**Self-Test 2** FN 451

**Regressions applications**

1. Variable X takes on the values shown in the following table for five observations. The table also shows the values for five other variables, Y1 through Y5. Which of the variables Y1 through Y5 have a zero correlation with variable X ?

| $X$ | $Y_1$ | $Y_2$ | $Y_3$ | $Y_4$ | $Y_5$ |
|---|---|---|---|---|---|
| 1 | 7 | 2 | 4 | 4 | 1 |
| 2 | 7 | 4 | 2 | 1 | 2 |
| 3 | 7 | 2 | 0 | 0 | 3 |
| 4 | 7 | 4 | 2 | 1 | 4 |
| 5 | 7 | 2 | 4 | 4 | 5 |

2. Use the data sample below to answer the following questions.

$$\sum_{i=1}^{n} X_i = 220 \qquad \sum_{i=1}^{n} (X_i - \overline{X})^2 = 440 \qquad \sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -568$$

$$\sum_{i=1}^{n} Y_i = 385 \qquad \sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 1120 \qquad n = 11$$

A. Calculate the sample mean, variance, and standard deviation for X .

B. Calculate the sample mean, variance, and standard deviation for Y .

C. Calculate the sample covariance between X and Y .

D. Calculate the sample correlation between X and Y .

3. Bouvier Co. is a Canadian company that sells forestry products to several Pacific Rim customers. Bouvier's sales are very sensitive to exchange rates. The following table shows recent annual sales (in millions of Canadian dollars) and the average exchange rate for the year (expressed as the units of foreign currency needed to buy one Canadian dollar).

| Year $i$ | Exchange Rate $X_i$ | Sales $Y_i$ |
|---|---|---|
| 1 | 0.40 | 20 |
| 2 | 0.36 | 25 |
| 3 | 0.42 | 16 |
| 4 | 0.31 | 30 |
| 5 | 0.33 | 35 |
| 6 | 0.34 | 30 |

a. Calculate the sample mean and standard deviation for X (the exchange rate) and Y (sales).

b. Calculate the sample covariance between the exchange rate and sales.

c. Calculate the sample correlation between the exchange rate and sales.

d. Calculate the slope coefficient for an estimated linear regression with the exchange rate as the independent variable and sales as the dependent variable and interpret the result.

4. Anurak is an energy analyst examining electricity, oil, and natural gas consumption in different regions over different seasons. She ran a regression explaining the variation in energy consumption as a function of temperature. The total variation of the dependent variable was 140.58, the explained variation was 60.16, and the unexplained variation was 80.42. She had 60 monthly observations.

A. Compute the coefficient of determination.

B. What was the sample correlation between energy consumption and temperature?

C. Compute the standard error of the estimate of Anurak's regression model.

D. Compute the sample standard deviation of monthly energy consumption.

5. You are trying to estimate the beta of a private firm that manufactures home appliances. You have managed to obtain betas for publicly traded firms that also manufacture home appliances.

| Firm | Beta | Debt | MV of Equity |
|---|---|---|---|
| Black & Decker | 1.40 | $ 2,500 | $ 3,000 |
| Fedders Corp. | 1.20 | $ 5 | $ 200 |
| Maytag Corp. | 1.20 | $ 540 | $ 2250 |
| National Presto | 0.70 | $ 8 | $ 300 |
| Whirlpool | 1.50 | $ 2900 | $ 4000 |

The private firm has a debt equity ratio of 25%, and faces a tax rate of 40%. The publicly traded firms all have marginal tax rates of 40%, as well.

a. Estimate the beta for the private firm. (Hint: Think about levered and unlevered beta problem from FN 311 class).

b. What concerns, if any, would you have about using betas of comparable firms?

6.

**Performance in Forecasting Quarterly Earnings per Share**

| | Number of Forecasts | Mean Forecast Error (Predicted–Actual) | Standard Deviations of Forecast Errors |
|---|---|---|---|
| Analyst A | 101 | 0.05 | 0.10 |
| Analyst B | 121 | 0.02 | 0.09 |

Investment analysts often use earnings per share (EPS) forecasts. One test of forecasting

quality is the zero-mean test, which states that optimal forecasts should have a mean forecasting error of 0. (Forecasting error = Predicted value of variable − Actual value of variable.)

