

## Lecture 10

### Options (Part 2)

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-1-

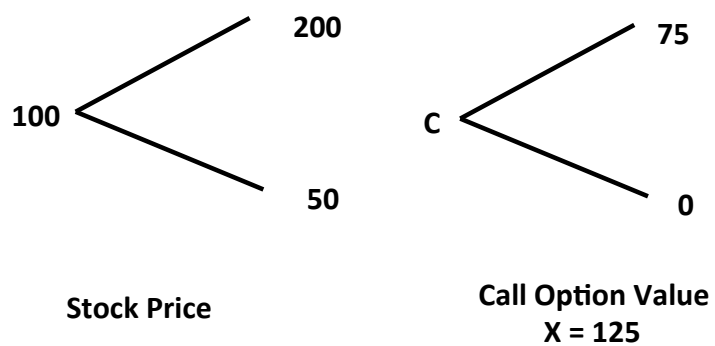
### Factors Influencing Option Values: Calls

| <u>Factor</u>             | <u>Effect on value</u> |
|---------------------------|------------------------|
| Stock price               | increases              |
| Exercise price            | decreases              |
| Volatility of stock price | increases              |
| Time to expiration        | increases              |
| Interest rate             | increases              |
| Dividend Rate             | decreases              |

## Factors Influencing Option Values: Puts

| <u>Factor</u>             | <u>Effect on value</u> |
|---------------------------|------------------------|
| Stock price               |                        |
| Exercise price            |                        |
| Volatility of stock price |                        |
| Time to expiration        |                        |
| Interest rate             |                        |
| Dividend Rate             |                        |

## Binomial Option Pricing



## Binomial Option Pricing

### Alternative Portfolio

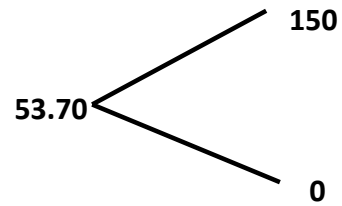
Buy 1 share of stock at \$100

Borrow \$46.30 (8% Rate)

Net outlay \$53.70

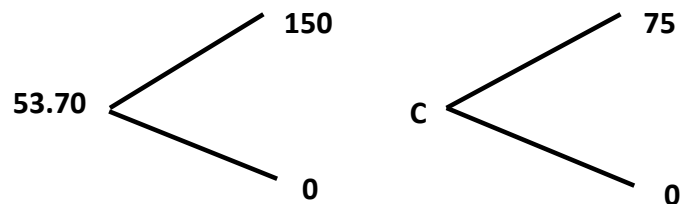
Payoff

|                |      |     |
|----------------|------|-----|
| Value of Stock | 50   | 200 |
| Repay loan     | - 50 | -50 |
| Net Payoff     | 0    | 150 |



Payoff Structure  
is exactly 2 times  
the Call

## Binomial Option Pricing



## Another View of Replication of Payoffs and Option Values

Alternative Portfolio - one share of stock and 2 calls written ( $X = 125$ )

|                 |          |             |
|-----------------|----------|-------------|
| Stock Value     | 50       | 200         |
| Call Obligation | <u>0</u> | <u>-150</u> |
| Net payoff      | 50       | 50          |

Hence  $100 - 2C = 46.30$  or  $C = 26.85$

Hedge Ratio =

## Generalizing the Two-State Approach

Assume that we can break the year into two six-month segments

In each six-month segment the stock could increase by 10% or decrease by 5%

Assume the stock is initially selling at 100

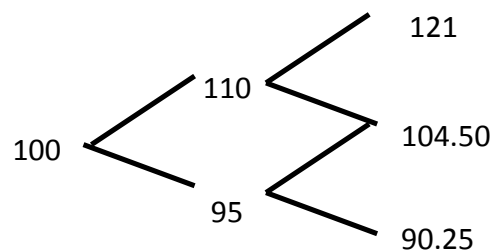
Possible outcomes

Increase by 10% twice

Decrease by 5% twice

Increase once and decrease once (2 paths)

