

Instructions

- (1) Please read the instruction carefully. Also take this habit with you into the exam room.
- (2) Please read each question carefully and answer the questions straightforwardly. Always provide economic reasons at least a paragraph for your analysis, or a graph when necessary, even when the question does not indicate so.
- (3) Handing and submitting assignments are only available via BE Moodle.

Answering the questions and preparing answer sheets

- (1) Answers are to be handwritten, in either digital or analog form, in a blank canvas or any clean paper. Make sure that your handwriting is clearly visible and readable.
- (2) There is no need to rewrite the question. Just indicate the question number clearly for each of the answer, such as 1.a).
- (3) Default decimal point is 4.
- (4) Choose precise wordings, especially when you want to interpret the meaning of a test, confidence interval, or coefficients.
- (5) When done, for the digital case, collage all the pages into a single PDF file. For those who write on sheets of paper, take photo of all pages then convert all of them into a single PDF file as well.
- (6) Name your PDF file as StudentID_YourNickname, such as 640123456_Bo.

Submitting your answers

- (1) Make sure your file does not exceed 10MB. This is the maximum file size for BE Moodle upload.
- (2) Login to BE Moodle, head into the course, then the assignment topic.
- (3) Choose your file to submit. Done. There will be timestamp for your upload date and time, so please make sure to not submit later than that.

Question 1.**Effects of Physical Attractiveness on Wage**

Hamermesh and Biddle (1994) used measures of physical attractiveness in a wage equation. Each person in the sample was ranked by an interviewer for physical attractiveness using five categories (homely, quite plain, average, good looking, and strikingly beautiful or handsome). Because there are so few people at the two extremes, the authors put people into one of three groups for the regression analysis: “average”, “below average”, and “above average”, where **the base or reference group is “average”**. Using data from the 1977 Quality of Employment Survey, after controlling for the usual productivity characteristics, the following two regressions were estimated using data on $n = 1,260$:

Estimate the model (1.1) reports in the Table 1.1

$$\log(wage_i) = \beta_1 + \beta_2 educ_i + \beta_3 exper_i + \beta_4 expersq_i + \beta_5 union_i + \beta_6 female_i + u_i \quad (1.1)$$

Table 1.1

Source	SS	df	MS	Number of obs	=	1,260
Model	166.011417	5	33.2022834	F(5, 1254)	=	149.25
Residual	278.96855	1,254	.222462959	Prob > F	=	0.0000
				R-squared	=	0.3731
				Adj R-squared	=	0.3706
Total	444.979967	1,259	.353439211	Root MSE	=	.47166

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]
educ	.0708503	.0052325			
exper	.0389808	.0043524			
expersq	-.0005986	.0000975			
union	.1924593	.0301994			
female	-.4421609	.0289766			
_cons	.443611	.078859			

Estimate the model (1.2) reports in the Table 1.2

$$\log(wage_i) = \beta_1 + \beta_2 educ_i + \beta_3 exper_i + \beta_4 expersq_i + \beta_5 union_i + \beta_6 female_i + \beta_7 belavg_i + \beta_8 abvavg_i + u_i \quad (1.2)$$

where $\log(wage_i)$ or $lwage$ = logarithm of hourly wage (in USD)

- $educ_i$ = years of schooling
- $exper_i$ = years of workforce experience
- $expersq_i$ = years of workforce experience squared
- $union_i$ = 1 if union member
- $female_i$ = 1 if female
- $belavg_i$ = 1 if in below average physical attractiveness
- $abvavg_i$ = 1 if in above average physical attractiveness

Assignment 2

Assigned on Nov 9th, 2021. Due date Nov 25th, 2021 before midnight.**Table 1.2**

Source	SS	df	MS	Number of obs	=	1,260
Model	168.697151	7	24.099593	F(7, 1252)	=	109.21
Residual	276.282816	1,252	.220673176	Prob > F	=	0.0000
				R-squared	=	0.3791
				Adj R-squared	=	0.3756
Total	444.979967	1,259	.353439211	Root MSE	=	.46976

lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]
educ	.0691306	.00525			
exper	.0395785	.0043428			
expersq	-.0006081	.0000971			
union	.1884632	.0301843			
female	-.4388235	.028877			
belavg	-.1388291	.0417749			
abvavg	.0070104	.0302809			
_cons	.4737302	.0795614			

Omitted for the purpose of this exam

Answer the following questions.

