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**EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020**

**Use 4 decimal places for numerical answers**

1. In Table 1.a. $X_i$ is total microeconomics exam point (total points are 100) and $Y_i$ is GPA of each student.

**Table 1.a**

| Student | $Y_i$ | $X_i$ |
|---|---|---|
| 1 | 2.8 | 63 |
| 2 | 3.4 | 72 |
| 3 | 3 | 78 |
| 4 | 3.5 | 81 |
| 5 | 3.6 | 87 |
| 6 | 3.0 | 75 |
| 7 | 2.7 | 75 |
| 8 | 3.7 | 90 |

| $X_i-\bar{X}$ | $Y_i-\bar{Y}$ | $(X_i-\bar{X})(Y_i-\bar{Y})$ | $(X_i-\bar{X})^2$ | $\hat{Y}_i$ | $\hat{U}_i$ |
|---|---|---|---|---|---|
| -14.625 | -0.4125 | 6.03 | 213.89 | 2.7313 | 0.0857 |
| -5.625 | 0.1875 | -1.05 | 31.64 | 3.0209 | 0.3791 |
| 0.375 | -0.2125 | -0.08 | 0.14 | 3.2253 | -0.2253 |
| 3.375 | 0.2875 | 0.97 | 11.39 | 3.3275 | 0.1725 |
| 9.375 | 0.3875 | 3.63 | 87.89 | 3.5319 | 0.0681 |
| -2.625 | -0.2125 | 0.56 | 6.89 | 3.1231 | -0.1231 |
| -2.625 | -0.5125 | 1.35 | 6.89 | 3.1231 | -0.4231 |
| 12.375 | 0.4875 | 6.03 | 153.14 | 3.6341 | 0.0659 |

$\bar{Y} = 3.2125$ $\bar{X} = 77.625$ $\Sigma = 17.4374$ $\Sigma = 511.875$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of $\beta_0$ and $\beta_1$. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$

$\hat{\beta}_1 = \frac{\Sigma (X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma (X_i - \bar{X})^2}$

$$\hat{\beta}_1 = \frac{17.4374}{511.875} = 0.0341$$

$$\hat{\beta}_0 = 3.2125 - (0.0341)77.625$$

$$= 0.5655$$

1.2 For each observation i, find $\hat{Y}_i$ and $\hat{u}_i$ . Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

$\hat{u}_i = y_i - \hat{y}_i$

| $y$ | $x$ | $\hat{y}$ | $\hat{u}$ |
|---|---|---|---|
| 2.8 | 63 | 2.7143 | 0.0857 |
| | 72 | 3.0209 | 0.3791 |
| | 78 | 3.2253 | -0.2253 |
| | 81 | 3.3275 | 0.1725 |
| | 87 | 3.5319 | 0.0681 |
| | 75 | 3.1231 | -0.1231 |
| | 75 | 3.1231 | -0.4231 |
| | 90 | 3.6341 | 0.0659 |

$\sum_{i=0}^{n} \hat{u}_i = 0$

1.3 Find $var(\hat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$\sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} =$$

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum (X_i - \bar{X})^2} =$$

$$Var(\hat{\beta}_0) = \frac{\sum x_i^2}{n\sum (x_i - \bar{X})^2} \cdot \sigma^2 =$$

$$Var(\hat{u}_i) = \frac{\sum (\hat{u}_i)^2}{n-2} =$$

$$\hat{y}_i : -8.81 + (0.8955)(X_i)$$

2. Data is listed in the table

$\bar{X} = 20$ $\bar{y} = 9.1$

| $X_i$ | $Y_i$ | $X_i - \bar{X}$ | $y_i - \bar{y}$ | $(x_i - \bar{x})(y_i - \bar{y})$ | $(x_i - \bar{x})^2$ | $\hat{y}_i$ | $\hat{u}_i$ |
|---|---|---|---|---|---|---|---|
| 10 | 0 | -10 | -9.1 | 91 | 100 | 0.145 | -0.145 |
| 12 | 2 | -8 | -7.1 | 56.8 | 64 | 1.936 | 0.064 |
| 14 | 5 | -6 | -4.1 | 24.6 | 36 | 3.727 | 1.273 |
| 16 | 6 | -4 | -3.1 | 12.4 | 16 | 5.518 | 0.482 |
| 18 | 7 | -2 | -2.1 | 4.2 | 4 | 7.309 | -0.309 |
| 22 | 10 | 2 | 0.9 | 1.8 | 4 | 10.891 | -0.891 |
| 24 | 10 | 4 | 0.9 | 3.6 | 16 | 12.682 | -2.682 |
| 26 | 15 | 6 | 5.9 | 35.4 | 36 | 14.473 | 0.527 |
| 28 | 16 | 8 | 6.9 | 55.2 | 64 | 16.264 | -0.264 |
| 30 | 20 | 10 | 10.9 | 109 | 100 | 18.055 | 1.945 |
| | | | | 394 | 440 | | |

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i, u_i \sim NIID(0, \sigma^2)$. Find estimators of $\beta_0$ and $\beta_1$ from the OLS method and interpret the meaning.