## Generalizing the Two-State Approach

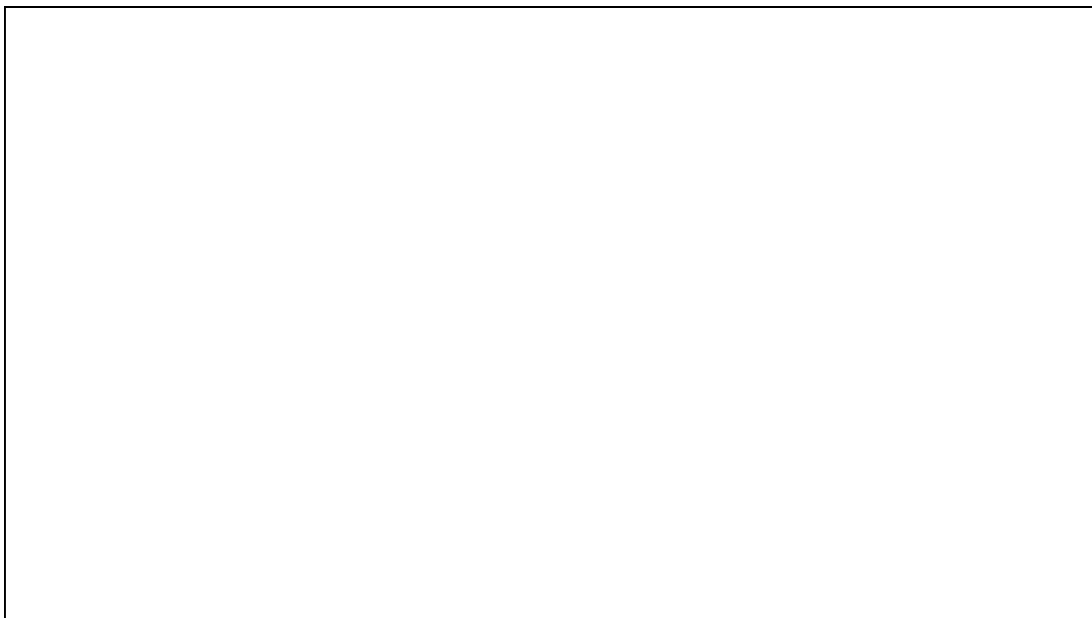


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### Example

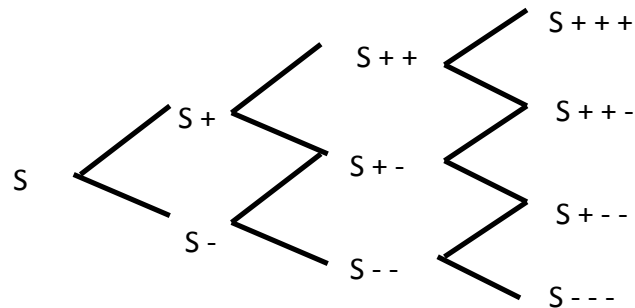
Suppose that the risk-free interest rate is 5% per 6 month period and we wish to value a call option with exercise price \$110 with the underlying stock price following the two-period price tree in the previous slide. Find the value of the call option.



## Expanding to Consider Three Intervals

- Assume that we can break the year into three intervals
- For each interval the stock could increase by 5% or decrease by 3%
- Assume the stock is initially selling at 100

## Expanding to Consider Three Intervals



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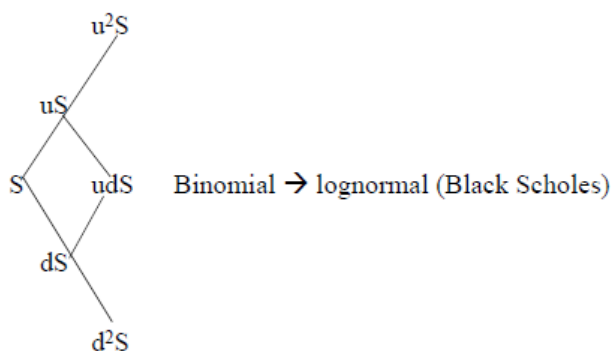
## Possible Outcomes with Three Intervals

| Event       | Probability | Stock Price                   |
|-------------|-------------|-------------------------------|
| 3 up        | 1/8         | $100 (1.05)^3 = 115.76$       |
| 2 up 1 down | 3/8         | $100 (1.05)^2 (.97) = 106.94$ |
| 1 up 2 down | 3/8         | $100 (1.05) (.97)^2 = 98.79$  |
| 3 down      | 1/8         | $100 (.97)^3 = 91.27$         |

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## Black-Scholes Option Pricing



## Black-Scholes Option Pricing

$$C_0 = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

$$P_0 = Xe^{-rT} N(-d_2) - S_0 N(-d_1)$$

Where

$$d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$r$  = risk-free rate (annualized, continuously compounded)

$T$  = time to maturity in years

$\sigma$  = standard deviation of annualized continuously compounded rate of return on stock in decimal

## Black-Scholes Option Valuation

$N(.)$  = Cumulative Standard Normal distribution

$\text{Ln}(.)$  = Natural logarithm

- Put price can also be obtained from put-call parity.

