Production and Costs in the Long run

EE311

Chayun Tantivasadakarn

Faculty of Economics, Thammasat University

New Concepts

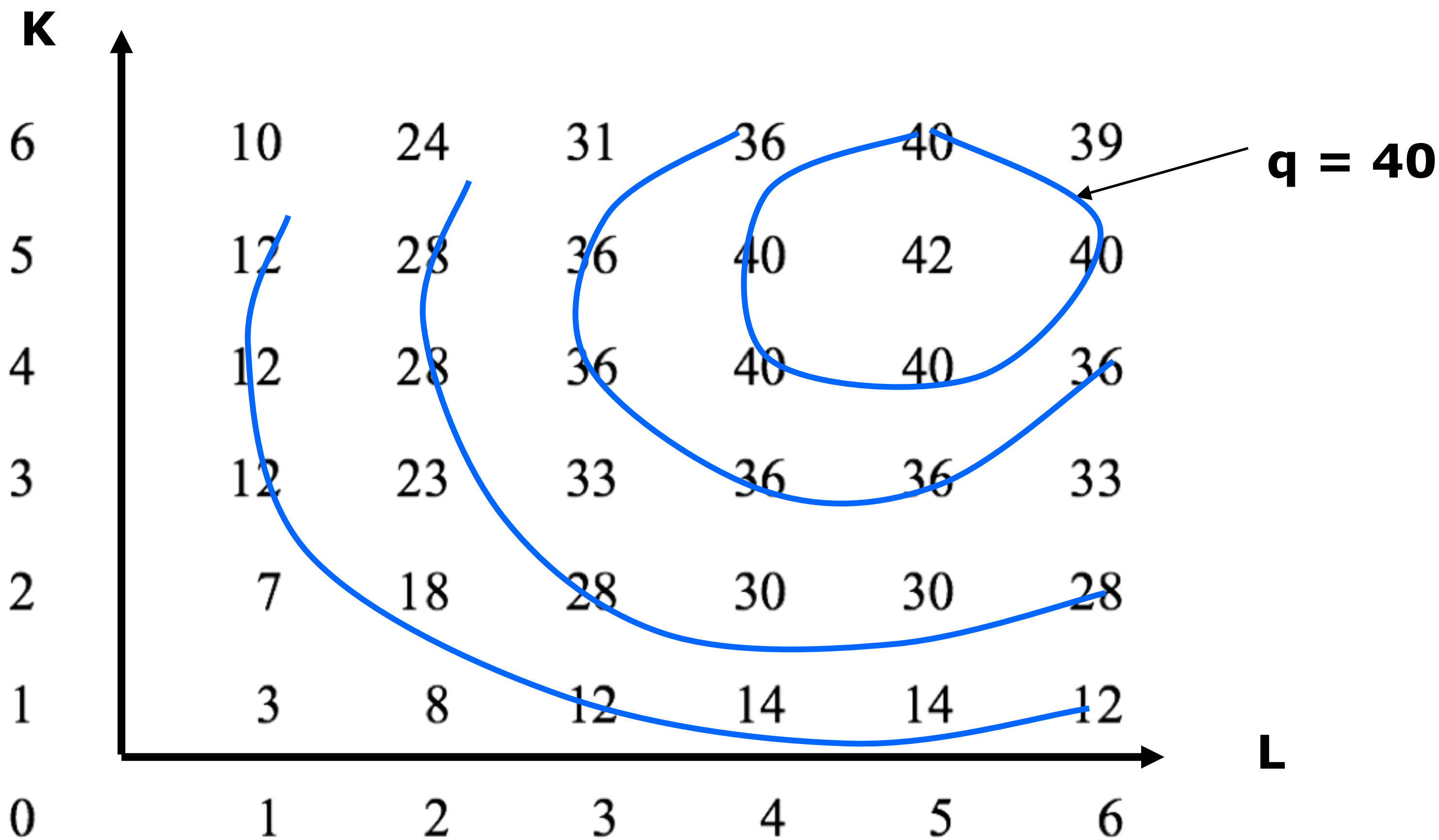
- Economies of Scale
- Economies of Scope
- Returns to Scale
- Learning Curve
- Elasticity of Substitution
- Elasticity of TC

Production: Two Variable Inputs

- In the long-run, capital and labor are both variable.

- We can look at the output we can achieve with different combinations of capital and labor

Production: Two Variable Inputs

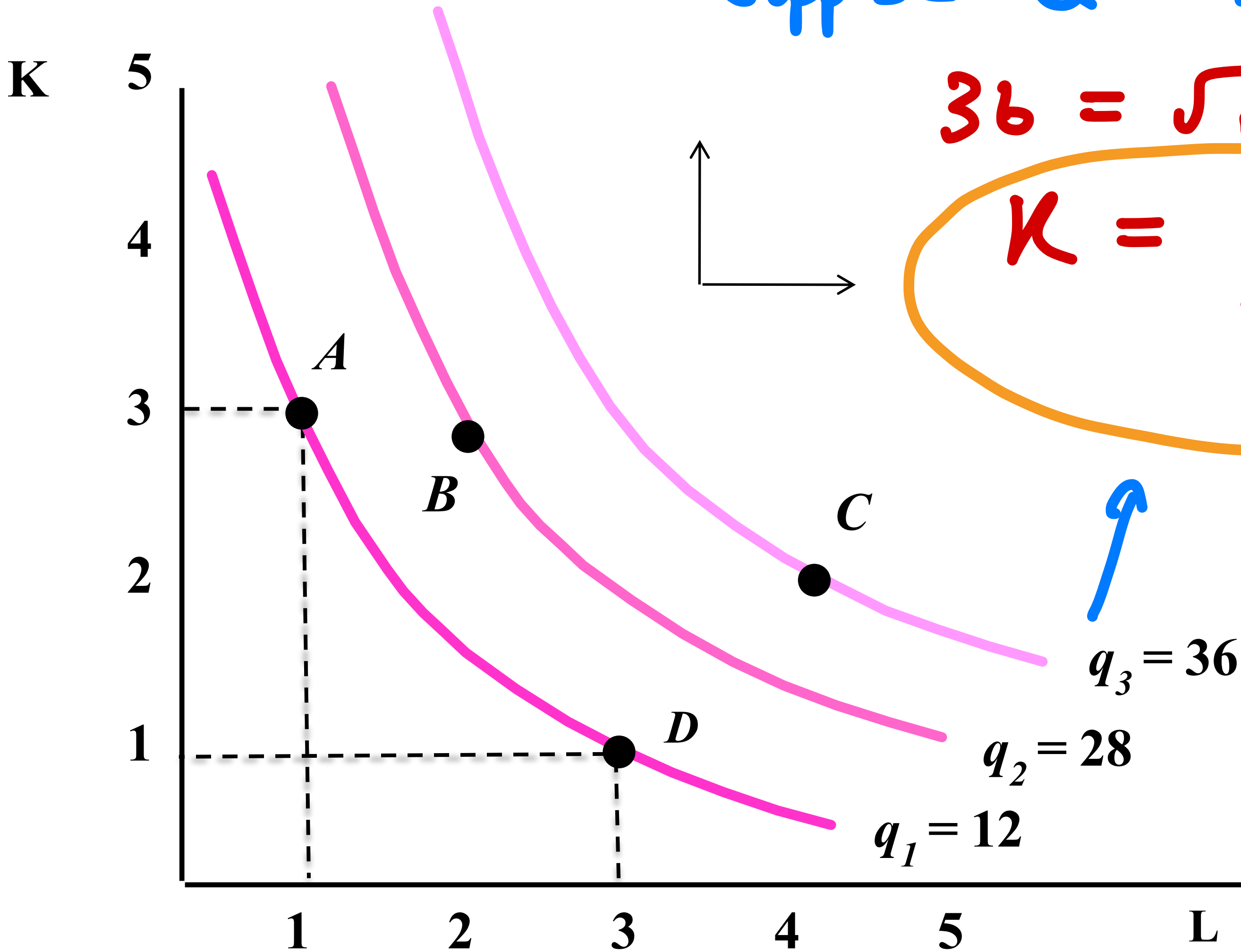


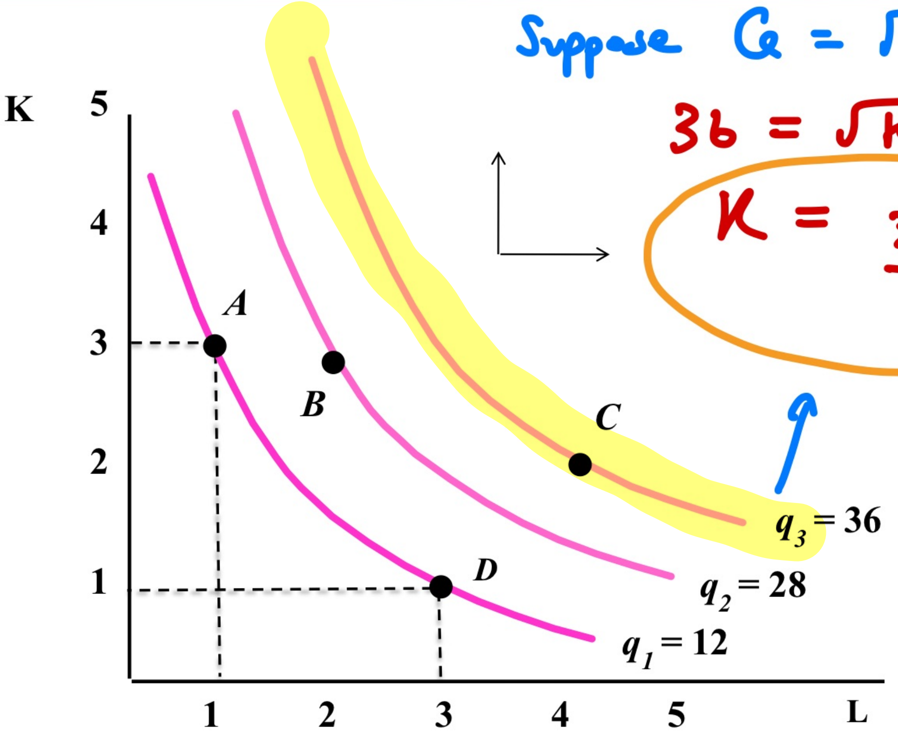
Isoquant Map

Suppose $Q = \sqrt{KL}$

$36 = \sqrt{KL}$

$K = \frac{36^2}{L}$





Suppose $q = \sqrt{KL}$

$36 = \sqrt{KL}$

$$K = \frac{36^2}{L}$$

$$K = 36^2 L^{-1}$$

Slope of q_3 isoquant

$$\frac{dK}{dL} = -36^2 L^{-2} = \frac{-36^2}{L^2} < 0$$

which depends on L

low L → steep

high L → flat

$q_3 = 36$

Derivation of an Isoquant

- From a production function $Q = K^{0.5}L^{0.5}$
- A a fixed Q, $K = Q^2/L$ $= \sqrt{KL}$
- Suppose Q = 100, the function for this isoquant is: $K = 10000/L$
- Any combination of K & L according to the above function will always give Q = 100 and the slope is declining as L increases

