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EE311: Microeconomics Theory

Supplement Slides on
Production and costs
in the Short run and in the long run

$Q = F(L, \bar{K})$

variable input

fixed input

SR production fn.

$Q = F(L, K)$

LR production fn.

Both are variable inputs

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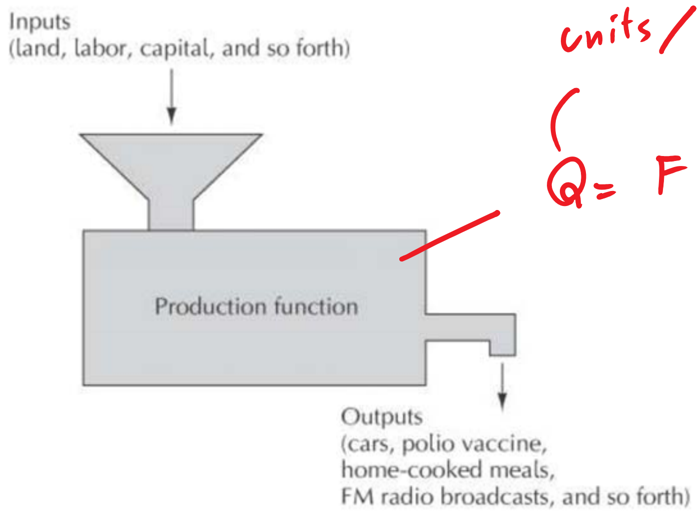
Figure 9.2: The Production Function



units / time period

$Q = F(\underbrace{L, K}_{\text{units / time period}})$

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Figure 9.3: A Specific Short-Run Production Function

$Q = 2KL$

$K_1 = 3$

$K_0 = 1$

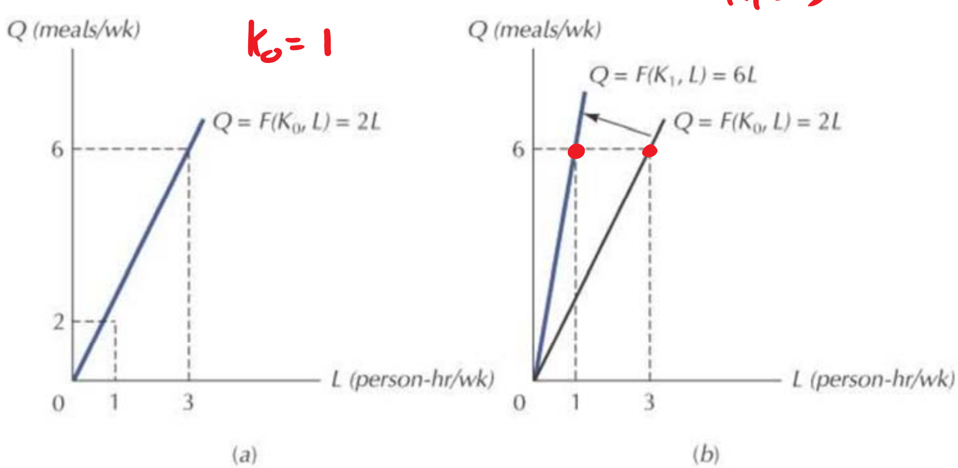

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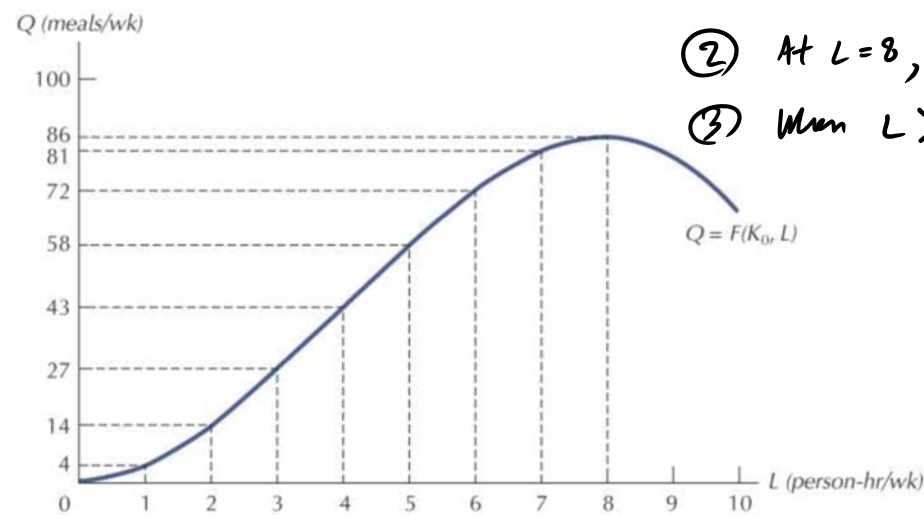
Figure 9.4: Another Short-Run Production Function

① When $L \uparrow$, $Q \uparrow$
($L=0 \rightarrow L=8$)

② At $L=8$, Q is max.

