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EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$
Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

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2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

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Student	Y_i	X_i	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$x_i y_i$
1	2.8	63	-14.625	-0.4125	6.0328125
2	3.4	72	-5.625	0.1875	-1.0546875
3	3.0	78	0.375	-0.2125	-0.0796875
4	3.5	81	3.375	0.2875	0.9703125
5	3.6	87	9.375	0.3875	3.6328125
6	3.0	75	-2.625	-0.2125	0.5578125
7	2.7	75	-2.625	-0.5125	1.3453125
8	3.7	90	12.375	0.4875	6.0328125
Σ	25.7	621			$\Sigma = 17.4375$

1.1 $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$

$$\hat{\beta}_1 = 3.2125 - 0.0340659\,(77.625)$$
$$= 0.568132$$

$\bar{y} = 3.2125$
$\bar{x} = 77.625$
$\Sigma x_i^2 = 511.875$

$$\hat{y}_i = 0.568132 + 0.0340659\,(x_i)$$

$$\hat{\beta}_2 = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2}$$
$$= \frac{17.4375}{511.875} = 0.0340659$$

with the exam score increases by 1 unit, GPA will increase by 0.0340659 units. With no score on exam, student will get GPA equal to 0.568132

1.2

$\hat{y}_1 = 2.714284$, $\hat{u}_1 = 0.085716$ $\quad$ $\hat{y}_2 = 3.020877$, $\hat{u}_2 = 0.379123$ $\quad$ $\hat{y}_3 = 3.225452$, $\hat{u}_3 = -0.225452$ $\quad$ $\hat{y}_4 = 3.32765$, $\hat{u}_4 = 0.17235$ $\quad$ $\hat{y}_5 = 3.532045$, $\hat{u}_5 = 0.067955$ $\quad$ $\hat{y}_6 = 3.123255$, $\hat{u}_6 = -0.123255$

$\hat{y}_7 = 3.123255$, $\hat{u}_7 = -0.423255$ $\quad$ $\hat{y}_8 = 3.634243$, $\hat{u}_8 = 0.065757$ $\quad$ $\Sigma \hat{u}_i = 0.001061$

1.3 $$\mathrm{Var}(\hat{u}_i) = \frac{\Sigma u_i^2}{n-2} = \frac{0.4348931}{6} = 0.0724822 = \sigma^2$$

$$\mathrm{Var}(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n \Sigma x_i^2}\sigma^2 = \frac{48717}{8(511.875)}(0.0724822) = 0.862299$$

$$\mathrm{Var}(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma x_i^2} = \frac{0.0724822}{511.875} = 0.0001416$$

2.1 $$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$$
$$= 9.1 - (17.909091)$$
$$= -8.809091$$

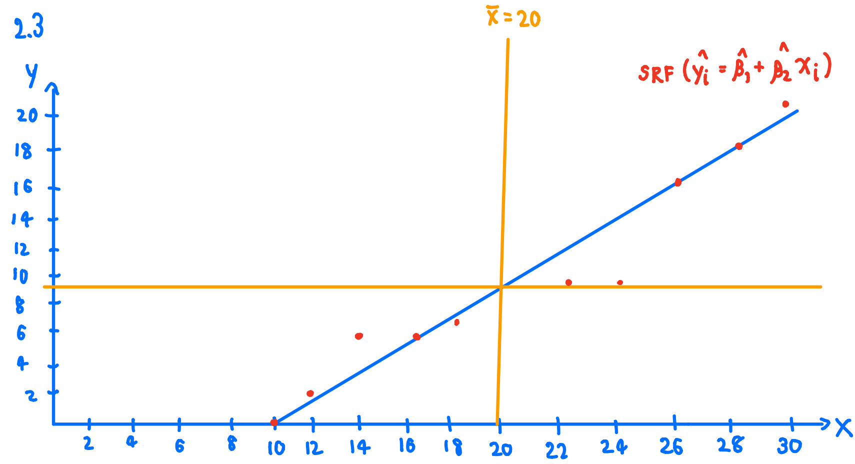
$$\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{394}{440} = 0.895455$$

