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Faculty of Economics **Thammasat university**

Midterm Exam **Semester 2/2015**

EE320 Introductory mathematical Economics

February, 8th 2016 **11:00 am – 01:00 pm**

Instructions:

1) There are ***<u>THREE</u>*** questions, each of which worth 40 points. You have 120 minutes to attempt all the questions.
2) This exam workbook has 20 pages in total.
3) Write down your seating number and student ID on *every single page* of this exam workbook.
4) You can only answer in the area provided in each part of the question.
5) Points indicated to each question are associated with the amount of time, in minutes, that you should allocate for attempting the question.
6) Simple calculator is allowed. This is a closed-book exam.
7) This exam is worth of 30% for the class's evaluation.
8) Good luck

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Question 1 *(40 points)* Suppose a monopolist faces with the market demand equation given by,

$$P = 40 + \frac{105}{Q} - 3Q^2$$

where P is the unit price and Q is the amount of quantity purchased. The monopolist is running the firm using the cost function given as follow,

$$C(Q) = 6Q^3 - 81Q^2 + 175Q + 10.$$

Consider the following questions.

1.1) (10 points) Determine the *level of revenue-maximizing output*, and calculate the value of the elasticity of demand at that level of output?

① $TR = 40Q + 105 - 3Q^3$

$\frac{dTR}{dQ} = 4Q - 9Q^2$

$\therefore \quad Q = \sqrt{\frac{40}{9}}, -\sqrt{\frac{40}{9}}$

which point $\frac{d^2TR}{dQ^2} = 4 - 18Q$

$\therefore \quad Q = \sqrt{\frac{40}{9}}$

② Elasticity of demand $= 1$ # by the property of maximum revenue

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1.2) (5 points) Construct the profit function

$\pi = TR - TC$

$= (40Q + 105 - 3Q^3) - (6Q^3 - 81Q^2 + 175Q + 10)$

$= -9Q^3 + 81Q^2 - 135Q + 95$

1.3) (15 points) Determine the profit-maximizing level of output. Confirm your result.

1-st $\frac{d\pi}{dQ} = -27Q^2 + 162Q - 135$

$\frac{d\pi}{dQ} = 0$ F.O.C.

$-27Q^2 + 162Q - 135 = 0$

$27Q^2 - 162Q + 135 = 0$

$(9Q - \cancel{3}\ 45)(3Q - 3) = 0$

$Q = 3, 5$ → which point.

2-nd test $\frac{d^2\pi}{dQ^2} = -54Q + 162$

$\therefore \left.\frac{d^2\pi}{dQ^2}\right|_{Q=5} < 0$ → $Q=5$ is profit max.

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1.4) (5 points) Calculate the level of maximized profit.

Calculate profit

$$\pi(Q) = -9(5)^3 + 81(5)^2 - 135(5) + 95$$
$$= -1125 + 2025 - 675 + 95$$
$$= 320 \quad \#$$

1.5) (5 points) Discuss the effect that would likely be happening if the government imposes a lump-sum tax on the monopolist.

$$\pi^{new}(Q) = \pi^{old}(Q) + \text{Lump-sum}$$
$$\therefore \frac{d\pi^{new}(Q)}{dQ} = \frac{d\pi^{old}(Q)}{dQ} \pm 0$$

① ∴ Q remains the same

② But profit drops by (−lump-sum Units)

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Question 2 *(40 points)* National income model

Consider a modified version of the IS-LM model where government spending is counter-cyclically tied to the level of GDP (Y).

$C = C_0 + C_1 Y_d - C_2 r, \qquad 0 < C_1 < 1$

$I = I_0 + I_1 Y - I_2 r, \qquad 0 < I_1 < 1$

$G = G_0 - G_1 Y, \qquad 0 < G_1 < 1$

$T = T_0$

$M^s = M_0$

$M^d = L_1 Y - L_2 r$

where C is the private consumption, I is the private investment, G is the government spending, T is tax, M^s is the level of money supply, M^d is the level of money demand, and r is the level of nominal interest rate. All the coefficients are <u>*non-negative*</u>, and that $C_1 + I_1 > G_1$. (All the variables are measured in the units of billion dollars.)

2.1) (5 points) Derive the IS and LM equation.