③ When $L>8$, Q starts to fall.



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Short-run Production Function

$\bar{K}$ $\frac{\Delta Q}{\Delta L} = MP_L$

$Q = F(L, \bar{K})$

- ***Law of diminishing returns:*** **if other inputs are fixed, the increase in output from an increase in the variable input must eventually decline.**

= Law of diminishing marginal product (MP_L)

Why can't all the world's people be fed from the amount of grain grown in a single flowerpot?

The law of diminishing returns suggests that no matter how much labor, fertilizer, water, seed, capital equipment, and other inputs were used, only a limited amount of grain could be grown in a single flowerpot. With the land input fixed at such a low level, increases in other inputs would quickly cease to have any effect on total output.

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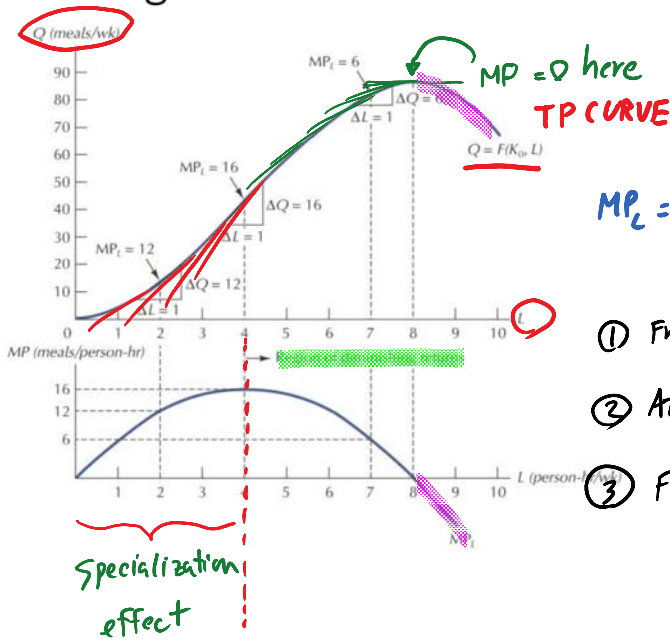
Figure 9.6: The Marginal Product of a Variable Input



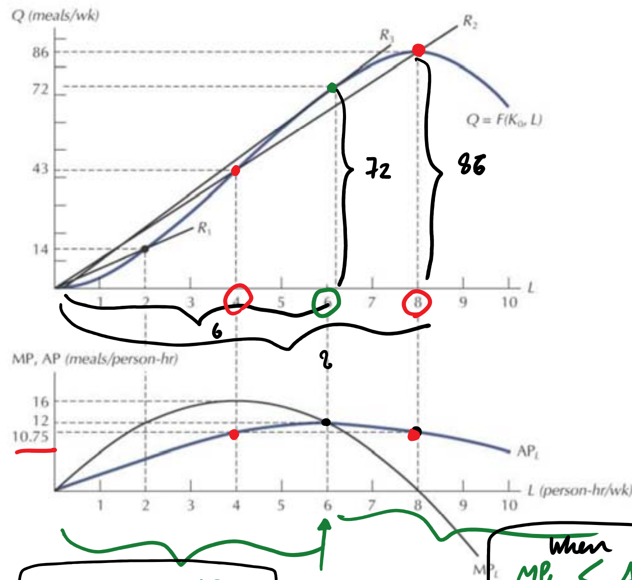
$MP_L = \frac{\Delta Q}{\Delta L}$ = slope of TP curve.

① From $L = 1 \rightarrow L = 4$, MP_L is rising.

② At $L = 4$, MP_L is at its maximum.

③ From $L = 4$ onward MP_L is falling
(Law of diminishing MP_L starts to operate.)

