EE 320 Introductory Mathematical Economics

Semester 1/2016

Assignment 5

Due date - December 2, 2016,

BE office

Question 1:

Given the production function

$$Q = f(K, L) = K^{0.5}L^{0.5}$$

Suppose that the per unit input prices for K and L are \$20 and \$5, respectively.

a. If the producer needs to maintain the output level at $\overline{Q} = 20$ units, what are the values K^* *and* L^* *that minimizes the total cost*, and what is the corresponding minimum cost? Use the bordered Hessian to verify that the second-order sufficient condition is met.
b. Suppose now that the output level is not fixed, but the producer has a budget constraint at \$400. Assume that this producer spends his entire budget on the production. Determine the values K^* *and* L^* *that maximizes the total output.* Also, use the bordered Hessian to verify that the second-order sufficient condition is met.

Question 2:

Pakorn's utility function depends on the consumption of two commodities, x and y, and it is given by

$$U(x, y) = 2xy$$

Suppose that his income is \$72, and the prices per unit of x and y are \$4 and \$6, respectively. Assume that Pakorn spends all of his income, and the values of x and y are both non-zero.

a. Use the Lagrange method to determine the values of x^* and y^* that maximize Pakorn's utility given an income constraint. Verify that the second-order sufficient conditions are satisfied.
b. Determine the maximum utility level and the Lagrange multiplier. Interpret the economic interpretation of the Lagrange multiplier.
c. Suppose that the income is now \$73. Approximate the new maximum utility level.

Question 3: Integration

Suppose that a monopolist faces the demand function $Q = 16 - P$. Its total cost function is given by $TC(Q) = 4Q + Q^2$.

a. Suppose that this monopolist cannot price-discriminate. Use integral to calculate the consumer and producer surplus at the profit-maximizing quantity and price
b. If the monopolist can now practice price-discrimination; that is, he can perfectly identify its new profit-maximizing output level. Also, use integral to calculate the consumer and producer surplus at this new quantity, and discuss the change in total welfare.

Question 4: Cost minimization problem

Consider a cost minimization problem where firm chooses for optimal combination of capital (K) and labor (L). Suppose that r and w are the prices per unit of capital and labor, respectively. And assume further that the production technology of this firm is given by $\sqrt{K} + L = Q$. Consider the following problem

a. Solve for the optimal combination of capital and labor.
b. State the condition under which both types of factor inputs are used by firm.
c. Derive the cost function.

Question 5 *Integration*

a. Suppose the demand and supply curves are $P = \frac{6000}{Q+50}$ and $P = Q + 10$. Find the equilibrium price and quantity, and compute the consumer and producer surplus.
b. Let $MR = 25 - 5x - 2x^2$ and $MC = 10 - 3x - x^2$, where x is the unit of output. Assume that fixed cost is $7. Determine the level of production that contributes to maximum profit and determine the level of maximized profit.

Question 6

The production function for a company's product is Q = $100L + 50K - L^2 - K^2$, where Q is the output that results from L units of labor and K units of capital. The unit costs of labor and capital are $6 and $3, respectively. Consider the following problem

a. If the company wants the total cost of inputs to be 30, determine the greatest output possible subject to this budget constraint.
b. Suppose market price of the product is $12 per unit. Determine the optimal combination of labor and capital that yields this company the highest level of profit.