Production: Two Variable Inputs

- Substituting Among Inputs

$MRTS_{L,K} = \frac{MPL}{MPK}$

  - Negative slope is the marginal rate of technical substitution (MRTS) = dK/dL

  - **MRTS: amount by which the quantity of one input can be reduced when one extra unit of another input is used, so that output remains constant.**

e.g.

$$MRTS = 4 = -\frac{\Delta K}{\Delta L} = \frac{MPL}{MPK}$$

L ↑ 1 unit ⟶ K ↓ 4 units

firm can replace 4 K with 1 L
(holding output constant)

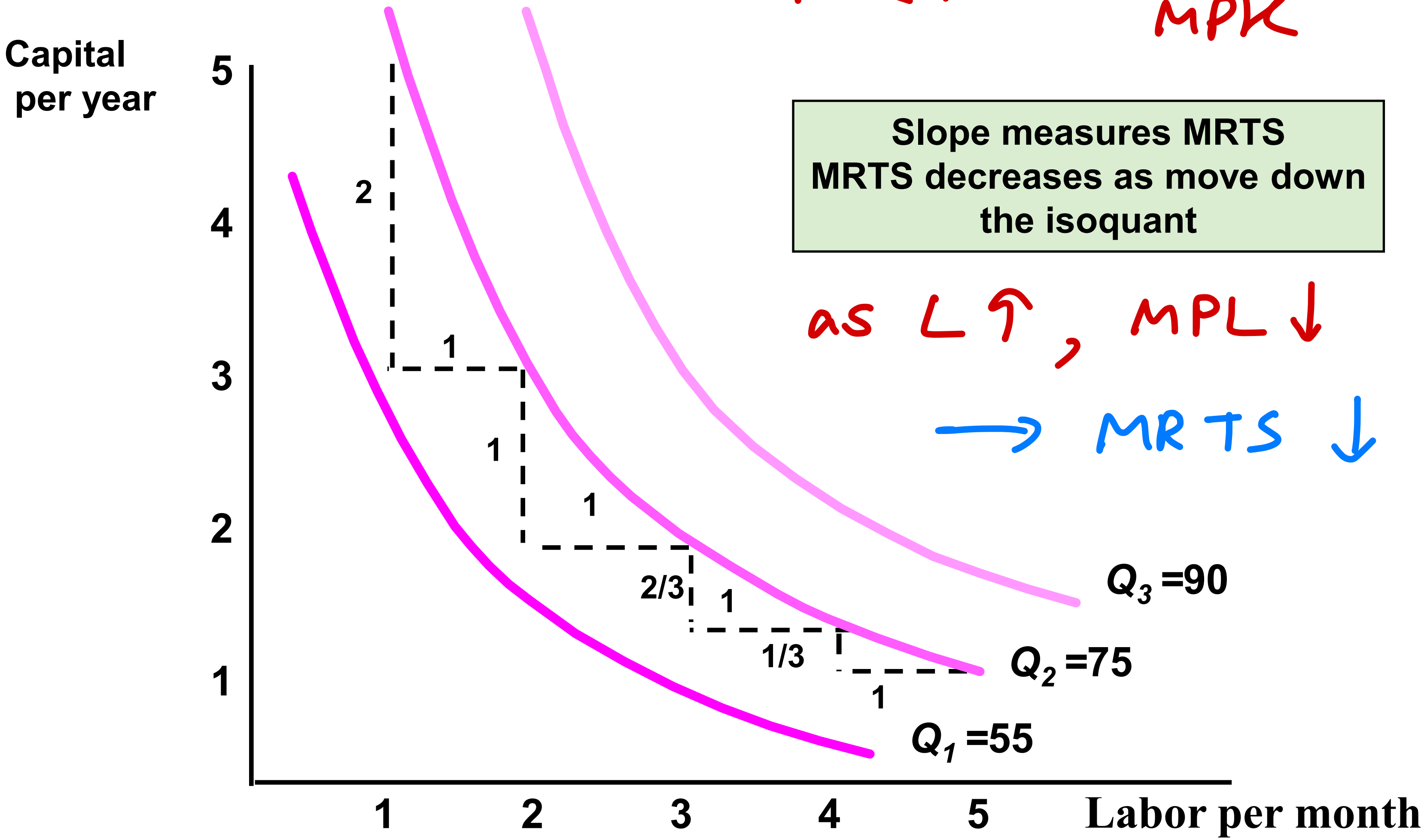
$$\frac{MPL}{MPK} = \frac{4}{1} \longrightarrow \begin{matrix} MPL = 4 \\ MPK = 1 \end{matrix}$$

MPL = 4 : L ↑ 1 unit ⟶ TP ↑ 4 units

MPK = 1 : K ↑ 1 unit ⟶ TP ↑ 1 unit

Marginal Rate of Technical Substitution

$MRTS = \frac{MPL}{MPK}$



Slope measures MRTS
MRTS decreases as move down the isoquant

as $L \uparrow$, $MPL \downarrow$

$\rightarrow MRTS \downarrow$

MRTS and Isoquants

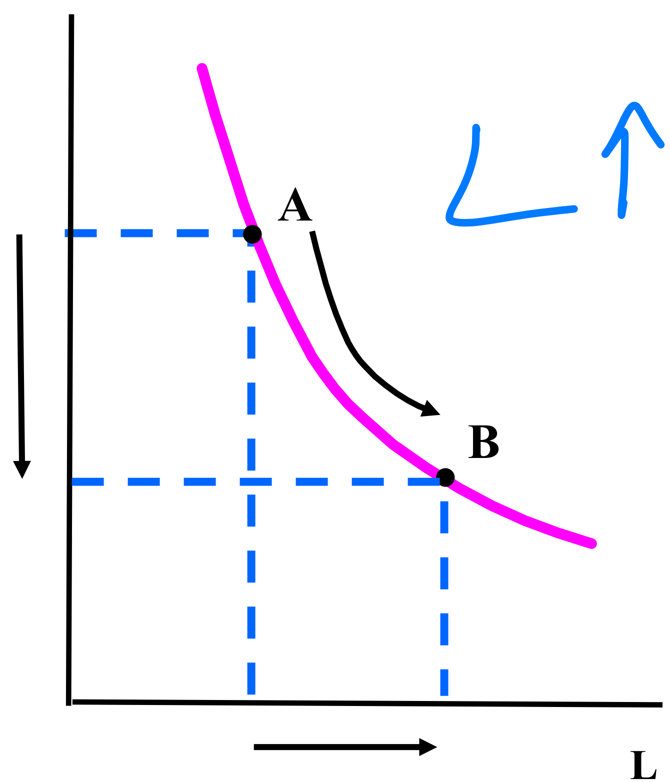
- We assume there is diminishing MRTS
  - Increasing labor in one unit increments from 1 to 5 results in a decreasing MRTS from 1 to 1/2.
  - Productivity of any one input is limited
- Diminishing MRTS occurs because of diminishing returns and implies isoquants are convex.
- There is a relationship between MRTS and marginal products of inputs