Figure 9.7: Total, Marginal, and Average Product Curves



- $MP_L = \dfrac{\Delta Q}{\Delta L}$
- $AP_L = \dfrac{Q}{L}$ (output per worker)

" labor productivity "

$\frac{86}{8} = 10.75$

When $MP_L > AP_L$, AP_L is rising

$AP_L = MP_L = 12$ here

When $MP_L < AP_L$, AP_L is falling.

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Table 6.1 Total Product, Marginal Product, and Average Product of Labor with Fixed Capital

Capital, $\bar{K}$	Labor, L	Output, Total Product of Labor, Q	Marginal Product of Labor, $MP_L = \Delta Q/\Delta L$	Average Product of Labor, $AP_L = Q/L$
8	0	0		
8	1	5	5	5
8	2	18	13	9
8	3	36	18	12
8	4	56	20	14
8	5	75	19	15
8	6	90	15	15
8	7	98	8	14
8	8	104	6	13
8	9	108	4	12
8	10	110	2	11
8	11	110	0	10
8	12	108	−2	9
8	13	104	−4	8

AP_L

MP_L

NEW AP_L

$MP_L > AP_L$

$MP_L = AP_L$

$MP_L < AP_L$

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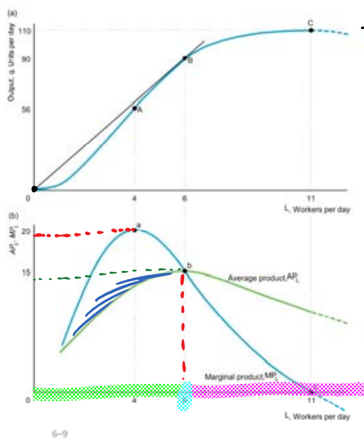
Figure 6.1
Production Relationships with Variable Labor



TP curve

- When $MP_L > AP_L$, AP_L is rising
- When $MP_L = AP_L$, AP_L will reach the max
- When $MP_L < AP_L$, AP_L is falling.

Math Note: FROM $AP_L = \frac{Q}{L}$

$$Q = AP_L \cdot L$$

Take derivative throughout w.r.t. L:

$$\frac{dQ}{dL} = \frac{d\,(AP_L \cdot L)}{dL}$$

$$\frac{dQ}{dL} = AP_L \cdot \underbrace{\cancel{\frac{dL}{dL}}}_{=1} + L\,\frac{d\,AP_L}{dL}$$

$$MP_L = AP_L + L \cdot \text{SLOPE OF } AP_L$$

$$\boxed{(MP_L - AP_L) = L \cdot \text{SLOPE OF } AP_L} \quad ***$$

If $MP_L - AP_L > 0$ or $MP_L > AP_L$, then slope of AP_L must be positive (= AP is rising)

IF $MP_L - AP_L < 0$ or $MP_L < AP_L$, then slope of AP_L must be negative (= AP is falling)

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The Practical Significance Of The Average marginal Distinction

- Suppose you own a fishing fleet consisting of a given number of boats, and can send your boats in whatever numbers you wish to either of two ends of an extremely wide lake, east or west. Under your current allocation of boats, the ones fishing at the east end return daily with 100 pounds of fish each, while those in the west return daily with 120 pounds each. The fish populations at each end of the lake are completely independent, and your current yields can be sustained indefinitely.

- ***Should you alter your current allocation of boats?***

EAST END

WEST END

AP = 100 < AP = 120

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TABLE 8.2

Average Product, Total Product, and Marginal Product (lb/day) for Two Fishing Areas

Number of boats	East end AP	East end TP	East end MP	West end AP	West end TP	West end MP
0	0	0		0	0	
			100			130
1	100	100		130	130	
			100			110
2	100	200		120	240	
			100			90
3	100	300		110	330	
			100			70
4	100	400		100	400	

The average catch per boat is constant at 100 pounds per boat for boats sent to the east end of the lake. The average catch per boat is a declining function of the number of boats sent to the west end.

(EAST, WEST)
2 2 → TOTAL OUTPUT = 2 · 100 + 2 · 120
= 200 + 240
= 440.