You have collected data (shown in the table above) for two analysts who cover two different industries: Analyst A covers the telecom industry; Analyst B covers automotive parts and suppliers.

a. With μ as the population mean forecasting error, formulate null and alternative hypotheses for a zero-mean test of forecasting quality.

b. For Analyst A, using both a t-test and a z-test, determine whether to reject the null at the 0.05 and 0.01 levels of significance.

c. For Analyst B, using both a t-test and a z-test, determine whether to reject the null at the 0.05 and 0.01 levels of significance.

**Solutions**

1. The four variables Y1 through Y4 have a zero correlation with X . Notice that although Y3 and Y4 are clearly nonlinearly related to X (decreasing and then increasing as the value of X increases), their overall linear relationship with X is zero. Variable Y5 has a correlation of 1.0 with X .

A. The sample mean, variance, and standard deviation of X are

$$\overline{X} = \sum_{i=1}^{n} X_i/n = 220/11 = 20$$

$$s_X^2 = \sum_{i=1}^{n} (X_i - \overline{X})^2/(n-1) = 440/10 = 44$$

$$s_X = \sqrt{s_X^2} = \sqrt{44} = 6.633$$

B. The sample mean, variance, and standard deviation of Y are

$$\overline{Y} = \sum_{i=1}^{n} Y_i/n = 385/11 = 35$$

$$s_Y^2 = \sum_{i=1}^{n} (Y_i - \overline{Y})^2/(n-1) = 1120/10 = 112$$

$$s_Y = \sqrt{s_Y^2} = \sqrt{112} = 10.583$$

C. The sample covariance between X and Y is

$$\text{Cov}(X, Y) = \sum_{i=1}^{n}[(X_i - \overline{X})(Y_i - \overline{Y})]/(n-1) = -568/10 = -56.8$$

D. The sample correlation between X and Y is

$$r = \frac{\text{Cov}(X, Y)}{s_X s_Y} = \frac{-56.8}{6.633 \times 10.583} = -0.809$$

3.

| Year $i$ | Exchange Rate $X_i$ | Sales $Y_i$ | $(X_i - \overline{X})^2$ | $(Y_i - \overline{Y})^2$ | $(X_i - \overline{X})(Y_i - \overline{Y})$ |
|---|---|---|---|---|---|
| 1 | 0.4 | 20 | 0.0016 | 36 | −0.24 |
| 2 | 0.36 | 25 | 0.0000 | 1 | 0.00 |
| 3 | 0.42 | 16 | 0.0036 | 100 | −0.60 |
| 4 | 0.31 | 30 | 0.0025 | 16 | −0.20 |
| 5 | 0.33 | 35 | 0.0009 | 81 | −0.27 |
| 6 | 0.34 | 30 | 0.0004 | 16 | −0.08 |
| Sum | 2.16 | 156 | 0.0090 | 250 | −1.39 |

A. The sample mean and standard deviation of the exchange rate are

$$\overline{X} = \sum_{i=1}^{n} X_i/n = 2.16/6 = 0.36$$

and

$$s_X = \sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2/(n-1)} = \sqrt{0.009/5} = 0.042426$$

The sample mean and standard deviation of sales are

$$\overline{Y} = \sum_{i=1}^{n} Y_i/n = 156/6 = 26$$

and

$$s_Y = \sqrt{\sum_{i=1}^{n}(Y_i - \overline{Y})^2/(n-1)} = \sqrt{250/5} = 7.0711$$

B. The sample covariance between the exchange rate and sales is

$$\text{Cov}(X, Y) = \sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})/(n-1) = -1.39/5 = -0.278$$

C. The sample correlation between the exchange rate and sales is

$$r = \frac{\text{Cov}(X, Y)}{s_x s_y} = \frac{-0.278}{(0.042426)(7.0711)} = -0.927$$

D. The intercept and slope can be derived from covariance of Y and X divided by variance of X.

$$\hat{b}_1 = \frac{\sum_{i=1}^{n}(Y_i - \overline{Y})(X_i - \overline{X})}{\sum_{i=1}^{n}(X_i - \overline{X})^2} = \frac{-1.39}{0.009} = -154.44$$

and

$$\hat{b}_0 = \overline{Y} - \hat{b}_1\overline{X} = 26 - (-154.44)(0.36) = 26 + 55.6 = 81.6$$

So the regression equation is $Y_i = 81.6 - 154.44X_i$.