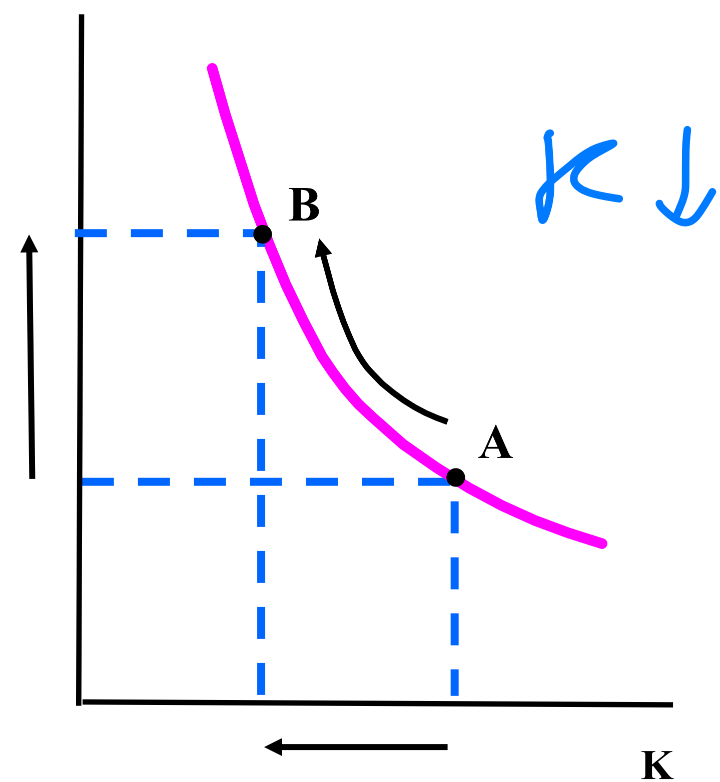
MRTS and Marginal Products

Diminishing

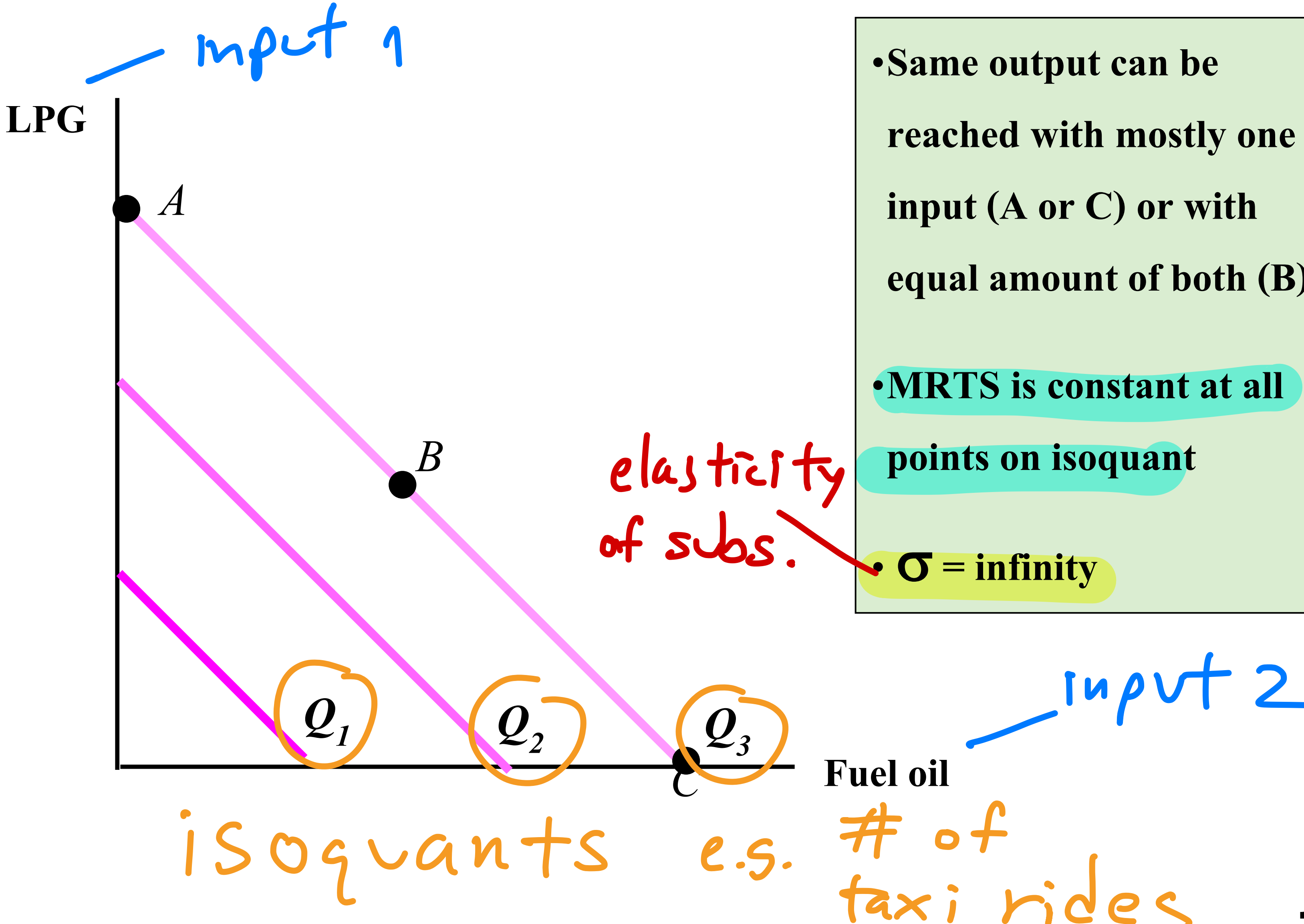
→ L↑, K↓

As we move down the isoquant, MP_L decreases while MP_K increases. Hence MRTS = MP_L / MP_K decreases



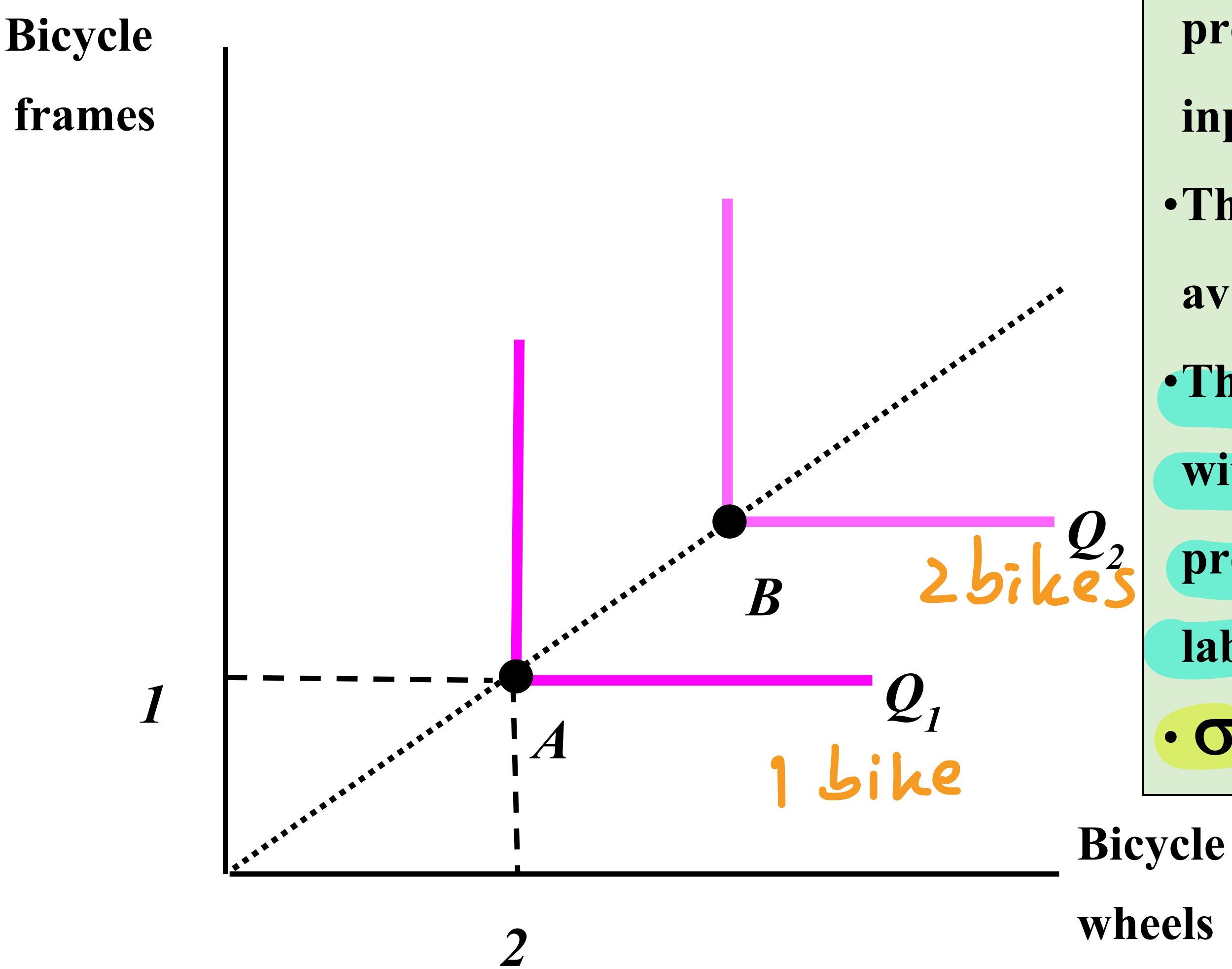


Perfect Substitutes



- Same output can be reached with mostly one input (A or C) or with equal amount of both (B)
- MRTS is constant at all points on isoquant
- σ = infinity

Fixed-Proportions Production Function



- Same output can only be produced with one set of inputs.
- There is no substitution available between inputs.
- The output can be made with only a specific proportion of capital and labor
- $\sigma = 0$

Returns to Scale

- **Long-run production is characterized by Returns to Scale (RTS).**
- Rate at which output increases as inputs are increased proportionately
  - Constant returns to scale
  - Increasing returns to scale
  - Decreasing returns to scale

a proportional increase in ALL inputs leads to equal / larger / smaller proportional increase in output

SR : Law of Dim. Ret. (fixed land)

× 1 × 1 $\longrightarrow$ = 5

× 1 × 2 $\longrightarrow$ = 9

× 1 × 3 $\longrightarrow$ = 12

MP = 4

MP = 3

LR : Returns to Scale (RTS) (land is not fixed)

× 1 × 1 $\longrightarrow$ = 5

× 2 × 2 $\longrightarrow$

= 10

< 10

> 10

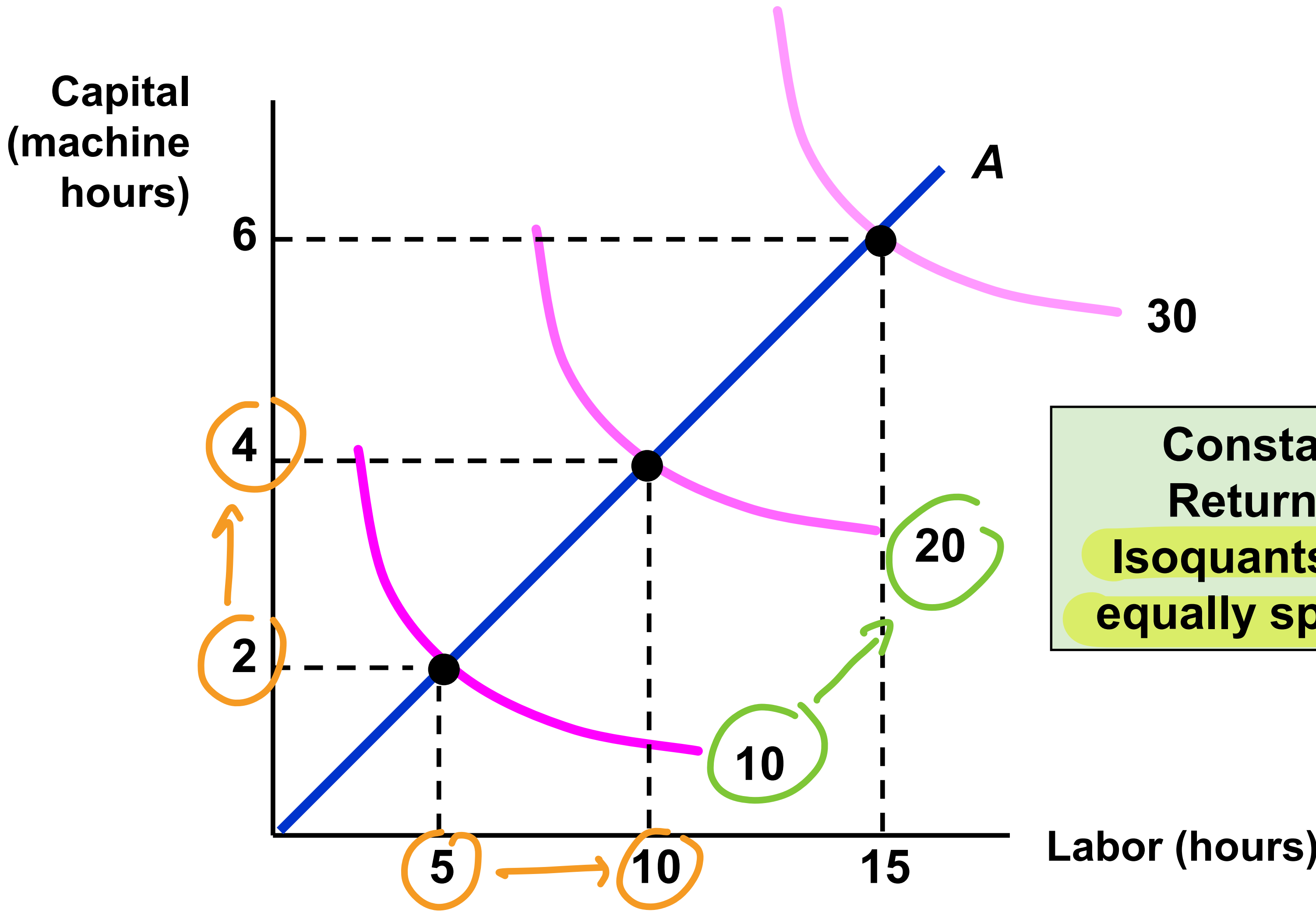
Returns to Scale

- Constant returns to scale: output doubles when all inputs are doubled
  - Size does not affect productivity
  - May have a large number of producers
  - Isoquants are equidistant apart

