Student ID.................................

EE431 Economics of Financial Markets and Institutions
Homework 3: Mean-Variance Analysis

Please submit at the BE office, 5th floor department of Economics building.

Deadline of submission : Wednesday, March 1st, 2016, before 15.00 hrs.

1. If the covariance of stock 1 with stock 2 is -.008, then what is the covariance of stock 2 with stock 1?

   ANSWER. -0.008. $COV(X,Y) = COV(Y,X)$.

   $$\begin{aligned} Proof : COV(X,Y) &= E(X - E(X))(Y - E(Y)) \\ &= E(Y - E(Y))(X - E(X)), \text{ (commutative property)} \\ &= COV(Y,X) \end{aligned}$$

2. Consider an economy with just two assets. The details of these are given below. Use the following information to answer the questions.

Security's name	Expected rate of returns (%)	Standard deviation (%)
X	3	10
Y	12	15

   Correlation coefficient (r_{XY}) between the rate of return of security X and the rate of return of security Y is 0. The risk free rate (R_f) is 1%.

   (a) Let the tangency portfolio M has

   - 0.3 of the asset invested in X and
   - 0.7 of the asset invested in Y.

   The tangency portfolio is the market portfolio. Write down the equation for the CML.

   ANSWER.

   $$CML: \; E(R_p) = R_f + \left[\frac{E(R_m) - R_f}{\sigma_m}\right]\sigma_p$$

   $E(R_m) = aE(X) + bE(Y) = (0.3 \times 0.03) + (0.7 \times 0.12) = 0.009 + 0.084 = 0.093 =$ **9.3**%

   ***

   $\sigma_p^2 = a^2\sigma_x + 2abr_{xy}\sigma_x\sigma_y + b^2\sigma_y^2$

   $$\begin{aligned} \sigma_p^2 &= (0.3)^2(0.10)^2 + 2(0.7)(0.3)(0)(0.10)(0.15) + (0.7)^2(0.15)^2 \\ &= 0.011925 \\ \sigma_m &= \mathbf{0.01092} *** \end{aligned}$$

   (b) State **Tobin's Separation Theorem**.

   ANSWER.

   - **Two-Fund Separation.** "Each investor will have a utility-maximizing portfolio that is a combination of risk-free asset and a portfolio of risky assets that is deterined by the line drawn from the risk-free rate of return tangent to the investor's efficient set or risky assets."

- (**James Tobin**) All investors will hold the same asset portfolio. There are **two steps in the investment selection process.**
  (1) choose the risky portfolio, determined by the line drawn from the risk-free rate of return tangent to the efficient frontier
  (2) blend the risky portfolio by borrowing and lending depending on whether we want more risk or less risk

(c) A = the last digit of your student ID. **If the last digit of your student ID = 0, use A = 5**.

Fill in the table below.

A	=...........................

Example.

A student with ID 540464031**9**, A = 9.

A student with ID 550464024**3**, A = 3

A student with ID 550464011**0**, A = 5.

(d) From previous question, suppose an investor wants A% expected rate of return,

i. how should the investor allocate his or her fund between the market portfolio and risk fee asset?

ii. What is the standard deviation of the portfolio?

iii. Graphically illustrate the investor's choice, the riskfree asset, the market portfolio and CML.

NOTE: PLEASE indicate clearly which part of question you are answering. DO this way in the exam.

i ANSWER.

$E(R_p) = aE(R_m) + (1 - a)R_f$

Solve for a

$a = \dfrac{E(R_p) - R_f}{E(R_m) - R_f} = \dfrac{A\% - 0.01}{0.093 - 0.01}$.

The value of a are as folows.

A=rate of return	a =proportion invested in Market Portfolio
0.01	0.00
0.02	0.12
0.03	0.24
0.04	0.36
0.05	0.48
0.06	0.60
0.07	0.72
0.08	0.84
0.09	0.96

ii. ANSWER.