1.a) Based on the regression results provided, write out the estimated coefficients in the form of regression equation (1.1). Interpret the estimated coefficients associated with $educ_i$. Based on Model (1.1), test whether education has an impact on logarithm of hourly wage. Show your work. (Use $\alpha = 0.05$)

1.b) What is the overall significance of the regression from Model (1.2)? What test do you use? (Use $\alpha = 0.05$)

1.c) If we are interested in testing whether “physical attractiveness” has an impact on logarithm of hourly wage at all, what kind of null/alternative hypothesis would we be testing? Perform the test and discuss your finding. (Use $\alpha = 0.05$)

1.d) Is there convincing evidence that women with above average looks earn more than women with average looks? Explain.

1.a) Based on the regression results provided, write out the estimated coefficients in the form of regression equation (1.1). Interpret the estimated coefficients associated with $educ_i$. Based on Model (1.1), test whether education has an impact on logarithm of hourly wage. Show your work. (Use $\alpha = 0.05$)

$$\log(wage_i) = \beta_1 + \beta_2 educ_i + \beta_3 exper_i + \beta_4 expersq_i + \beta_5 union_i + \beta_6 female_i + u_i \quad (1.1)$$

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lwage	Coefficient	Std. err.	t	P> t	[95% conf. interval]
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expersq	-.0005986	.0000975			
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① estimated coefficients

$$\log(\widehat{wage}) = 0.4436 + 0.0785 edu + 0.3898 exper + (-0.0006) expsq + 1.9246 union + (-0.4422) female$$

② Interpret the estimated coefficients associated with $educ_i$

$B = 0.0709$ if education increase by 1 year, wage will increase by $0.0709 \times 100 \approx 7\%$
(log-linear)

③ Test hypothesis (t-test)

step 1 State your hypothesis

$$H_0 : \beta_{edu} = 0$$

$$H_a : \beta_{edu} \neq 0$$

step 2 calculate test statistics

$$t = \frac{\hat{\beta}_{edu} - \beta_{edu}}{se \beta_{edu}} = \frac{0.07 - 0}{0.00523} = 13.38 \quad \text{reject } H_0$$

step 3 Pick an α and state decision rules

$$\alpha = 0.05$$

$$0.05 / 2 = 0.025$$

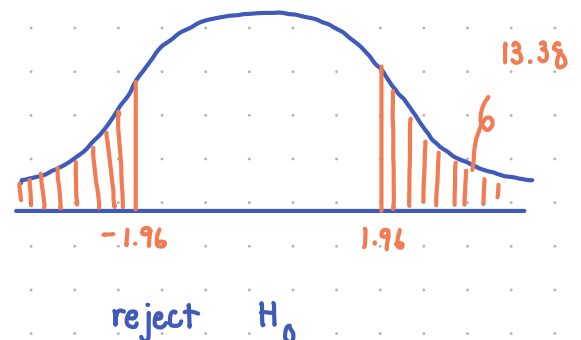
$$df = 1260 - 6$$

$$= 1254$$

$$t_{lower} = -1.96$$

$$t_{upper} = 1.96$$

step 4 Concluding



1.b) What is the overall significance of the regression from Model (1.2)? What test do you use?
(Use $\alpha = 0.05$)

Table 1.2

Source	SS	df	MS	Number of obs	=	1,260
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step 1 state your hypothesis

$$H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$$

H_a : otherwise

step 2 calculate test statistics

$$F_{cal} = \frac{ESS/df}{RSS/df} = \frac{ESS/k-1}{RSS/n-k}$$

$$= \frac{24.0996}{0.2207} = 109.1962$$

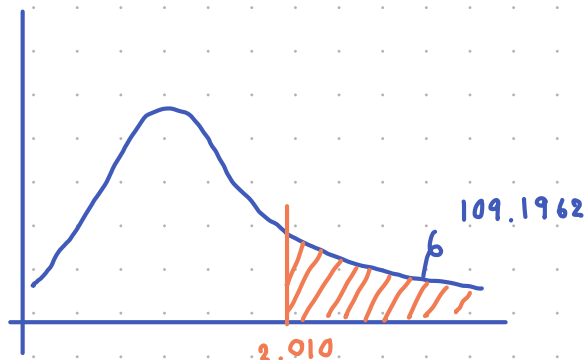
step 3 Pick an α and state decision rules

$$\alpha = 0.05 \quad F_{upper, \alpha}(7, 1252)$$

$$F\text{-table} = 2.010$$

step 4 Conclude

$\therefore$ Reject H_0 , use F-test



1.c) If we are interested in testing whether “physical attractiveness” has an impact on logarithm of hourly wage at all, what kind of null/alternative hypothesis would we be testing? Perform the test and discuss your finding. (Use $\alpha = 0.05$)