ALL inputs ↑ 20 % → output ↑ 20 %

(Constant RTS)

Returns to Scale



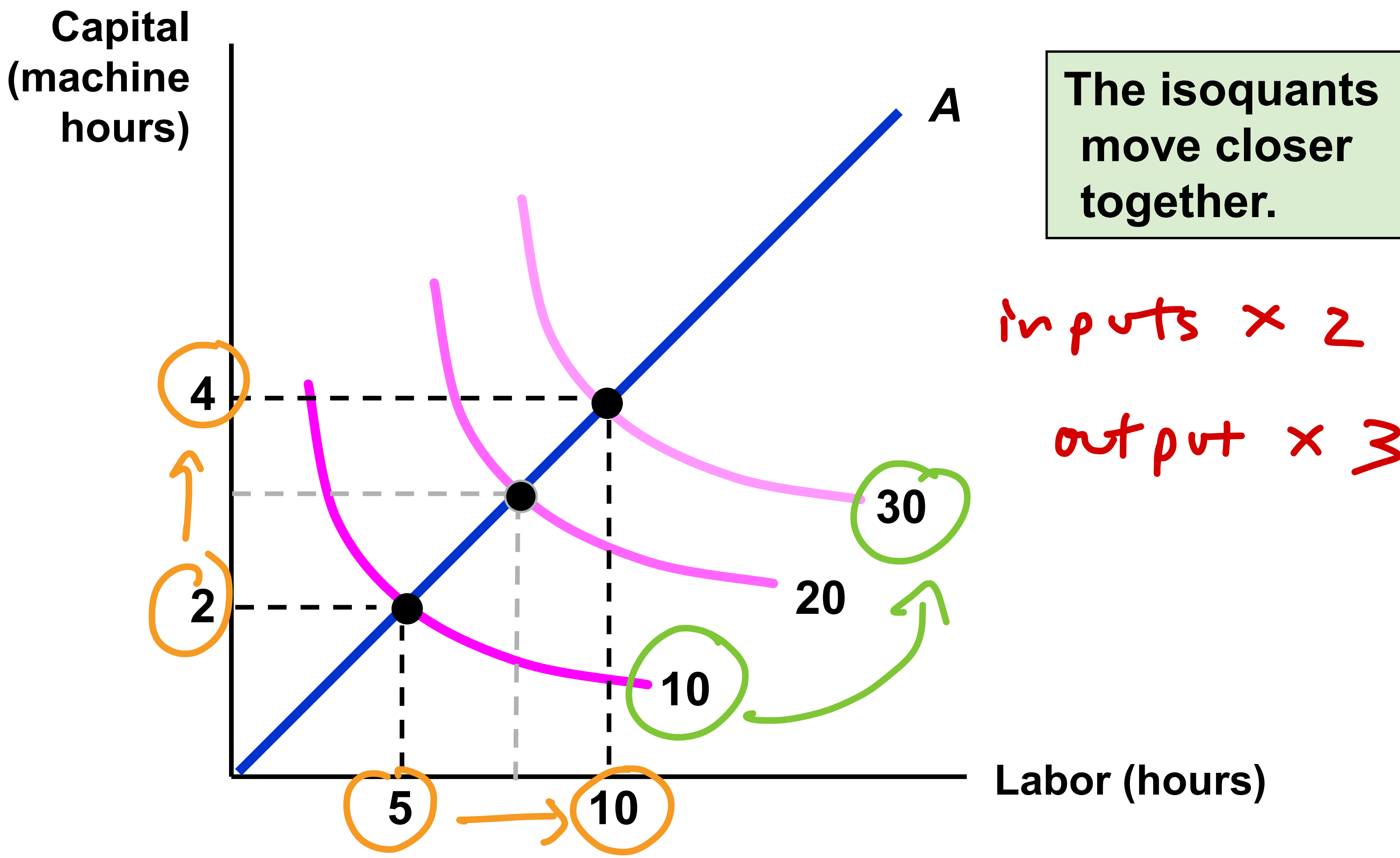
Constant Returns:
Isoquants are equally spaced

Returns to Scale

- Increasing returns to scale: output more than doubles when all inputs are doubled
  - Larger output associated with lower cost (cars)
  - One firm is more efficient than many (utilities)
  - The isoquants get closer together
- Reasons
  - Specialization and division of labor
  - Technical increasing returns to scale

- It leads to a declining cost per unit

Increasing Returns to Scale



The isoquants move closer together.

Returns to Scale

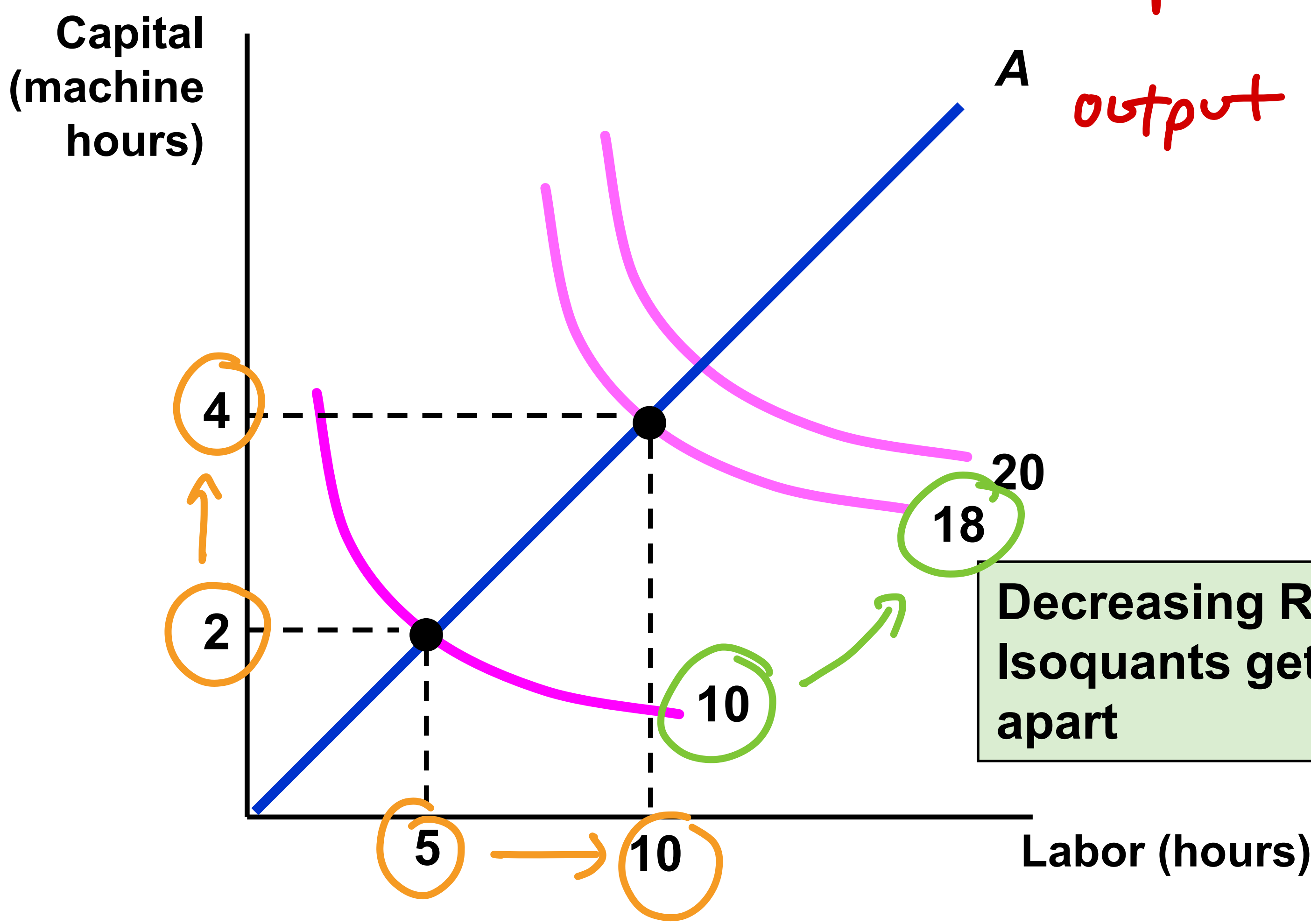
- Decreasing returns to scale: output less than doubles when all inputs are doubled
  - Decreasing efficiency with large size
  - Reduction of entrepreneurial abilities
  - Isoquants become farther apart
- Reasons
  - Coordination problems, limitation of manager

coordination or communication problems

Returns to Scale

inputs × 2

output < × 2



Decreasing Returns: Isoquants get further apart

output $= f(K, L)$

inputs

$f(\lambda K, \lambda L)$ inputs increase by the proportion of λ

$\lambda f(K, L)$ output increases by the proportion of λ

Constant RTS : $f(\lambda K, \lambda L) = \lambda f(K, L)$

Increasing RTS : $f(\lambda K, \lambda L) > \lambda f(K, L)$

decreasing RTS : $f(\lambda K, \lambda L) < \lambda f(K, L)$

Constant RTS

e.g. $Q = f(K, L) = \sqrt{KL}$

suppose $K = L = 2 \longrightarrow Q = 2$

$K = L = 4 \longrightarrow Q = 4$

$$f(2K, 2L) = \sqrt{(2K)(2L)}$$

$$= 2\sqrt{KL} = 2f(K, L)$$

$$f(\lambda K, \lambda L) = \lambda f(K, L)$$

Increasing RTS

e.g. $Q = f(K, L) = KL$

suppose $K = L = 2 \longrightarrow Q = 4$

$K = L = 4 \longrightarrow Q = 16$

$$f(2K, 2L) = (2K)(2L)$$

$$= 4KL = 4f(K, L)$$

$$> 2f(K, L)$$

$$f(\lambda K, \lambda L) > \lambda f(K, L)$$

Returns to Scale: Example

$m = \lambda$

$$Q_0 = AL_0^{\alpha}K_0^{\beta}$$

$$Q_1 = A(mL)_0^{\alpha}(mK)_0^{\beta}, \quad m > 1$$

$$= Am^{\alpha}L_0^{\alpha}m^{\beta}K_0^{\beta}$$

$$= m^{\alpha+\beta}AL_0^{\alpha}K_0^{\beta} = m^{\alpha+\beta}Q_0$$

$\alpha + \beta > 1$, **Increasing returns to scale**

$\alpha + \beta = 1$, **Constant returns to scale**

$\alpha + \beta < 1$, **Decreasing returns to scale**

$$f(K, L) = K^{\alpha} L^{\beta}$$

$$f(\lambda K, \lambda L) = (\lambda K)^{\alpha} (\lambda L)^{\beta}$$

$$= \lambda^{\alpha} K^{\alpha} \lambda^{\beta} L^{\beta}$$

$$= \lambda^{\alpha+\beta} K^{\alpha} L^{\beta}$$

$$= \lambda^{\alpha+\beta} f(K, L)$$

$$f(\lambda K, \lambda L) = \lambda^{\alpha+\beta} f(K, L) \boxed{\begin{matrix} = \\ > \\ < \end{matrix}} \lambda f(K, L)$$