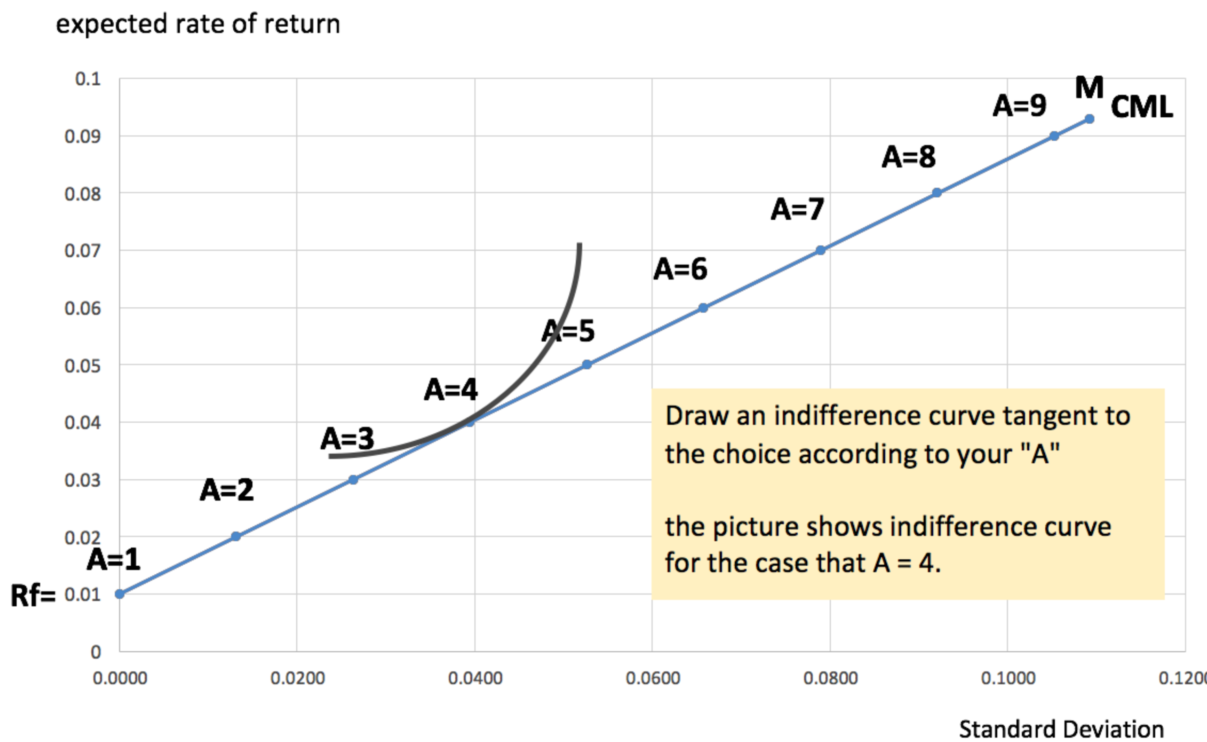
standard deviation of portfolio = $\sigma_p = a\sigma_m$.

The value of standard deviation of the portfolio are as follows.

A =rate of return	a	$a\sigma_m$= standard deviation of the portfolio
0.01	0.00	0.0000
0.02	0.12	0.0132
0.03	0.24	0.0263
0.04	0.36	0.0395
0.05	0.48	0.0526
0.06	0.60	0.0658
0.07	0.72	0.0789
0.08	0.84	0.0921
0.09	0.96	0.1053

iii. ANSWER.

Investor's choice depends on "A".



(e) From question (d), how much the investor invest in stock X and how much investor invest in stock Y.

ANSWER.

For all investors, weight of stock X and the weight of stock Y in the investor's **portfolio of risky assets** (portfolio M) are equal to 0.3 are 0.7, respectively.

Weight of stock X in the entire portfolio (risk-free and risky asset) will be equal to weight of market portfolio $\times$ weight of stock X in the market portfolio $= a \times 0.3$.

Weight of stock Y in the entire portfolio (risk-free and risky asset) will be equal to weight of market portfolio $\times$ weight of stock Y in the market portfolio $= a \times 0.7$.