(1, 3) → TOTAL OUTPUT = 1 · 100 + 3 · 110
= 100 + 330
= 430. −10

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The Practical Significance Of The Average marginal Distinction

- *The general rule for allocating an input efficiently in such cases is to allocate the next unit of the input to the production activity where its marginal product is highest.*

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Table 7.1 Variation of Short-Run Cost with Output

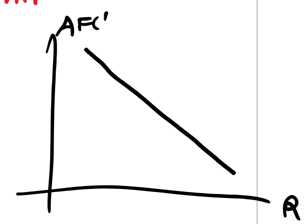
(FC', VC', TC' | AFC' AVC' AC' | MC') — 7 terms

(1) (2) (3) = (1)+(2)

Output, q	Fixed Cost, FC'	Variable Cost, + VC	Total Cost, = TC	Marginal Cost, MC	Average Fixed Cost, AFC = FC/q	Average Variable Cost, AVC = VC/q	Average Cost, AC = C/q
0	48	0	48				
1	48	25	73	25	48	25	73
2	48	46	94	21	24	23	47
3	48	66	114	20	16	22	38
4	48	82	130	16	12	20.5	32.5
5	48	100	148	18	9.6	20	29.6
6	48	120	168	20	8	20	28
7	48	141	189	21	6.9	20.1	27
8	48	168	216	27	6	21	27
9	48	198	246	30	5.3	22	27.3
10	48	230	278	32	4.8	23	27.8
11	48	272	320	42	4.4	24.7	29.1
12	48	321	369	49	4.0	26.8	30.8

0 → 25: 25; 25 → 46: 21

unit cost or cost per unit



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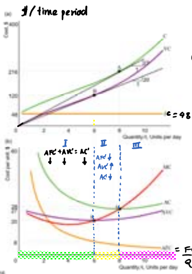
- $\boxed{TC' = FC' + VC'}$ — (1) $\quad (TC' = TFC' + TVC')$
- $\frac{TC'}{Q} = \frac{FC'}{Q} + \frac{VC'}{Q} \rightarrow \boxed{AC' = AFC' + AVC'}$ — (2)
- $\boxed{MC' = \frac{\Delta TC'}{\Delta Q}}$ — (3A)

$$MC' = \frac{\Delta TC'}{\Delta Q} = \frac{\Delta (FC' + VC')}{\Delta Q} = \cancel{\frac{\Delta FC'}{\Delta Q}}^{\,0} + \frac{\Delta VC'}{\Delta Q}$$

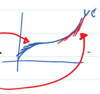
$\boxed{MC' = \frac{\Delta VC'}{\Delta Q}}$ — (3B)

Figure 7.1
Short-Run
Cost Curves



① FC is constant throughout
② notice that when Q = 0, we also have to pay FC = 48
③ When Q = 0, VC = 0

④ higher the Q, the higher the VC.
⑤ VC increases at decreasing rate first and then at increasing rate.



⑥ TC behaves like VC too!
⑦ $\frac{FC}{Q}$ falls sharply at the low level of output and falls steadily at high level of output ("Spreading effect")

When $\boxed{0 < Q < 6}$, $\underset{\downarrow}{AFC} + \underset{\downarrow}{AVC} = \underset{\downarrow}{AC}$

Thanks to spreading effect and specialization effect, AC ↓

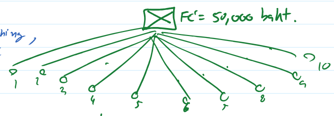
when $\boxed{6 < Q < 8}$, $\underset{\downarrow}{AFC} + \underset{\uparrow}{AVC} = \underset{\downarrow}{AC}$

As spreading effect still dominates Diminishing Return Effect, AC is still falling.

$\boxed{\text{When } Q > 8}$ $\underset{\downarrow}{AFC} + \underset{\uparrow}{AVC} = \underset{\uparrow}{AC}$

Now, DMR Effect > Spreading effect and then AC ↑. :(

FC = 50,000 baht.



when Q = 1, $\frac{FC}{Q} = \frac{50000}{1} = 50{,}000$ baht/unit

when Q = 2, $\frac{FC}{Q} = \frac{50000}{2} = 25{,}000$ baht/unit

when Q = 10, $\frac{FC}{Q} = \frac{50000}{10} = 5000$ baht/unit.

when Q = 100, $\frac{FC}{Q} = \frac{50000}{100} = 500$ baht/unit

$$AC = \underset{\downarrow}{AFC} + AVC$$
↗ ↗

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Fact#8 (on AVC)

$\boxed{AVC = \frac{VC}{Q}}$ $\frac{\text{baht}}{\text{unit of output}}$

AVC or $\frac{VC}{Q}$ is U-shaped.