Table 1.1

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physical
Att

Omitted for the purpose of this exam

F testing

$$F = \frac{[R^2_{UR} - R^2_R]/q}{[1 - R^2_{UR}]/[n - k_{UR}]}$$

step 1 state hypothesis

$$H_0 : \beta_{be} = \beta_{ab} = 0$$

H_1 : otherwise

step 2 calculate test

$$F = \frac{[R^2_{UR} - R^2_R]/q}{[1 - R^2_{UR}]/[n - k_{UR}]}$$

$$= \frac{[0.3791 - 0.3731]/2}{[1 - 0.3791]/[1260 - 8 - 1]}$$

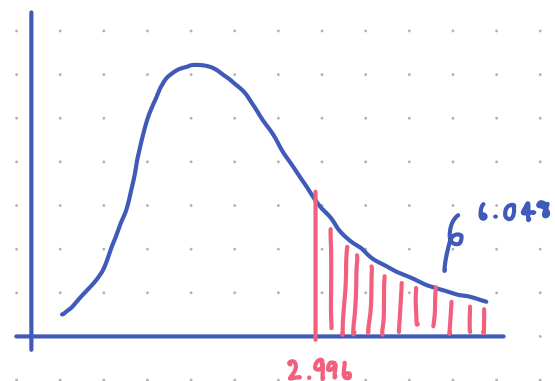
$$= \frac{0.003}{[0.6209]/1251} = \frac{0.003}{0.000496} = 6.048$$

step 3 Pick an α and state decision

$$\alpha = 0.05 \quad df = 1251$$

$$F_{upper} = 2.996$$

step 4 conclude



Reject H_0

$\therefore$ Physical Attractiveness affect to wage
at $\alpha = 0.05$

1.d) Is there convincing evidence that women with above average looks earn more than women with average looks? Explain.

Table 1.2

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from 1.c) table 1.2 is better

$$\beta_{\text{female}} = -0.43 \rightarrow \text{male} = 0.43 \times 100 = 43\%$$

step 1 state Hypothesis

$$H_0: \beta_{\text{female}} \geq 0$$

$$H_1: \beta_{\text{female}} < 0$$

step 3 Pick an α and state decision

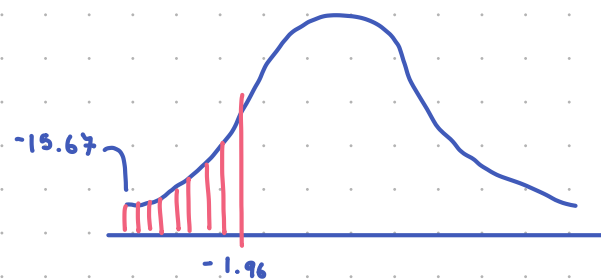
$$\alpha = 0.05 \quad df = n - k = 1260 - 8 = 1252$$

$$t = -1.96$$

step 2 cal t-test

$$t = \frac{-0.4388 - 0}{0.028} = -15.67$$

step 4 conclude



reject H_0

$\therefore$ wage of female is less than male at 0.05 significant.

Question 2.

A household expenditure model is given by

$$hhexp_i = \beta_1 + \beta_2 area_i + \beta_3 child_i + u_i$$

where $hhexp_i$ = household expenditure per month
 $area_i$ = a dummy variable for household location:
 (0 if in a municipal area and 1 if otherwise)
 $child_i$ = number of children in household i , aged under 15

Using socio-economic dataset collected in 2018 with 14,908 households, the result is given below with **t value in parentheses**. Answer the following questions.

$$\widehat{hhexp}_i = 9,736 - 2,835area_i + 881child_i + \hat{u}_i$$

(43.83) (-15.8) (6.82)