$y = C + I + G$

$y = C_0 + C_1(y - T_0) - C_2 r + I_0 + I_1 y - I_2 r + G_0 - G_1 y$

IS: $\rightarrow y = \dfrac{1}{1-(C_1+I_1-G_1)}\left[C_0 - C_1 T_0 + I_0 + G_0 - (C_2 + I_2) r\right]$

LM: $\rightarrow \quad M_0 = L_1 y - L_2 r$

$\rightarrow$ ~~r~~

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IS : $\left[1-(C_1+I_1-G_1)\right]y+(C_2+I_2)r = (C_0-C_1T_0+I_0+G_0)$

LM : $L_1y-L_2r = M_0$

2.2) (5 points) Write the IS-LM equation in terms of the matrix representation, *and* state the condition that warrants the *uniqueness* of the solution.

$$\begin{pmatrix} 1-(C_1+I_1-G_1) & C_2+I_2 \\ L_1 & -L_2 \end{pmatrix}\begin{pmatrix} y \\ r \end{pmatrix} = \begin{pmatrix} C_0-C_1T_0+I_0+G_0 \\ M_0 \end{pmatrix}$$

Uniqueness iff $|A| \neq 0$; i.e. A is non-singular

$-\left(L_2(1-(C_1+I_1-G_1))+(C_2+I_2)L_1\right) \neq 0$

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2.3) (10 points) Solve for the equilibrium GDP and interest rate (Y^*, r^*) using the Cramer's rule. (*Caution*: you will get **ZERO** if you don't use the Cramer's rule.)

$$y^* = \frac{\begin{vmatrix} C_0 - C_1T_0 + I_0 + G_0 & C_2 + I_2 \\ M_0 & -L_2 \end{vmatrix}}{\det(A)}$$

$$= \frac{-L_2(C_0 - C_1T_0 + I_0 + G_0) - M_0(C_2 + I_2)}{\det(A)}$$

$$= \frac{L_2(C_0 - C_1T_0 + I_0 + G_0) + M_0(C_2 + I_2)}{(C_2 + I_2)L_1 + L_2[1 - (C_1 + I_1 - G_1)]}$$

$$r^* = \frac{\begin{vmatrix} 1 - (C_1 + I_1 - G_1) & C_0 - C_1T_0 + I_0 + G_0 \\ L_1 & M_0 \end{vmatrix}}{\det(A)}$$

$$= \frac{-M_0(1 - (C_1 + I_1 - G_1)) + L_1(C_0 - C_1T_0 + I_0 + G_0)}{(C_2 + I_2)L_1 + L_2(1 - (C_1 + I_1 - G_1))}$$

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2.4) (10 points) Calculate the multiplier of G_0 and M_0 *on* Y^* and r^*. Discuss whether the multiplier is bigger or smaller than the case that government spending is strictly exogenous.

$$A = \frac{\Delta y^*}{\Delta G_0} = \frac{1}{(C_2+I_2)L_1+L_2(1-(C_1+I_1-G_1)]} > 0$$

$$B = \frac{\Delta y^*}{\Delta M_0} = \frac{C_2+I_2}{(C_2+I_2)L_1+L_2[1-(C_1+I_1-G_1)]} \geq 0$$

$$C = \frac{\Delta r^*}{\Delta G_0} = \frac{L_1}{(C_2+I_2)L_1+L_2[1-(C_1+I_1-G_1)]} \geq 0$$

$$D = \frac{\Delta r^*}{\Delta M_0} = -\frac{(1-(C_1+I_1-G_1))}{(C_2+I_2)L_1+L_2(1-(C_1+I_1-G_1))} < 0.$$

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The government is seeking for some advices on fiscal and monetary policy implementation. The goal of the government is to (i*) increase the real GDP (Y) by $1 billion*, while (ii) *keeping the current level of interest rate stayed the same.* (That is, the government was thinking that the country is running into an unemployment situation, but the level of interest rate is now optimal.) Following the storyline given here and all your work that you have done before, answer the next two questions.

2.5) (5 points) Can the government successfully achieve both goals by simply relying on *a single type of policy implemented*? That is, would it work to either change the *government expenditure* or *money supply*, but not both at the same time, to achieve the two goals? If yes, *under which conditions*?

Goals: $\Delta y > 0$ and $\Delta r = 0$

(i) Fiscal policy $\frac{\Delta r^*}{\Delta G} = 0$ only if $L_1 = 0$

(ii) Monetary policy $\frac{\Delta r^*}{\Delta M} = 0$ not possible as the question assume $1 - (C_1 + I_1 - G_1) > 0$

So, it is possible only if demand for money is independent of income!

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2.6) (5 points) If the condition that you assumed in (2.5) does not hold, what would you recommend to the government so that both goals can be simultaneously achieved? (Hint: think about an appropriate mixture of the two policies.)

If $L_1 = 0$ is not true.