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{X}$$

$$\hat{\beta}_1 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$\hat{\beta}_1 = \frac{394}{440} = 0.8955$$

$$\hat{\beta}_0 = 9.1 - (0.8955)(20)$$

$$= -8.81$$

$$\bar{X} = \frac{10+12+14+16+18+22+24+26+28+30}{10}$$

$$= 20$$

$$\bar{y} = 0+2+5+6+7+10+10+15+16+20$$

$$= 9.1$$

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$ . Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

$\hat{u}_i = y_i - \hat{y}_i$

| $X_i$ | $y_i$ | $\hat{y}_i$ | $\hat{u}_i$ |
|---|---|---|---|
| 10 | 0 | 0.145 | -0.145 |
| 12 | 2 | 1.936 | 0.064 |
| 14 | 5 | 3.727 | 1.273 |
| 16 | 6 | 5.518 | 0.482 |
| 18 | 7 | 7.309 | -0.309 |
| 22 | 10 | 10.891 | -0.891 |
| 24 | 10 | 12.682 | -2.682 |
| 26 | 15 | 14.473 | 0.527 |
| 28 | 16 | 16.264 | -0.264 |
| 30 | 20 | 18.055 | 1.945 |

$\sum_{i=0}^{N} (y_i - \hat{y}_i) = 0$

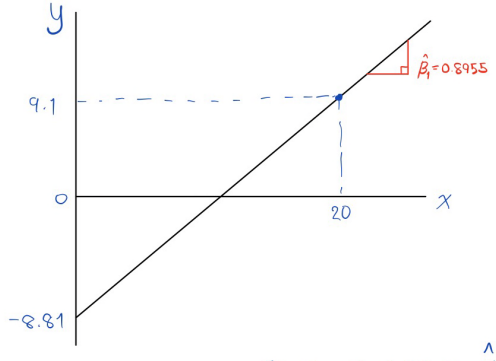
$\sum_{i=0}^{N} \hat{u}_i = 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

(20, 9.1)

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 X_i$

$\hat{y} = -8.81 + 0.8955 X_i$




(Estimated $y = \hat{y}$)

2.4 If $X_i$ =16, what is the predicted Y?

$X_i = 16 \quad \hat{y} = 5.518$

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_0)$, $var(\hat{\beta}_1)$

$\sigma^2 = \frac{\sum \hat{u}_i^2}{n-2} =$

$Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum (X_i - \bar{X})^2} =$

$Var(\hat{\beta}_0) = \frac{\sum X_i^2}{n \sum (X_i - \bar{X})^2} \cdot \sigma^2 =$

$Var(\hat{u}_i) = \frac{\sum (\hat{u}_i)^2}{n-2} =$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of $\beta_1$. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{n} (X - \bar{X})^2}$$

suppose $x_i = X_i - \bar{X}$ $\quad k_i = \frac{x_i}{\sum_i x_i^2}$

$$\beta_1 = \sum_{i=1}^{n} (Y_i - \bar{Y}) k_i \quad \text{OR} \quad \hat{\beta}_1 = \frac{\sum_{i=1}^{n} (Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{n} (X - \bar{X})^2}$$

$$= \sum_{i=1}^{n} \left( \hat{\beta}_0 - \beta_1 X_i + u_i - \beta_0 - \beta_1 \bar{X} \right) k_i$$

$$= \sum_{i=1}^{n} \beta_1 (X_i - \bar{X}) k_i + \Sigma_i u_i k_i$$

$$= \beta_1 \sum_{i=1}^{n} x_i k_i + \Sigma_i u_i k_i$$

$$= \beta_1 \sum_{i=1}^{n} x_i \frac{x_i}{\Sigma_i x_i^2} + \Sigma_i u_i k_i$$

$$= \beta_1 \frac{\Sigma_i x_i^2}{\Sigma_i x_i^2} + \Sigma_i u_i k_i$$

$$E(\hat{\beta}_1) = E\left[\beta_1 + \Sigma_i u_i k_i\right]$$

SLR 4 : $E(u_i \mid X_i) = 0$

$$E(\beta_1) = \beta_1 + \Sigma_i k_i E(u_i) \quad (E(u_i) \to 0)$$

$$\therefore E(\hat{\beta}_1) = \beta_1$$