- 2.a)** Do all the signs for each coefficient make economic sense? Explain.
- 2.b)** Test each parameter separately if they are significantly different from zero or not. (Use $\alpha = 0.01$)
- 2.c)** Find the expected value of a household expenditure not living in a municipal area with 3 children aged under 15.
- 2.d)** When an interaction term is included in this model, the result becomes with **t value in parentheses**.

$$\widehat{hhexp}_i = 9,693 - 2,742area_i + 910child_i - 64(area_i * child_i) + \hat{u}_i$$

(34.38) (-6.55) (5.17) (-0.25)

Draw a diagram for this model displaying sampled regression functions (SRF) with expected value of household expenditure on the vertical axis and number of children on the horizontal axis, taking **only significant parameter(s)** into account. Indicate the intercept and slope for each SRF where applicable. Testing of significance can be shortened.

Question 2.

A household expenditure model is given by

$$hhexp_i = \beta_1 + \beta_2 area_i + \beta_3 child_i + u_i$$

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$$\widehat{hhexp}_i = 9,736 - 2,835 area_i + 881 child_i + \hat{u}_i$$

(43.83) (-15.8) (6.82)

2.a) Do all the signs for each coefficient make economic sense? Explain.

$\beta_{area} = -2835$, living in municipal = 0
otherwise = 1

β_{area} tell the different between living in municipal and otherwise so it makes economic sense because living in municipal $hhexp$ should be greater than living in other area (β_{area} negative is make sense!)

$\beta_{child} = 881$

House hold will have a lot of $hhexp$ if they have many kids (school, foods)
Sign of coefficient is positive. make sense!

$\therefore$ All signs for each coefficient are make economic sense

2.b) Test each parameter separately if they are significantly different from zero or not. (Use $\alpha = 0.01$)

step 1 state hypothesis

For β_1 $H_0 : \beta_1 = 0$
 $H_a : \beta_1 \neq 0$

For β_2 $H_0 : \beta_2 = 0$
 $H_a : \beta_2 \neq 0$

For β_3 $H_0 : \beta_3 = 0$
 $H_a : \beta_3 \neq 0$

step 2 Calculate

$$t_{cal}(\beta_i) = \frac{\hat{\beta}_i - \beta_i}{se \hat{\beta}_i}$$

For β_1 $t_{cal}(\beta_1) = 43.83$

For β_2 $t_{cal}(\beta_2) = -15.8$

For β_3 $t_{cal}(\beta_3) = 6.89$

step 3 pick α and state decision

$$\alpha = 0.01 \rightarrow \alpha/2 = 0.005$$

$$df = n - k = 14908 - 3 = 14905$$

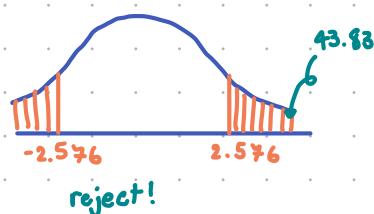
$$\text{from } t\text{-table} = 2.576$$

$$t_{\text{lower}} = -2.576$$

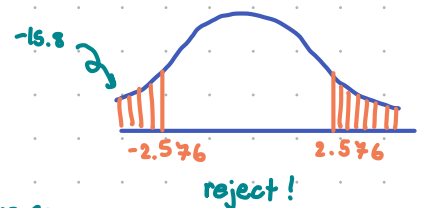
$$t_{\text{upper}} = 2.576$$

step 4 Conclude

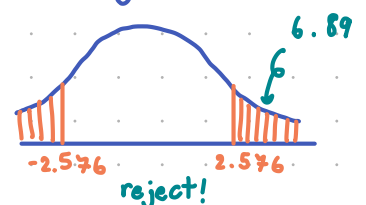
For β_1



For β_2



For β_3



2.c) Find the expected value of a household expenditure not living in a municipal area with 3 children aged under 15.

$$\begin{aligned}\hat{hhexp} &= 9736 - 2835(1) + 881(3) \\ &= 9736 - 2835 + 2643 \\ &= 9544\end{aligned}$$

∴ The expected value of household expenditure is 9544.

2.d) When an interaction term is included in this model, the result becomes with **t value in parentheses**.

$$\begin{aligned}\widehat{hhexp}_i &= 9,693 - 2,742area_i + 910child_i - 64(area_i * child_i) + \hat{u}_i \\ &\quad (34.38) \quad (-6.55) \quad (5.17) \quad (-0.25)\end{aligned}$$

Draw a diagram for this model displaying sampled regression functions (SRF) with expected value of household expenditure on the vertical axis and number of children on the horizontal axis, taking **only significant parameter(s)** into account. Indicate the intercept and slope for each SRF where applicable. Testing of significance can be shortened.

