x+1)^2 \sin(\frac{1}{3x+3}) & \le (x+1)^2 \end{array}$$

Since
$\lim\limits_{x \to -1^+} [-(x+1)^2] = -(-1+1)^2 = 0$, and
$\lim\limits_{x \to -1^+} (x+1)^2 = (-1+1)^2 = 0$,
then by the Squeeze Theorem,

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (x+1)^2 \sin(\frac{1}{3x+3}) = 0.$$

Since $\lim\limits_{x \to -1^-} f(x) \neq \lim\limits_{x \to -1^+} f(x)$, then the limit $\lim\limits_{x \to -1} f(x)$ does not exist (DNE).
Note: We **cannot** use

$$\lim_{x \to -1^+} (x+1)^2 \sin(\frac{1}{3x+3}) = \left(\lim_{x \to -1^+} (x+1)^2 \right) \left(\lim_{x \to -1^+} \sin(\frac{1}{3x+3}) \right)$$

since $\lim\limits_{x \to -1^+} \sin(\frac{1}{3x+3})$ does not exist.

$\blacksquare$