Cost in the Long Run

no fixed cost

- In the long run a firm can change all of its inputs
- Assumptions
  - Two Inputs: Labor (L) & capital (K)
  - Price of input: wage rate (w), rental rate (r)
- The Isocost Line
  - A line showing all combinations of L & K that can be purchased for the same cost
  - Total cost of production is sum of firm's labor cost, wL and its capital cost rK

$$TC = wL + rK$$

= Budget Line

$P_x X + P_y Y = I$

Isocost

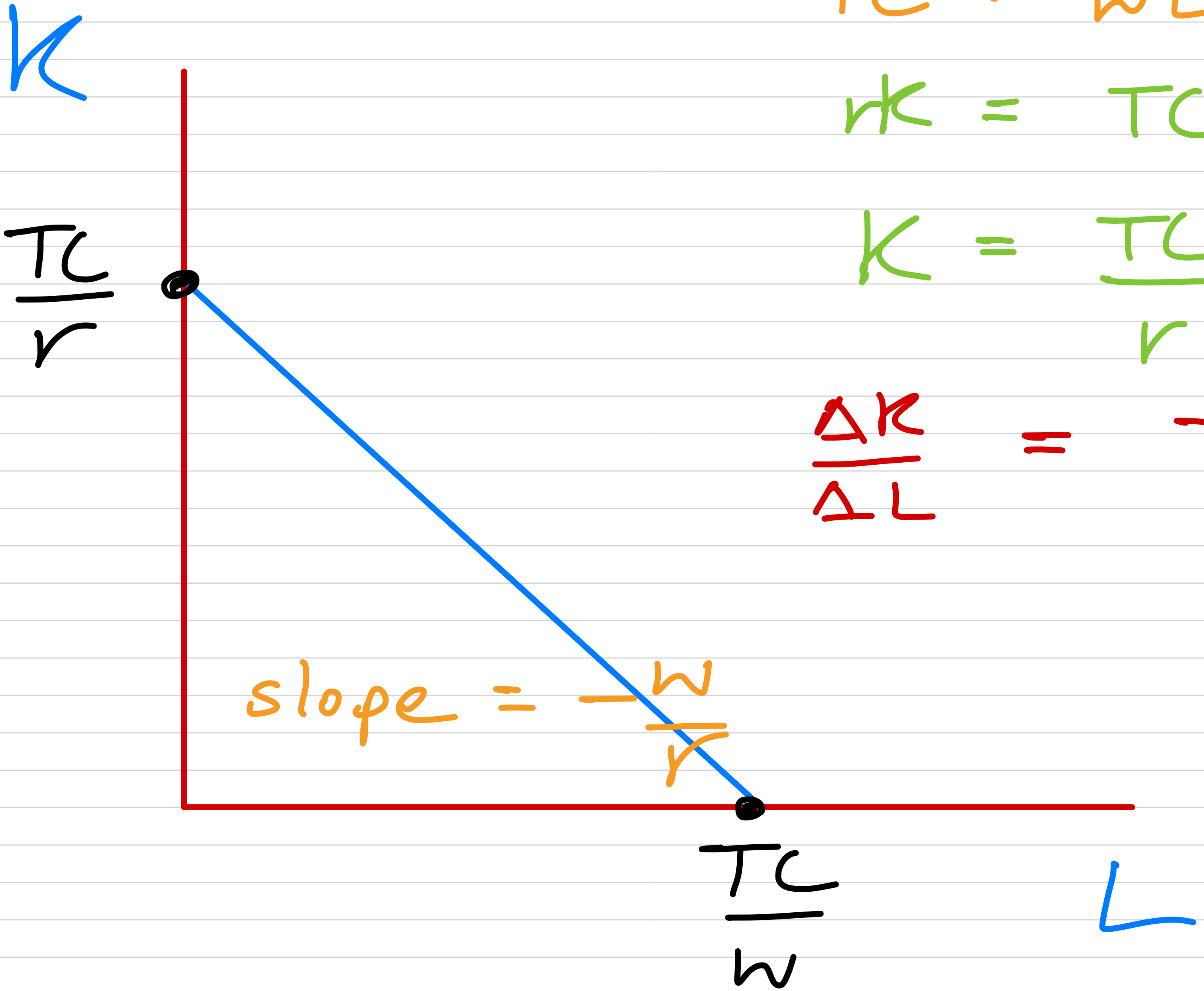
$y = mx + c$

$TC = wL + rK$

$rK = TC - wL$

$K = \frac{TC}{r} - \frac{w}{r}L$

$\frac{\Delta K}{\Delta L} = -\frac{w}{r}$



Cost in the Long Run

- Rewriting TC as an equation for a straight line:
  - $K = TC/r - (w/r)L$
  - Slope of the isocost: $$\Delta K / \Delta L = -\left(w / r\right)$$
    - $-w/r$ is the ratio of the wage rate to rental cost of capital.
    - This shows the rate at which capital can be substituted for labor with no change in cost.

Least Cost Combination of Inputs

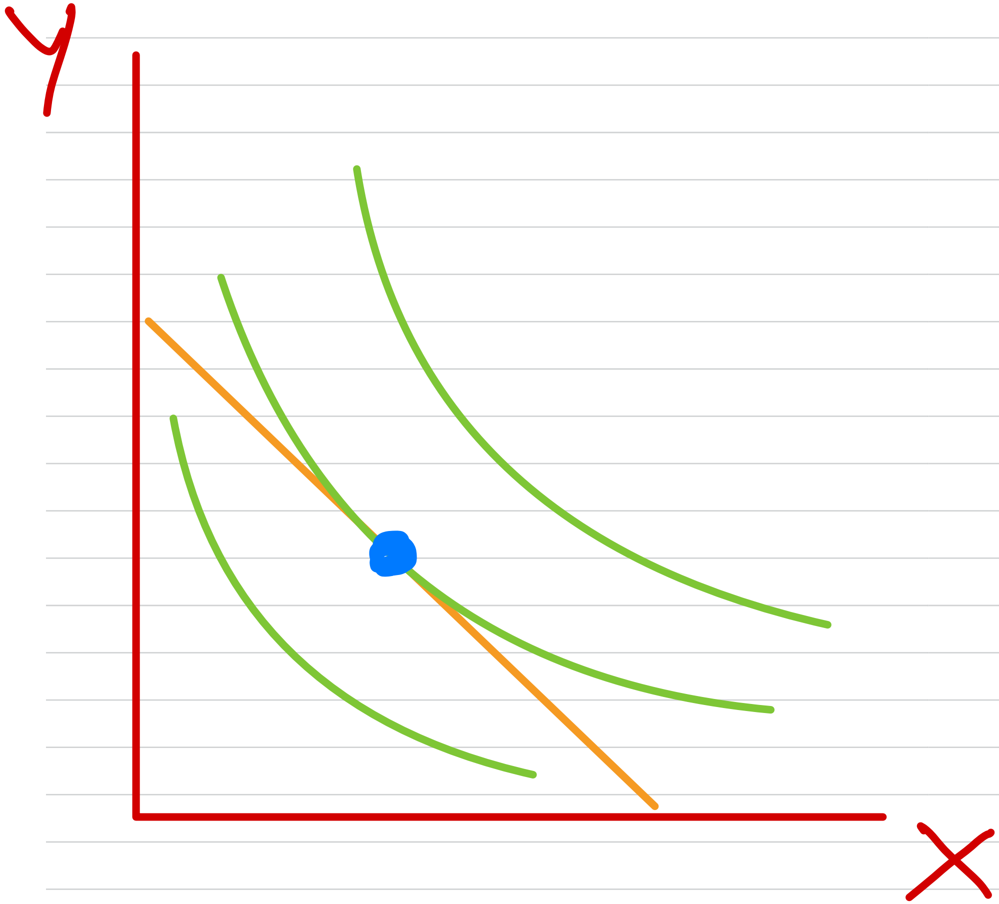
- We will address how to minimize cost for a given level of output by combining isocosts with isoquants max profit $\equiv$ min cost
- We choose the output we wish to produce and then determine how to do that at minimum cost
  - Isoquant is the quantity we wish to produce
  - Isocost is the combination of K and L that gives a set cost

Cost Minimization

→ firm chooses K^* & L^* that minimize its cost to produce a given output.