Recall from Statistics

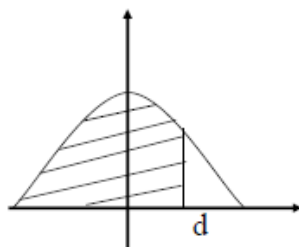
Suppose  $Z$  is standard normal then

$$P[Z \leq d] = N(d)$$

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## Black Scholes Option Pricing



To get  $N(d)$  you can use the table of cumulative normal distribution or the function NORMSDIST in excel.

## Call Option Example

Suppose  $S_0 = \$50$

$X = \$45$

$r = 6\%$

$\sigma = 20\%$

$T = 0.25$  years

Calculation:

$$d_1 = \frac{\ln\left(\frac{50}{45}\right) + \left(0.06 + \frac{(0.20)^2}{2}\right) 0.25}{(0.20)\sqrt{0.25}}$$

$$= 1.2536$$

$$d_2 = 1.2536 - \sigma\sqrt{T} =$$

$$= 1.2536 - 0.1 = 1.1536$$

$$N(d_1) = 0.8950$$

$$N(d_2) = 0.8757$$

$$C = 50(0.8950) - 45(0.9851)(0.8757)$$

$$\begin{array}{ccccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ S & N(d_1) & X & e^{-rT} & N(d_2) \end{array}$$

$$= 44.7500 - 38.8193$$

$$= 5.9307 \cong \$5.93$$

## Binomial Tree versus Black-Scholes

|                    |        | BS price | Binomial price | N   |
|--------------------|--------|----------|----------------|-----|
| stock price        | 100    | 3.7146   | 3.0147         | 2   |
| strike price       | 100    | 3.7146   | 3.347          | 4   |
| volatility         | 0.15   | 3.7146   | 3.466          | 6   |
| T                  | 1 year | 3.7146   | 3.563          | 10  |
| Treasury           | 0.05   | 3.7146   | 3.663          | 30  |
| Put option price ? |        | 3.7146   | 3.699          | 100 |
|                    |        | 3.7146   | 3.7115         | 500 |

## Binomial Tree versus Black-Scholes

- Black-Scholes is easy to use and quite accurate for European options, especially at the money options
- Black-Scholes cannot be used to value American options that have optimal early exercise while binomial tree can be modified to value American options
- Binomial tree method requires more programming and longer execution time for accuracy
- Binomial method is easy to modify for pricing exotic options

## Assumptions in Black-Scholes

- Perfect Capital markets
- The stock pays no dividends until after option expiration date
- Both the interest rate and variance rate of the stock are constant and are known functions of time
- Stock prices are continuous, meaning that sudden extreme jumps such as those in the aftermath of an announcement of a takeover attempt are ruled out.

## Implied Volatility

- Is the volatility that makes the option price calculated using Black-Scholes model equal to the market price of the option?
- How to obtain it?
  - Solve the non-linear Black-Scholes pricing equation for the volatility given an option price
- Why do we care?
  - You want to know what the market consensus is for future volatility
  - Investors can then judge whether they think the actual stock standard deviation exceeds the implied volatility.
  - Used as an “investor fear gauge” as implied volatility is correlated with crisis

## Adjusting the Black-Scholes Model for Dividends

- The call option formula applies to stocks that does not pay dividends
- One approach to incorporate dividends is to replace the stock price with a dividend adjusted stock price

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## Using the Black-Scholes Formula

### Hedge ratio or delta

- The change in option price for a \$1 increase in the stock price
- This is the slope of the curve (option value graphed as a function of the stock value) at the current stock price.
- Represents the number of stocks required to hedge against the price risk of holding one option
- For Black Scholes:

### Option Elasticity

Percentage change in the option's value given a 1% change in the value of the underlying stock