From $0 < Q < 6$, $\frac{VC}{Q}$ is falling.

at $Q = 6$, $\frac{VC}{Q}$ bottoms out.

From $Q > 6$, $\frac{VC}{Q}$ is rising.

Q: Why AVC is U-shaped? $Q = F(\bar{K}, L)$

A: $AVC = \frac{VC}{Q} = \frac{w \cdot L}{Q}$ where w = wage rate or wage paid to workers.

$$= w \cdot \frac{L}{Q}$$

$$= w \cdot \frac{1}{\frac{Q}{L}}$$

$$= w \cdot \frac{1}{AP_L}$$

So $\boxed{AVC = \frac{w}{AP_L}}$ → If w is fixed, If AP_L ↑, AVC or $\frac{VC}{Q}$ must be falling

If AP_L ↓, AVC or $\frac{VC}{Q}$ must be rising.

Ex

$\boxed{L_1 = 10 \quad Q_1 = 1000 \quad w = 300}$ → $AVC_1 = \frac{VC_1}{Q_1} = \frac{w \cdot L}{Q} = \frac{300 \cdot 10}{1000} = \frac{3\cancel{000}}{1\cancel{000}} = 3$ baht/unit

$AP_L = \frac{Q_1}{L_1} = \frac{1000}{10} = 100$ units / person

$\boxed{L_2 = 11, \quad Q_2 = 1200 \quad w = 300}$

$AVC_2 = \frac{VC_2}{Q_2} = \frac{w \cdot L_2}{Q_2} = \frac{300 \cdot 11}{1200} = \frac{3300}{1200} = 2.75$ baht/unit

$AP_L = \frac{Q_2}{L_2} = \frac{1200}{11} = 109.09$ units / person

When labor becomes more productive, $\frac{VC}{Q}$ will fall.

- IF $\underbrace{MP_L > AP_L}_{\text{due to specialization \& division of labor}} \rightarrow AP_L \uparrow \rightarrow AVC \text{ or } \frac{VC}{Q} \downarrow$

- IF $\underbrace{MP_L < AP_L}_{\text{due to Law of Diminishing } MP_L} \rightarrow AP_L \downarrow \rightarrow AVC \text{ or } \frac{VC}{Q} \uparrow$ ☹

due to Diminishing Return effect

Fact#9 (on AC or ATC)

$$\boxed{AFC + AVC = AC}$$

or

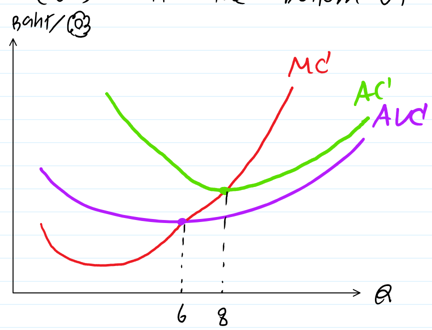
$$\boxed{\frac{FC}{Q} + \frac{VC}{Q} = \frac{TC}{Q}}$$

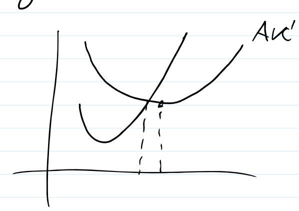
AC is U-shaped (why?)

Fact #10 (on MC)

$$\boxed{MC = \frac{dTC}{dQ}} \quad \text{or} \quad \boxed{MC = \frac{dVC}{dQ}}$$

- MC is U-shaped too.
- MC curve cuts at the bottom of AVC first and then cuts at the bottom of AC accordingly.





MC & AVC

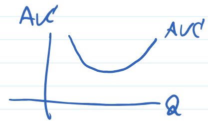
When $0 < Q < 6$, MC < AVC and then AVC will be going down.
At $Q = 6$, MC = AVC and then AVC will bottom out
From $Q > 6$, MC > AVC and then AVC will be going up.