Test significant $\alpha = 0.05$ from t-table = 1.96
 → insignificant (absolute of t-test of interaction term < 1.96)

$$\therefore \hat{hhexp} = 9693 - 2742 area + 910 child$$

Municipal area = 0

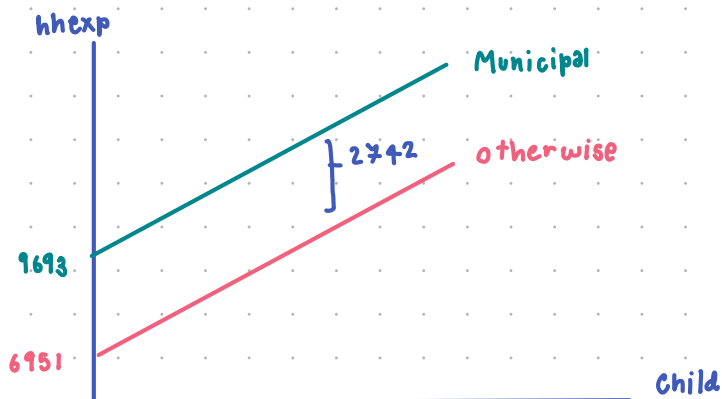
$$\hat{hhexp} = 9693 - 2742(0) + 910 child$$

$$\hat{hhexp} = 9693 + 910 child$$

otherwise area = 1

$$\hat{hhexp} = 9693 - 2742(1) + 910 child$$

$$\hat{hhexp} = 6951 + 910 child$$



Question 3.

Assume a multiple linear regression model as

$$hours_i = \beta_1 + \beta_2 sex_i + \beta_3 age_i + \beta_4 agesq_i + \beta_5 weekot_i + u_i$$

where $hours_i$ is hours worked in a week
 sex_i is a dummy variable: 0 = male and 1 = otherwise
 age_i is age of observation i
 $agesq_i$ is age square observation i
 $weekot_i$ is nominal overtime paid per week

Answer the following questions.

3.a) A VIF and tolerance table (postestimation) is given below

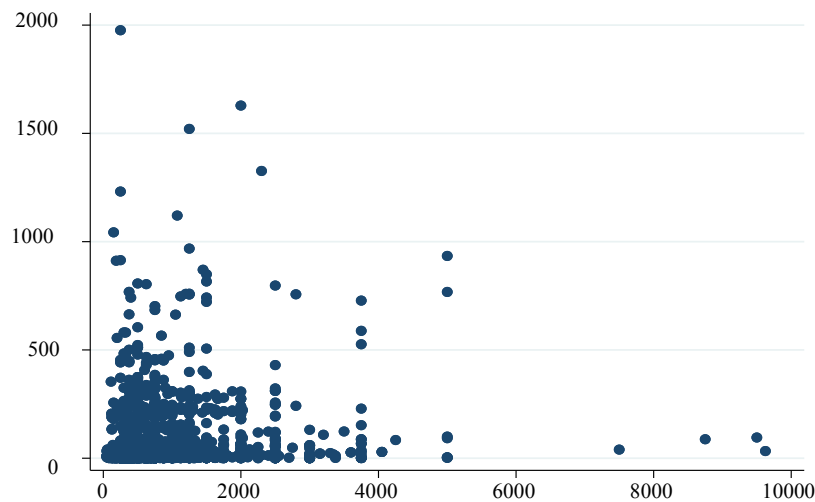
Variable	VIF	1/VIF
2.sex	1.02	0.979129
age	50.61	0.019759
agesq	50.68	0.019731
weekot	1.01	0.985618
Mean VIF	25.83	

Given that you are exploring multicollinearity assumption, which pair of variables that you suspect they might be linearly correlated? Provide clear explanation what criteria (ion) that you rely on making that judgement.

3.b) From **(3.a)**, do you consider removing one of the variables from the model? Why or why not and which one that you choose to remove, if that is the case?

3.c) The graph provided below is a scatter plot between $\hat{u}_i^2$ (vertical axis) and $weekot_i$ (horizontal axis). Using the graphical method, do you conclude that heteroscedasticity is present in this model or not. Explain clearly to support your answer.