The slope -154.44 means that for one foreign currency unit increase in C$ exchange rate (this means C$ is appreciating against foreign currency), sales drop by 154.44 million.

4.

A. The coefficient of determination is

$$\frac{\text{Explained variation}}{\text{Total variation}} = \frac{60.16}{140.58} = 0.4279$$

B. For a linear regression with one independent variable, the absolute value of correlation between the independent variable and the dependent variable equals the square root of the coefficient of determination, so the correlation is $\sqrt{0.4279} = 0.6542$. (The correlation will have the same sign as the slope coefficient.)

C. The standard error of the estimate is

$$\left(\sum_{i=1}^{n}\frac{(Y_i - \hat{b}_0 - \hat{b}_1X_i)^2}{n-2}\right)^{1/2} = \left(\frac{\text{Unexplained variation}}{n-2}\right)^{1/2} = \sqrt{\frac{80.42}{60-2}} = 1.178$$

D. The sample variance of the dependent variable is

$$\sum_{i=1}^{n}\frac{(Y_i - \overline{Y})^2}{n-1} = \frac{\text{Total variation}}{n-1} = \frac{140.58}{60-1} = 2.3827$$

The sample standard deviation is $\sqrt{2.3827} = 1.544$.

5.

| Company | Beta | Debt | Equity | D/E | |
|---|---|---|---|---|---|
| Black & Decker | 1.40 | 2500 | 3000 | 83.33% | |
| Fedders | 1.20 | 5 | 200 | 2.50% | |
| Maytag | 1.20 | 540 | 2250 | 24.00% | |
| National Presto | 0.70 | 8 | 300 | 2.67% | |
| Whirlpool | 1.50 | 2900 | 4000 | 72.50% | |
| | 1.20 | | | 37.00% | |
| Unlevered beta = 1.20 / (1+(1-.4)(.37)) = | | | 0.98 | | |
| a. Beta for private firm = 0.98 (1+ (1-.4) (.25)) = | | | | 1.13 | |
| b. The firms might not be directly comparable in terms of business mix. Furthermore, the risk of a firm may be affected by its size; if the private firm is much smaller, this beta might not adequately reflect its risk. Finally, this beta reflects only market risk; the private firm may care about firm-specific risk as well. | | | | | |

6.

A. $H_0$: $\mu = 0$ versus $H_a$: $\mu \neq 0$.

B. The $t$-test is based on $t = \dfrac{\overline{X} - \mu_0}{s/\sqrt{n}}$ with $n - 1 = 101 - 1 = 100$ degrees of freedom. At the 0.05 significance level, we reject the null if $t > 1.984$ or if $t < -1.984$. At the 0.01 significance level, we reject the null if $t > 2.626$ or if $t < -2.626$. For Analyst A, we have $t = (0.05 - 0)/(0.10/\sqrt{101}) = 0.05/0.00995 = 5.024938$ or 5.025. We clearly reject the null hypothesis at both the 0.05 and 0.01 levels.

The calculation of the $z$-statistic with unknown variance, as in this case, is the same as the calculation of the $t$-statistic. The rejection point conditions for a two-tailed test are as follows: $z > 1.96$ and $z < -1.96$ at the 0.05 level; and $z > 2.575$ and $z < -2.575$ at the 0.01 level. Note that the $z$-test is a less conservative test than the $t$-test, so when the $z$-test is used, the null is easier to reject. Because $z = 5.025$ is greater than 2.575, we reject the null at the 0.01 level; we also reject the null at the 0.05 level.

In summary, Analyst A's EPS forecasts appear to be biased upward—they tend to be too high.