MC - AVC RELATIONSHIP

MC – AVC RELATIONSHIP

Proof

$$AVC = \frac{VC}{Q}$$



$$VC = AVC \cdot Q$$

$$\frac{dVC}{dQ} = AVC \cdot \underbrace{\frac{dQ}{dQ}}_{=1} + Q \cdot \frac{dAVC}{dQ}$$

$$MC = AVC + Q \cdot (\text{SLOPE OF } AVC)$$

$$\boxed{MC - AVC = Q \cdot \text{SLOPE OF } AVC}$$

$MC - AVC > 0 \iff$ SLOPE OF AVC is positive (rising part of AVC)

$MC - AVC < 0 \iff$ SLOPE OF AVC is negative (Falling part of AVC)

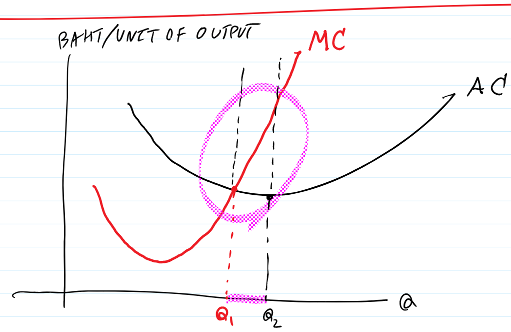
For MC & AC, D-I-Y

Solution : $\boxed{MC - AC = Q \cdot \text{SLOPE OF } AC}$

IF $MC < AC$, $AC \downarrow$

IF $MC > AC$, $AC \uparrow$

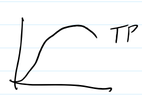
IF $MC = AC$, SLOPE OF $AC = 0$



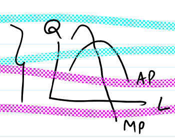
SUMMARY Production and cost in the short run.

Production

$Q = F(L, \bar{K})$



- TP
- $AP_L = \frac{Q}{L}$
- $MP_L = \frac{\Delta Q}{\Delta L}$



- Law of Diminishing Marginal product of labor
- AP & MP Relationship

cost

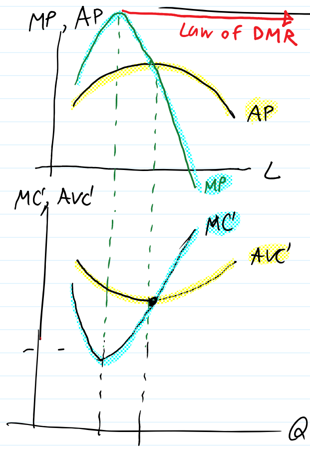
- $TC = FC + VC$ (3 curves)
- $AC = AFC + AVC$ (curves)
- $MC = \frac{\Delta TC}{\Delta Q}$ (MC 1 curve)
- U – SHAPED AC
- MC & AVC relationship
- MC & AC relationship

$$MC = \frac{dVC}{dQ} = \frac{d\,w \cdot L}{dQ} = w \cdot \frac{dL}{dQ} = w \cdot \frac{1}{\frac{dQ}{dL}} = \frac{w}{MP_L}$$

$$\boxed{AVC = \frac{w}{AP_L}} \qquad \boxed{MC = \frac{w}{MP_L}}$$

$$\boxed{AVC' = \frac{w}{AP_L}}$$

$$\boxed{MC' = \frac{w}{MP_L}}$$

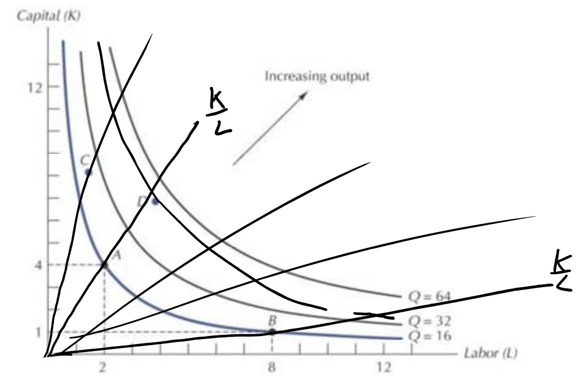


→ Production

→ cost

- MC' is a mirror image of MP_L
- AVC' is a mirror image of AP_L

Figure 9.8: Part of an Isoquant Map for the Production Function



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$Q = F(L, K)$

both now are variable inputs.

Min Total cost of production
subject to $Q = \overline{Q}$

"COST MINIMIZATION PROBLEM"

$$\min_{L,K} \quad TC = w \cdot L + r \cdot K$$
$$\text{s.t.} \quad Q = F(L, K) = Q_0$$

$(L^*, K^*) \to \min TC$

$L^* = ?$, $K^* = ?$