Assignment 2

Assigned on Nov 9th, 2021. Due date Nov 25th, 2021 before midnight.

3.d) An auxiliary model here is estimated and the result is given in the table below.

$$\hat{u}_i^2 = \beta_1 + \beta_2 \text{sex}_i + \beta_3 \text{age}_i + \beta_4 \text{agesq}_i + \beta_5 \text{weekot}_i + v_i$$

Source	SS	df	MS	Number of obs	=	2,032
Model	829063.863	4	207265.966	F(4, 2027)	=	9.52
Residual	44148135	2,027	21780.037	Prob > F	=	0.0000
				R-squared	=	0.0184
				Adj R-squared	=	0.0165
Total	44977198.8	2,031	22145.3465	Root MSE	=	147.58

uhat2	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
2.sex	-5.648899	6.630832	-0.85	0.394	-18.65286	7.355058
age	-2.490434	2.37094	-1.05	0.294	-7.140168	2.1593
age2	.044175	.0301279	1.47	0.143	-.0149098	.1032599
weekot	.0229916	.0043502	5.29	0.000	.0144603	.0315229
_cons	83.8484	44.4418	1.89	0.059	-3.307973	171.0048

From the table, setup the hypotheses and perform the Breusch-Pagan test to check that heteroscedasticity is present in the original model or not.

3.a) A VIF and tolerance table (postestimation) is given below

Variable	VIF	1/VIF
2.sex	1.02	0.979129
age	50.61	0.019759
agesq	50.68	0.019731
weekot	1.01	0.985618
Mean VIF	25.83	

Given that you are exploring multicollinearity assumption, which pair of variables that you suspect they might be linearly correlated? Provide clear explanation what criteria (ion) that you rely on making that judgement.

$$VIF = \frac{1}{1 - r^2_{x_j}}$$

VIF > 5 [have at chance] multicollinearity

VIF > 10 Multicollinearity

from the table VIF of age and agesq > 10
so it must be Multicollinearity.

3.b) From (3.a), do you consider removing one of the variables from the model? Why or why not and which one that you choose to remove, if that is the case?

Detecting Multicollinearity

from 3c.) I found that age and agesq are multicollinearity
(VIF > 10) so I choose to remove β_{age} because

If don't remove some variable

- Wrong sign of value Coefficient
- Variance and sted are wrong!
- T-Test wrong!

3.c) The graph provided below is a scatter plot between $\hat{u}_i^2$ (vertical axis) and $weekot_i$ (horizontal axis). Using the graphical method, do you conclude that heteroscedasticity is present in this model or not. Explain clearly to support your answer.

yes, heteroscedasticity is detected in this model because as weekot increase, there are increase in $\hat{u}^2$ as well.

3.d) An auxiliary model here is estimated and the result is given in the table below.

$$\hat{u}_i^2 = \beta_1 + \beta_2 \text{sex}_i + \beta_3 \text{age}_i + \beta_4 \text{agesq}_i + \beta_5 \text{weekot}_i + v_i$$

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Total	44977198.8	2,031	22145.3465	Root MSE	=	147.58

uhat2	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
2.sex	-5.648899	6.630832	-0.85	0.394	-18.65286 7.355058
age	-2.490434	2.37094	-1.05	0.294	-7.140168 2.1593
age2	.044175	.0301279	1.47	0.143	-.0149098 .1032599
weekot	.0229916	.0043502	5.29	0.000	.0144603 .0315229
_cons	83.8484	44.4418	1.89	0.059	-3.307973 171.0048

From the table, setup the hypotheses and perform the Breusch-Pagan test to check that heteroscedasticity is present in the original model or not.

H_0 : the model is Homoscedasticity

H_a : otherwise

$$F_{cal} = \frac{R^2 \hat{u}_i^2 / k}{(1 - R^2 \hat{u}_i^2) / (n - k - 1)} = \frac{(0.01821) / 5}{(1 - 0.01821) / (2032 - 5 - 1)} = \frac{0.00368}{0.00048} = 7.6667$$

$$F_{cri}(5, 2026) = 2.2141$$

$$\alpha = 0.05$$

$$\therefore F_{cal} > F_{cri}$$

$\therefore$ we can reject H_0 , this means that 95 times of observation, Heteroscedast is present.