$$\hat{y}_i = -8.8091 + 0.8955\,(x_i)$$

2.2

$\hat{y}_1 = 0.1459$	$\hat{u}_1 = -0.1459$
$\hat{y}_2 = 1.9369$	$\hat{u}_2 = 0.0631$
$\hat{y}_3 = 3.7279$	$\hat{u}_3 = 1.2721$
$\hat{y}_4 = 5.5189$	$\hat{u}_4 = 0.4811$
$\hat{y}_5 = 7.3099$	$\hat{u}_5 = -0.3099$
$\hat{y}_6 = 10.8919$	$\hat{u}_6 = -0.8919$
$\hat{y}_7 = 12.6829$	$\hat{u}_7 = -2.6829$
$\hat{y}_8 = 14.4739$	$\hat{u}_8 = 0.5261$
$\hat{y}_9 = 16.2649$	$\hat{u}_9 = -0.2649$
$\hat{y}_{10} = 18.0559$	$\hat{u}_{10} = 1.9441$
	$\Sigma\hat{u}_i = -0.009 \approx 0$

2.3



$\hat{y}_i = -8.8091 + 0.8955(20) = 9.1009 \approx 9.1 = \bar{y}$

2.4 $x_i = 18$, $\hat{y}_i = 7.3099$ from 2.2

2.5 $\mathrm{Var}(\hat{u}_i) = \sigma^2 = \dfrac{\Sigma\hat{u}_i^2}{n-2} = \dfrac{14.09092}{8} = 1.761365$

$\mathrm{Var}(\hat{\beta}_2) = \dfrac{\sigma^2}{\Sigma x_i^2} = \dfrac{1.761365}{440} = 0.004$

$\mathrm{Var}(\hat{\beta}_1) = \dfrac{\Sigma X_i^2}{n} \cdot \dfrac{\sigma^2}{\Sigma x_i^2} = \dfrac{444}{10}(0.004) = 1.776$

3. As $\hat{\beta}_1$ is an estimator of β_1, $E(\hat{\beta}_1)$ should be equal to β_1

$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2\bar{x}$, $\hat{\beta}_2 = \dfrac{\Sigma x_i y_i}{\Sigma x_i^2}$ let $k = \dfrac{x_i}{\Sigma x_i^2}$

$\hat{\beta}_1 = \bar{y} - \Sigma k y_i \bar{x}$ and $y_i = \beta_1 + \beta_2 x_i + u_i$

$$\hat{\beta}_1 = \frac{\Sigma y_i}{n} - \Sigma k Y_i \bar{x} = \Sigma\left(\frac{1}{n} - k\bar{x}\right) y_i$$

$$= \Sigma\left(\frac{1}{n} - k\bar{x}\right)(\beta_1 + \beta_2 x_i + u_i)$$

$$= \Sigma\left(\frac{\beta_1}{n} + \frac{\beta_2 x_i}{n} + \frac{u_i}{n} - k\bar{x}\beta_1 - k\bar{x}\beta_2 x_i - k\bar{x}u_i\right)$$

$$= \Sigma\frac{\beta_1}{n} + \beta_2\bar{x} + \frac{\Sigma u_i}{n} - \beta_2\bar{x} - \bar{x}\Sigma k_i u_i$$

take $E(\cdot) \rightarrow E(\hat{\beta}_1) = E(\beta_1) + \bar{x}E(\Sigma k_i u_i) \rightarrow$ assumption 3 : zero mean value of disturbance treat X as given $E(u_i|x_i)$

$$E(\hat{\beta}_1) = E(\beta_1) + \bar{x}\Sigma k_i \underbrace{\cancel{E(u_i)}}_{}^{0}$$

$$E(\hat{\beta}_1) = \beta_1$$

$\hat{\beta}_1$ is an unbiased estimator of actual β_1

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