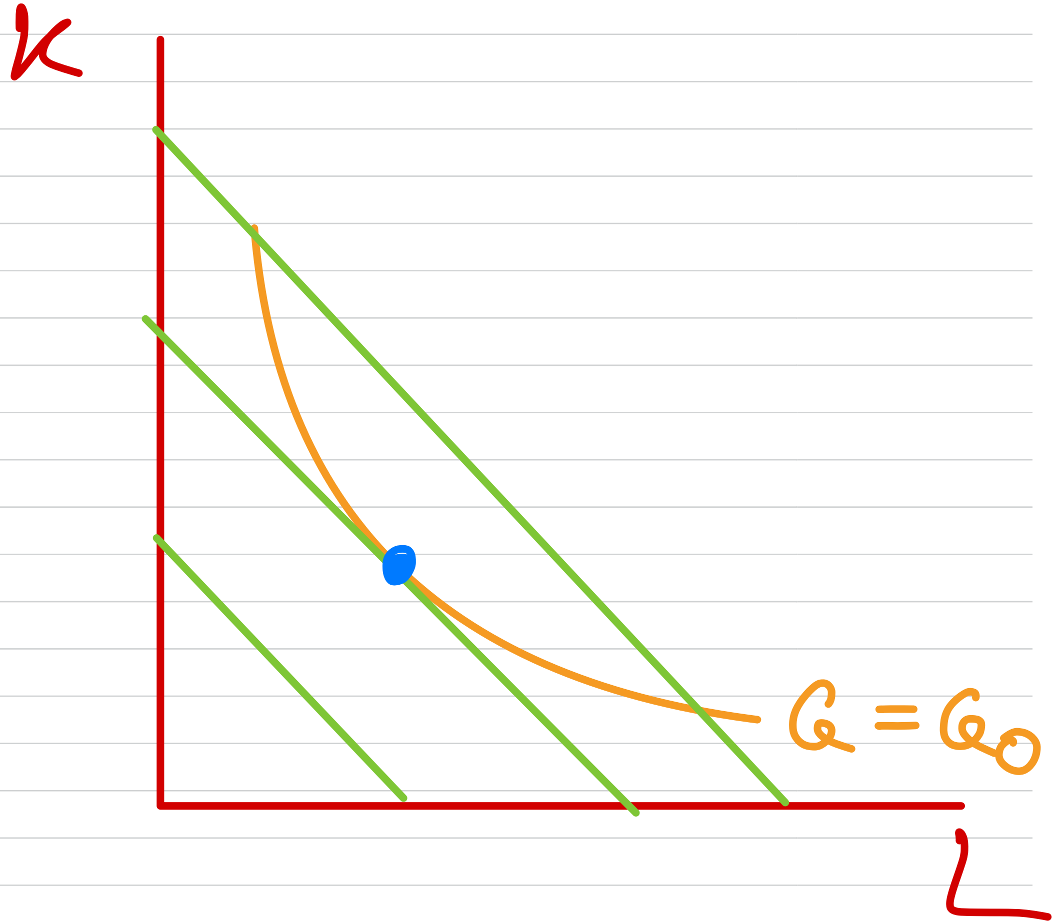
U-max

consumer chooses X, Y
to max $U = f(X, Y)$
s.t. $P_X X + P_Y Y = I$



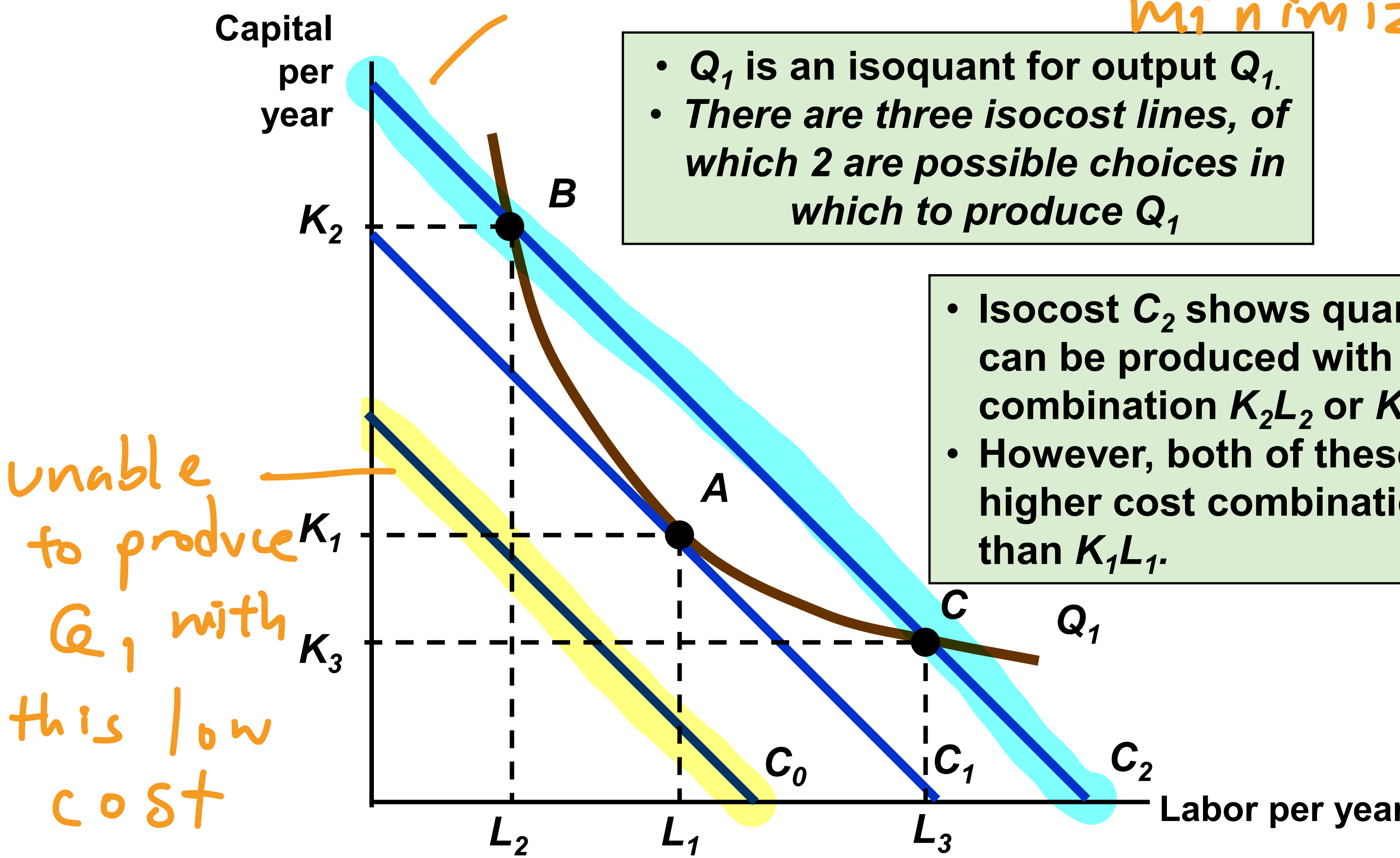
Cost-min

firm chooses L, K
that min $TC = wL + rK$
s.t. $Q = Q_0$



Least Cost Combination of Inputs

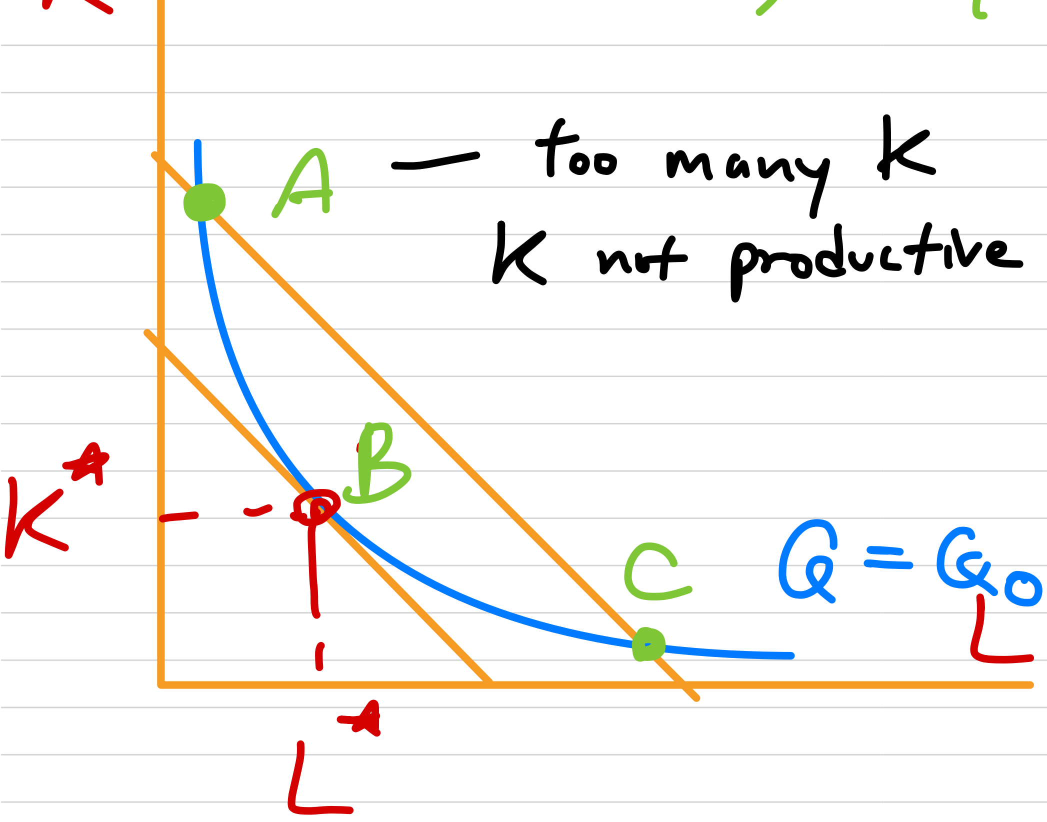
able to make Q_1 but cost is not minimized



- Q_1 is an isoquant for output Q_1.
- *There are three isocost lines, of which 2 are possible choices in which to produce Q_1*

- Isocost C_2 shows quantity Q_1 can be produced with combination K_2L_2 or K_3L_3.
- However, both of these are higher cost combinations than K_1L_1.

at A, isoquant steeper than isocost



$$MRTS > \frac{w}{r}$$

$$\frac{MPL}{MPK} > \frac{w}{r}$$

$$\downarrow$$

$$\frac{MPL}{w} > \frac{MPK}{r}$$

→ one \$ spent on L gives more extra output than one \$ spent on K

→ firm should use more L & less K until $\frac{MPL}{w} = \frac{MPK}{r}$.

Cost in the Long Run

$$MRTS = -\Delta K / \Delta L = MP_L / MP_K$$

$$\text{Slope of isocost line} = \Delta K / \Delta L = -w/r$$

$$MP_L / MP_K = w/r \text{ when firm minimizes cost}$$

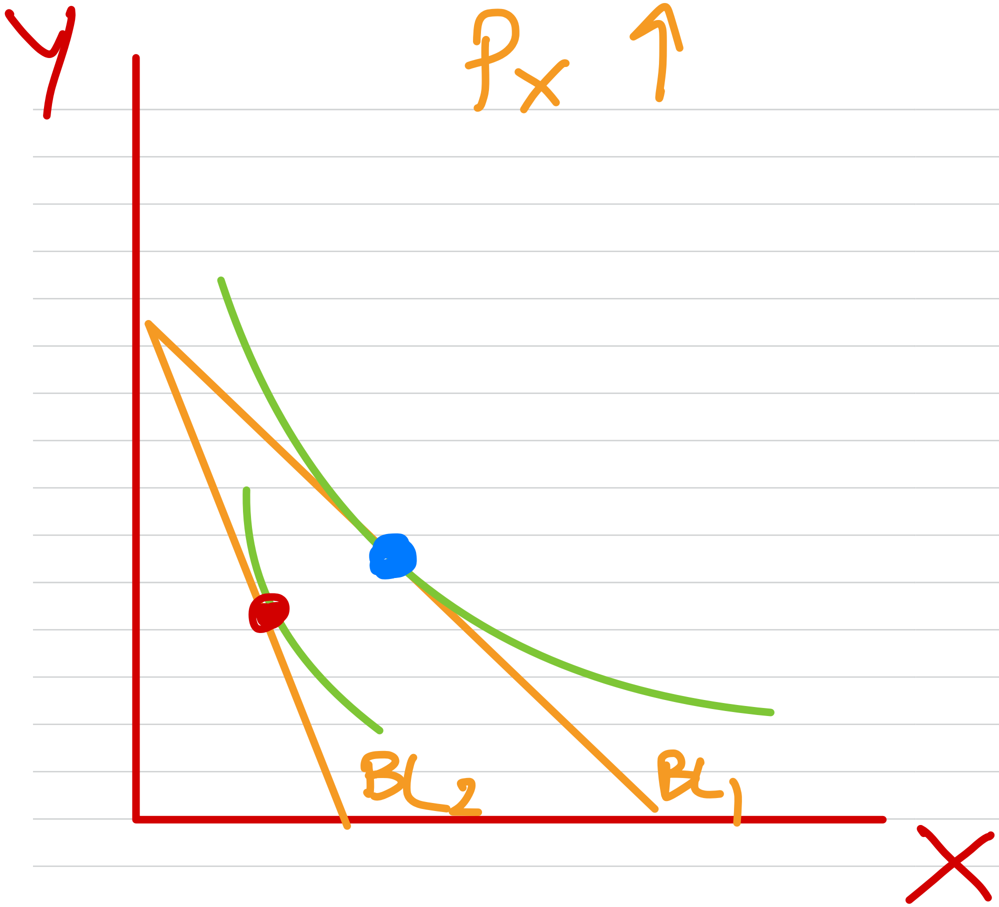
- The minimum cost combination can then be written as:

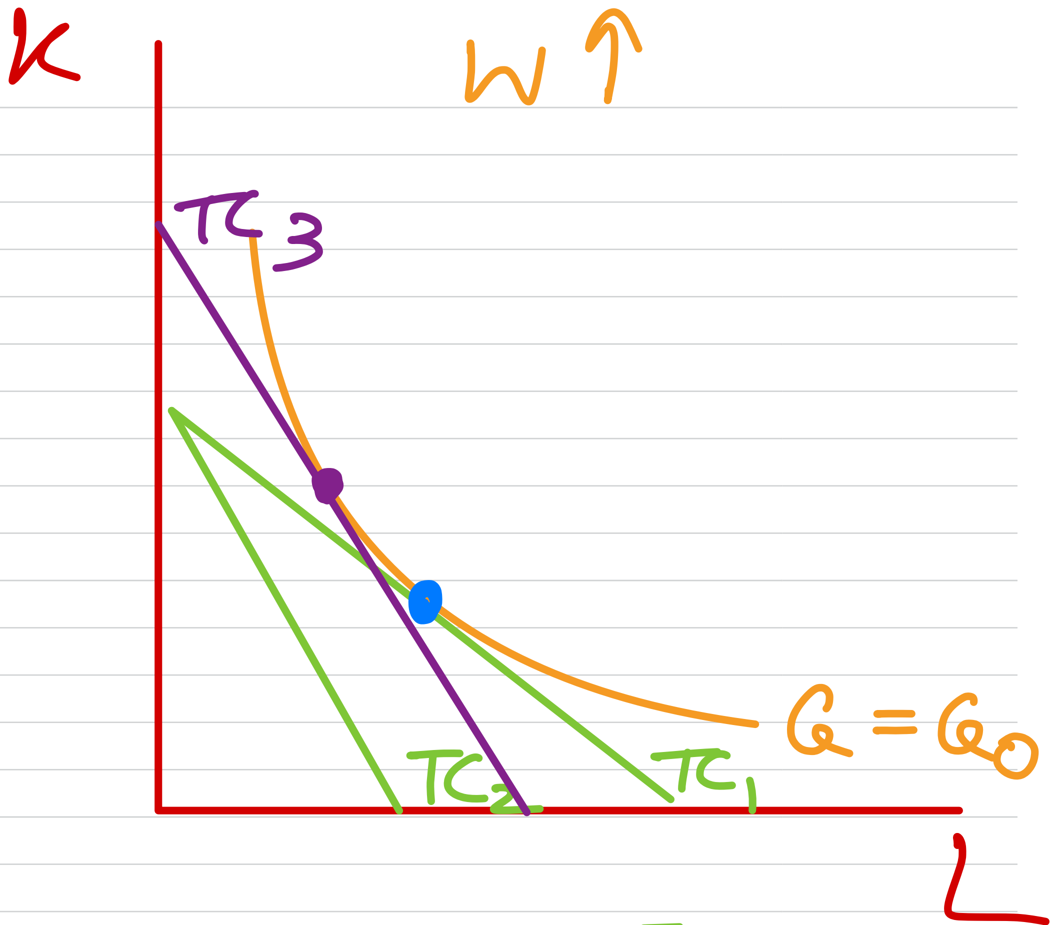
$$MP_L / w = MP_K / r$$

- Minimum cost for a given output will occur when each dollar of input added to the production process will add an equivalent amount of output.

Input Substitution When an Input Price Change

- If the price of labor changes, then the slope of the isocost line change, w/r
- It now takes a new quantity of labor and capital to produce the output
- If price of labor increases relative to price of capital, and capital is substituted for labor



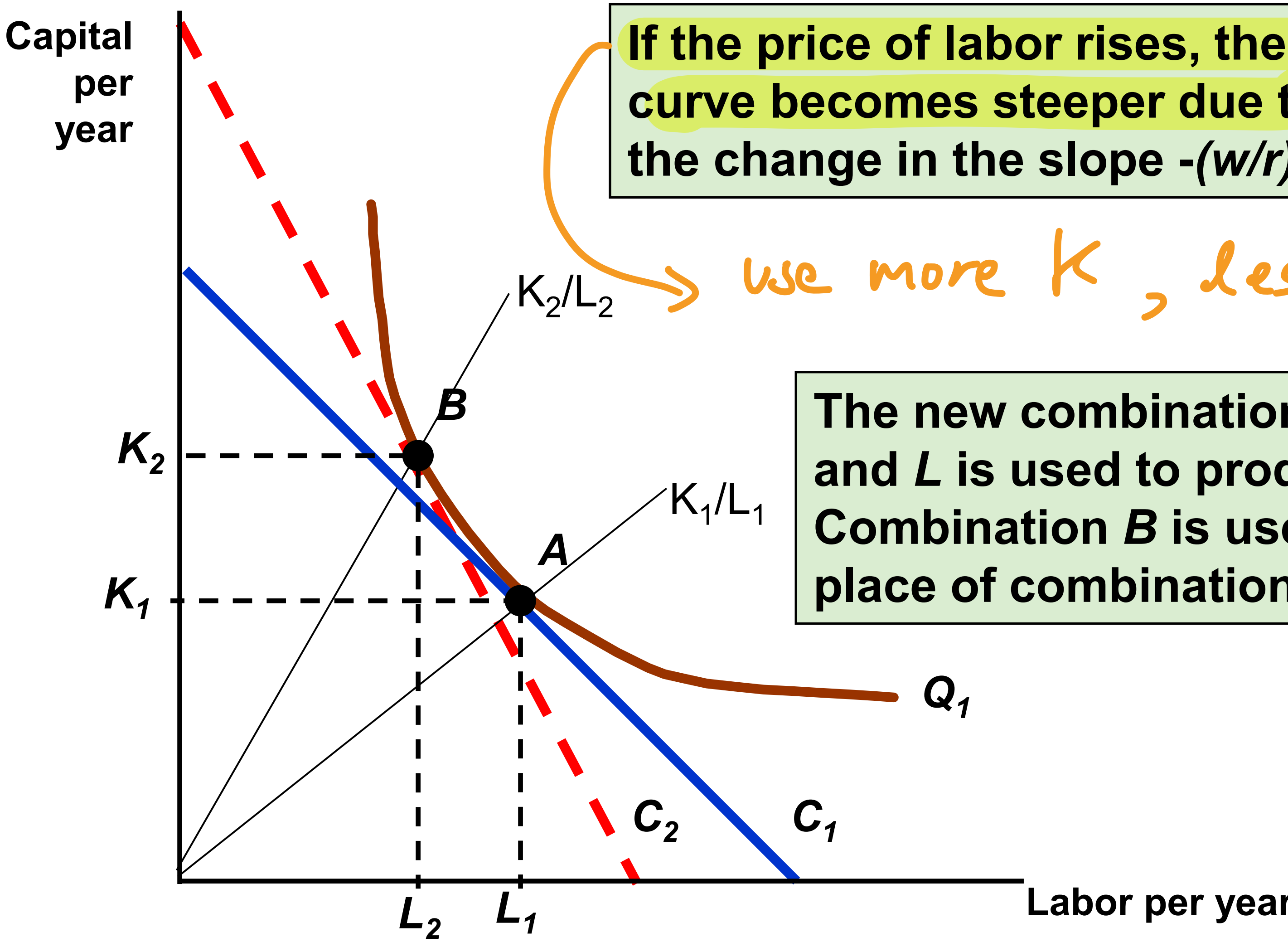


Income at BL_1
= Income at BL_2

$TC_1 = TC_2$
but TC_2 cannot make Q_0

$TC_3 > TC_1 = TC_2$
but TC_3 can make Q_0
at higher w.

Input Substitution When an Input Price Change



If the price of labor rises, the isocost curve becomes steeper due to the change in the slope *-(w/r).*

use more K, less L

The new combination of *K* and *L* is used to produce Q_1. Combination *B* is used in place of combination *A.*

Elasticity of substitution

shows how easy / difficult for the firm to replace one input for another

- $\%\Delta$ in capital-labor ratio divided by the $\%\Delta$ in the slope of the isoquant

$$\sigma = \frac{\%\Delta\left(K/L\right)}{\%\Delta\,\text{MRTS}} = \frac{d\left(K/L\right)}{d\left(\text{MRTS}\right)}\,\frac{\text{MRTS}}{K/L} = \frac{\partial\ln\left(K/L\right)}{\partial\ln\left(\text{MRTS}\right)}$$

$\sigma = 0$ means fixed proportion perfect complements

$\sigma = \infty$ means perfect substitutes

Example

** take derivative w.r.t. ln MRTS

From Cobb-Douglas production function: $Q = K^{\alpha}L^{\beta}$,

$$MRTS = \frac{MP_L}{MP_K} = \frac{\beta K^{\alpha}L^{\beta-1}}{\alpha K^{\alpha-1}L^{\beta}} = \frac{\beta K}{\alpha L}$$