$\Delta y = \text{multiplier}_G \cdot \Delta G + \text{multiplier}_m \cdot \Delta M_0$

$\Delta r = \text{multiplier}_m \cdot \Delta M_0 + \text{multiplier}_G \, \Delta G_0$

$$\Rightarrow \quad A \Delta G_0 + B \Delta M_0 = 1$$
$$C \cdot \Delta G_0 + D \Delta M_0 = 0$$

$$\therefore \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \Delta G_0 \\ \Delta M_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \Delta G_0 \\ \Delta M_0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \#$$

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Question 3 *(40 points)*

Consider a market with *10 identical consumers*. Each of the consumer's demand function is given by:

$$P = 1 + k_1 P_x + k_2 Y - Q_j^d,$$

where P is the unit price of the product sold in this market, Q_j^d is the amount of quantity demanded by the j-th consumer, P_x is the price of product x, and Y is the level of income. Assume further that the industry is controlled by two producers, each of whom has the following supply function:

$$P = 5 + k_3 W + Q_1^s, \qquad \text{and}$$

$$P = 20 + k_4 T + 2Q_2^s,$$

where Q_1^s and Q_2^s are the amount of quantity supplied by the first and second producer, respectively. W is the price of gasoline and T is the level of technology. All the parameters are positive.

Use the information given to answer the following questions:

3.1) (5 points) Derive the market demand equation

$Q_j^d = 1 + k_1 P_x + k_2 y - P$

$\sum Q_j^d = 10(1 + k_1 P_x + k_2 y - P)$

$= 10(1 + k_1 P_x + k_2 y) - 10P$ #

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Now, I supplement an additional piece of information to be used in the remaining parts of this question. That is, I assume that $0 < 5 + k_3W < 20 + k_4T$.

3.2) (5 points) What does the given condition mean in terms of the relative cost advantages between the two firms? Given your interpretation, derive the market supply equation.

$5+k_3W$ is min price that activate first firm.

$20+k_4T$ is min price that activates second firm.

$\therefore$ 1-st firm is cost advantage firm.

$$Q_s = \begin{cases} 0 & ; \quad 0 \leq P \leq 5+k_3W \\ -(5+k_3W)+P & ; \quad 5+k_3W < P \leq 20+k_4T \\ -\left(5+k_3W+\frac{20+k_4T}{2}\right)+\frac{3}{2}P & ; \quad P > 20+k_4T. \end{cases}$$

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3.3) (10 points) Given that condition (i) holds, what additional restrictions do we need to impose so that we have two firms that stay active in the equilibrium?

Set $q^d = Q^s$ where $p > 20 + k_4 T$

$$\frac{3}{2}p - \left(5 + k_3 W + \frac{20 + k_4 T}{2}\right) =$$

$$10(1 + k_1 P_x + k_2 y) - 10p$$

$$11.5p^* = 10(1 + k_1 P_x + k_2 y) + \left(5 + k_3 W + \frac{20 + k_4 T}{2}\right)$$

$$p^* = \frac{2}{23}\left[10(1 + k_1 P_x + k_2 y) + \left(5 + k_3 W + \frac{20 + k_4 T}{2}\right)\right]$$

$\therefore$ $p^* > 20 + k_4 T$

① $\frac{2}{23}\left[10(1 + k_1 P_x + k_2 y) + \left(5 + k_3 W + \frac{20 + k_4 T}{2}\right)\right] > 20 + k_4 T$ and,

② $\cancel{10}(1 + k_1 P_x + k_2 y) > p^*$ to ensure positive demand

If this condition doesn't hold,

Sol^n might not exist for positive "Q".

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Continue with the information given above, but now consider a specific case where the value of coefficients and exogenous variables are given in the following table:

Coefficients	k_1	k_2	k_3	k_4
Value	1	1	1	1

Variables	Y	P_x	w	T
Value	5	5	10	5

3.4) (10 points) Solve for the equilibrium price *and* quantity.

$Q^d = 10(11) - 10P$

$$Q^s = \begin{cases} 0 & 0 < P \le 15 \\ -15 + P & 15 < p \le 25 \\ -27.5 + \frac{3}{2}P & P > 25. \end{cases}$$

Notice that p that make $Q^d = 0$ is 11

11 < 15 → min price that make firm 1 active ⇒ $\boxed{\text{Market doesn't exist!}}$

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3.5) (10 points) Suppose that the government provides a subsidy of \$9.5 for each unit of output that the consumers have purchased. Discuss the implication of this subsidy program by relating the result ~~in question~~ to ~~the question that~~ you obtained from the previous question.

what . you have

$P^d = P^s - 9.5$

$Q^d = 110 - 10(P^s - 9.5)$

$= 205 - 10P^s$

$Q^d = Q^s$

$-15 + P^s = 205 - 10P^s$

$11P^s = 220$

$P^s = 20 \quad - \in [15, 25]$

$\therefore \; P^s = 20 \quad P^d = 20 - 9.5 = 10.50$

$Q^d = 10(11) - 10(10.50)$

$= 10(0.5) = 5$ Units. equ^{m}

∃ only one firm in the market
1-st firm in the market.

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