Thm. 2.1. If f is differentiable at x_0

f is increasing at $x_0 \Rightarrow f'(x_0) \geq 0.$

$\not\Leftarrow$

~~at x_0:~~

Thm 2.2 If f is differentiable at x_0 " at x^*

$f''(x^*) > 0$ $\quad$ f'

$\cancel{f'(x_0) > 0} \Rightarrow f$ is strictly increasing at x_0

$\not\Leftarrow$

f strictly increasing at x_0, but $f'(x_0) \not> 0.$

Cor. 2.2. $f'(x_0) < 0 \Rightarrow f$ is strictly decreasing at x_0

Thm 2.3 f diff in $(a,b) \subseteq S$

in (a,b)

f is increasing in $(a,b) \Leftrightarrow f'(x) \geq 0,\ x \in (a,b)$

Cor. 2.3. f is decreasing in $(a,b) \Leftrightarrow f'(x) \leq 0,\ x \in (a,b)$

Thm 2.4 f diff in $(a,b) \subseteq S$ $\quad (x^*, x^*+\varepsilon)$

$f'(x) > 0,\ x \in (a,b) \Rightarrow f$ is strictly increasing at $x \in (a,b)$

$\not\Leftarrow$

Cor 2.4 $f'(x_0) < 0,\ x \in (a,b) \Rightarrow f$ is strictly decreasing $x \in (a,b)$

$\boxed{f''(x) < 0,\ x \in S} \Rightarrow f'$ strictly decreasing

$\Leftrightarrow$ f strictly concave

$\Leftarrow$ f strictly concave

Thm 2.10. $f \in C^1$

f is concave $\Leftrightarrow$ f' is decreasing function $\Leftrightarrow$ $f''(x) \le 0$, $x \in S$ (Thm 2.12)

Thm 2.11 $f \in C^1$

f is strictly concave $\Leftrightarrow$ f' is strictly decreasing function

$\Leftarrow$ $f''(x) < 0$, $x \in S$

(Thm 2.12)

Thm 2.8 $f : S \to \mathbb{R}$, $f \in C^1$ – differentiable.

f concave (convex) $\Leftrightarrow$ $f(y) - f(x) \underset{\ge}{\le} f'(x)(y-x)$

$x, y \in S$

(f concave) $\Leftrightarrow$ $\dfrac{f(y)-f(x)}{y-x} \le f'(x)$, $x<y$

$\dfrac{f(y)-f(x)}{y-x} \ge f'(x)$, $x>y$

Thm 2.9 $f: S \to \mathbb{R}$, $f \in C^1$ – differentiable.

f strictly concave $\iff$ $f(y)-f(x) < f'(x)(y-x)$

$x, y \in S$

(strictly f concave) $\iff$ $\dfrac{f(y)-f(x)}{y-x} < f'(x)$, $x<y$

$\dfrac{f(y)-f(x)}{y-x} > f'(x)$, $x>y$

Thm 2.10. $f: S \to \mathbb{R}$, $f \in C'$

2.11 f strictly concave $\iff$ f' is strictly decreasing fn

Thm 2.12. $f: S \to \mathbb{R}$, $f \in C^2$

f concave $\iff$ $f''(x) \le 0$, $x \in S$

$f''(x) < 0, x \in S \implies f$ strictly concave

Methods of proofs in Mathematics

Math: $A \to B$

— given A is true, how we show B will follow.

① **Deduction** - step by step reasoning using the know math fact. to deduce the next following. fact.

If $x^2 > 4$ then x is ....

2) **Induction**. 3 steps. to prove a statement involving an integer n.

1) Prove the easiest case   Eg $1+2+3+\dots+n = \frac{n(n+1)}{2}$

ie $n=1$.

$$1 = \frac{n(n+1)}{2} = \frac{(1)(1+1)}{2} = 1$$

2) State the Induction Hypothesis that it is true up to any case n $-\frac{1}{12}$ $= 1+2+3+4+\dots\dots$

$$\boxed{\frac{n(n+1)}{2}} = 1+2+\dots\dots+n.$$

$1-1+1-1+1-1+\dots\dots$

$= \frac{1}{2}$

3) Prove for the case of $n+1$:

$$\underbrace{1+2+3+\cdots+n}+n+1 = \frac{n(n+1)}{2}+(n+1)$$

$$= (n+1)\left(\frac{n}{2}+1\right)$$

$$= \frac{(n+1)(n+2)}{2}.$$

Ex. $1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}.$ ✓

3) Prove by Contradiction.

Want to show $A \to B$

i.e. whenever A is true B must follow.

– Assume $\boxed{A \to B}$ is false.

i.e. there is at least one situation that A is true but B is false.

→ This will lead to a contradiction

{ of known math fact
{ of our own assumption A.

Ex. $\sqrt{2}$ is irrational.

rational number $= \frac{n}{m}$, n & m are integers.

Suppose $\sqrt{2}$ is rational.

$$\sqrt{2} = \frac{n}{m} \Rightarrow 2 = \frac{n^2}{m^2}$$

$$4k^2 = n^2 = 2m^2 \Rightarrow n^2 \text{ is even}$$

n is even

$n = 2k$ for some integer k

$$4k^2 = 2m^2$$

$$m^2 = 2k^2 \Rightarrow m \text{ is even.}$$

$$\sqrt{2} = \frac{n}{m} = \frac{n_1}{m_1} = \frac{n_2}{m_2} \text{-------}\rightarrow \text{forever.}$$

integer

$n_1 = n/2$ – integer

$m_1 = m/2$ – integer.

integer

$n_2 = n_1/2$

$m_2 = m_1/2$

Never prove by ~~show~~ examples. something is true.

$$1+2+3 = 6 = \frac{3(3+1)}{2} \checkmark$$

$$\cancel{1+2+\cdots 6{,}000}$$

But you can disprove anything by showing an counterexample.

Ex $\quad f'(x_0) \geq 0 \Rightarrow f$ is increasing at x_0.