Proportion of stock X and stock Y in the entire portfolio are as follows.

(1) A=Return	(2)=0.3*(4) Proportion of X in the entire portfolio	(3)=0.7*(4) Proportion of Y in the entire portfolio	(4) proportion of market portfolio = a	(5)=1-(4) proportion of risk-free in the entire portfolio = (1-a)
0.01	0.00	0.00	0.00	1.00
0.02	0.04	0.08	0.12	0.88
0.03	0.07	0.17	0.24	0.76
0.04	0.11	0.25	0.36	0.64
0.05	0.14	0.34	0.48	0.52
0.06	0.18	0.42	0.60	0.40
0.07	0.22	0.51	0.72	0.28
0.08	0.25	0.59	0.84	0.16
0.09	0.29	0.67	0.96	0.04

This means that, if you have $1 to invest, the column (2) shows the amount you will invest in X, the column (3) shows the amount you will invest in Y and the column (5) shows the amount you will invest in risk-free asset. Examples are as follows.

- A = 0.01 , you will invest 0 in X, 0 in Y and 1 in risk free asset.
- A = 0.02 , you will invest 0.04 in X, 0.08 in Y and 0.88 in risk free asset.
- A = 0.03 , you will invest 0.07 in X, 0.17 in Y and 0.76 in risk free asset.

(f) From question (d), what are the weight of stock X and the weight of stock Y in the investor's **portfolio of risky assets** (portfolio M)?

ANSWER. For all investors, weight of stock X and the weight of stock Y in the investor's **portfolio of risky assets** (portfolio M) are equal to 0.3 are 0.7, respectively.

3. You own 50 shares of stock A, which has a price of $12 per share, and 100 shares of stock B, which has a price of $6 per share. What is the portfolio weight for stock B in your portfolio?

ANSWER. $Weight_i = \frac{\text{Value of asset i}}{\text{Value of the portfolio}}$

Value of stock A = $\$12 \times 50 = 600$.

Value of stock B = $\$6 \times 100 = 600$

Value of portfolio = Value of stock A + Value of stock B = 600 + 600 = 1,200

$$\text{Weight for stock A} = \frac{\text{Value of stock A}}{\text{Value of the portfolio}} = \frac{600}{1200} = 0.5$$

$$\text{Weight for stock B} = \frac{\text{Value of stock B}}{\text{Value of the portfolio}} = \frac{600}{1200} = 0.5$$

4. Imagine you have a portfolio of two risky stocks, stock X and Stock Y.

(a) What does diversification mean?

ANSWER. In portfolio theory, diversification refers to spreading your funds across different assets. Benefit from diversification occurs, when you spread your funds across different assets, you can get a better risk and return trade-off. Mathematically, the risk of the portfolio is lower than the weighted average of the component standard deviation. Benefits from diversification occurs when $-1 \leq r_{x,y} < 1$ (X and Y are less than perfectly correlated). Investing in both X and Y (a portfolio comprising both X and Y) is better than investing in only the single asset X or investing in only the single asset Y and the portfolio standard deviation is less than the weighted average of the component standard deviations.

(b) Suppose it turns out that the portfolio have no benefit from diversification.
What is the reason for getting no benefit from diversification?

ANSWER. This means that we invest in both X and Y and then we cannot get a better risk and return trade-off. The portfolio consisting both X and Y has standard deviation exactly equal to the weighted avarage of the standard deviation of X and standard deviation of Y. The reason is that X and Y are perfectly correlated. In other words, correlaiton between X and Y is equal to 1.

(c) Suppose it turns out that the portfolio have benefit from diversification.
What is the reason for getting some benefits from diversification?

ANSWER. This means that we invest in both X and Y and then we can get a better risk and return trade-off. The portfolio consisting both X and Y has standard deviation less than the weighted avarage of the standard deviation of X and standard deviation of Y. The reason is that X and Y are less than perfectly correlated. In other words, correlaiton between X and Y is less than 1; $-1 \leq r_{xy} < 1$.