$$\frac{K}{L} = \frac{\alpha}{\beta} MRTS$$

take natural log

$$\ln\frac{K}{L} = \ln\frac{\alpha}{\beta} + \ln MRTS$$

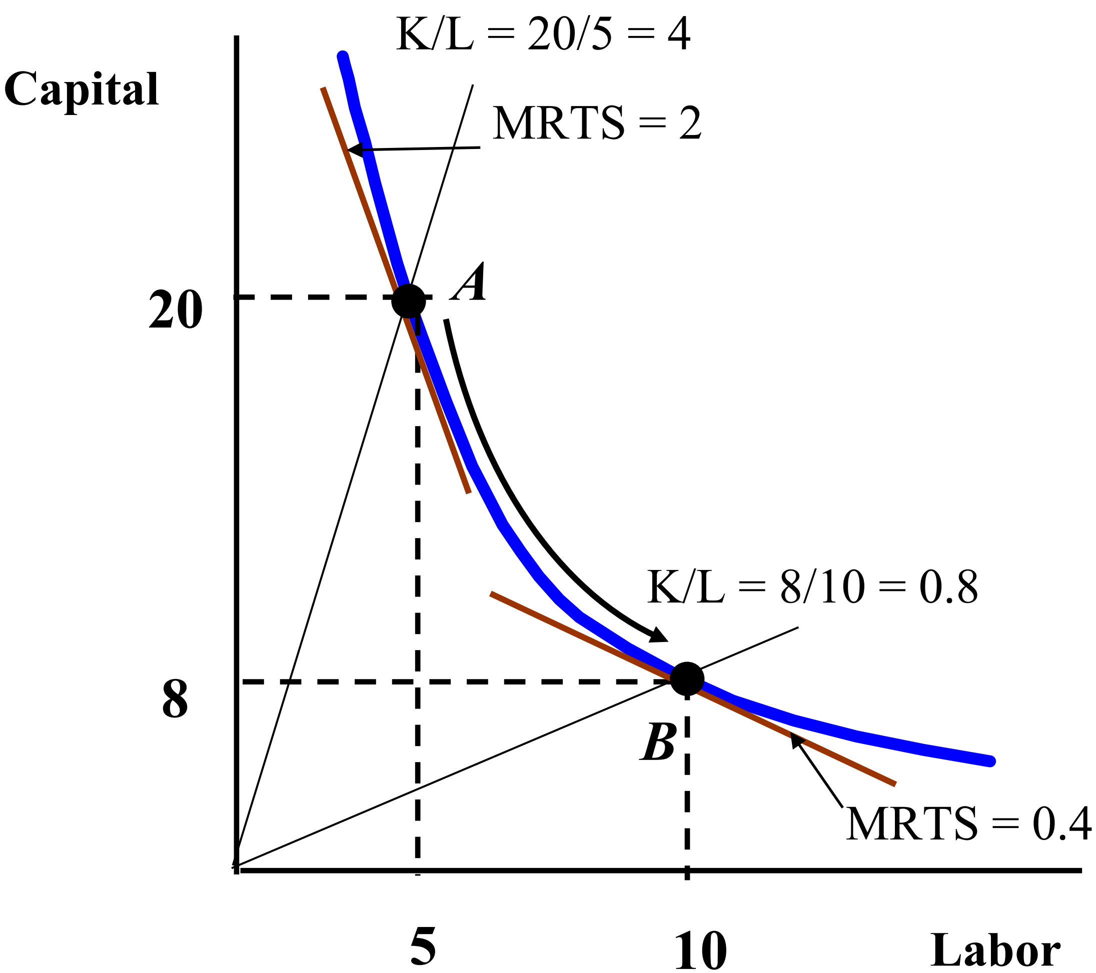
***

always

$$\sigma = 1$$

L & K are imperfect substitutes

Elasticity of substitution

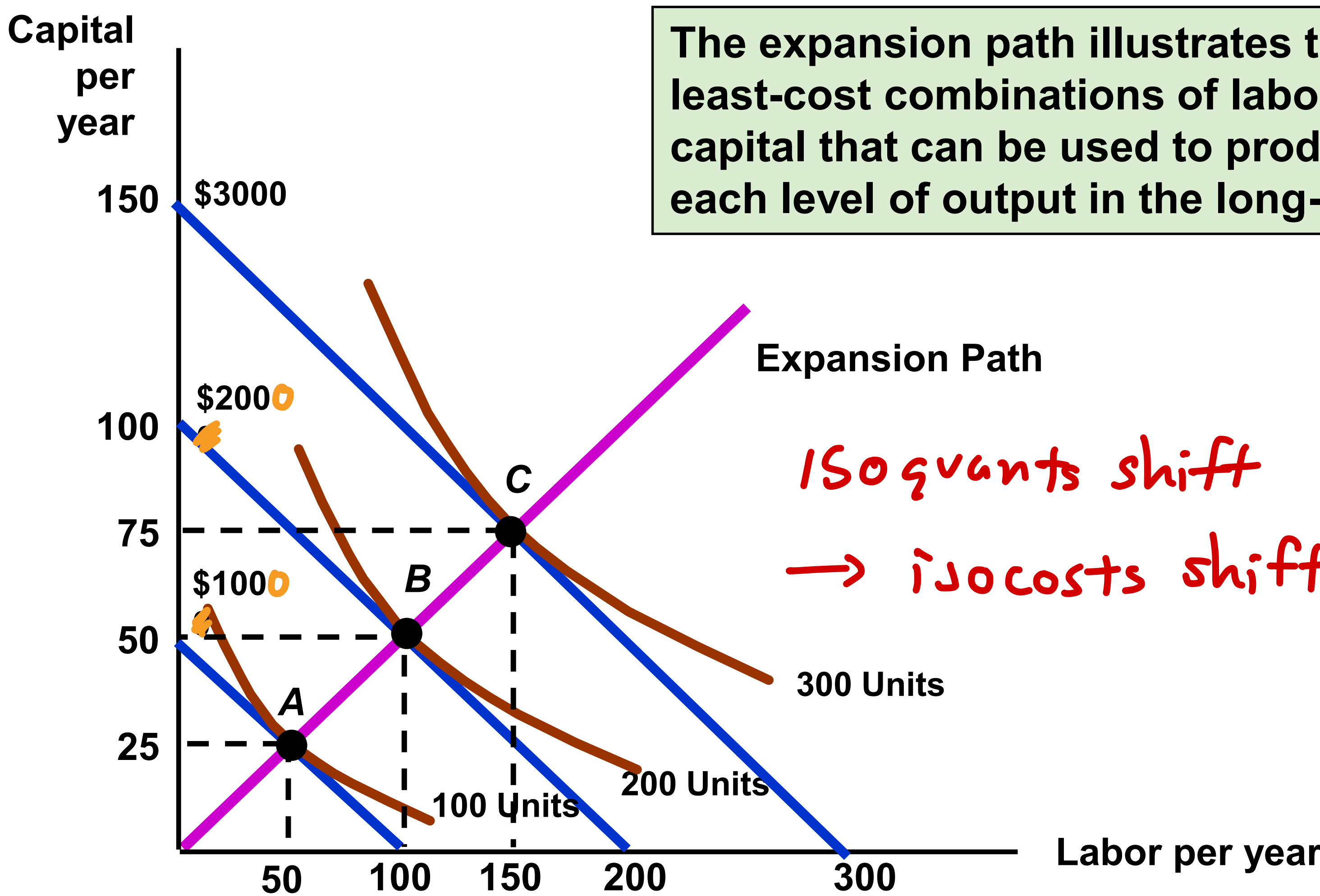


- **Point *A* is more capital-intensive, and *B* is more labor-intensive.**
- **Moving from A to B, K/L decreases by 80% and MRTS also decreases by 80% --> σ = 1**

Cost in the Long Run

- Cost minimization with Varying Output Levels
  - For each level of output, there is an isocost curve showing minimum cost for that output level
  - A firm’s expansion path shows the minimum cost combinations of labor and capital at each level of output.

A Firm’s Expansion Path



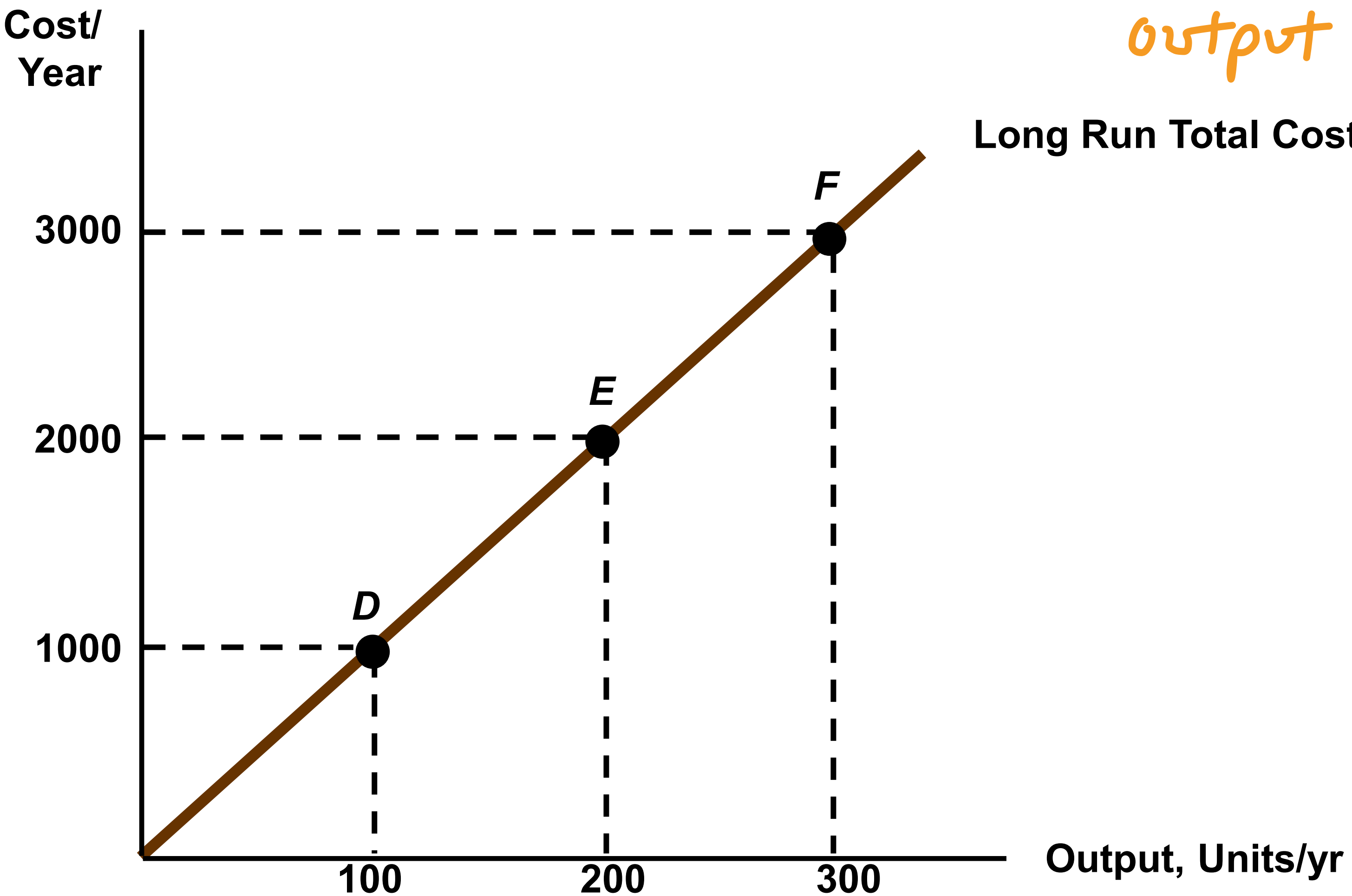
The expansion path illustrates the least-cost combinations of labor and capital that can be used to produce each level of output in the long-run.

Isoquants shift

→ isocosts shift

A Firm's Long-Run Total Cost Curve: CRTS case

constant RTS : inputs × 2
output × 2



Expansion Path & Long-run Costs

- Firms expansion path has same information as long-run total cost curve
- To move from expansion path to LR cost curve
  - Find tangency with isoquant and isocost
  - Determine min cost of producing the output level selected
  - Graph output-cost combination

Long-Run Versus Short-Run Cost Curves

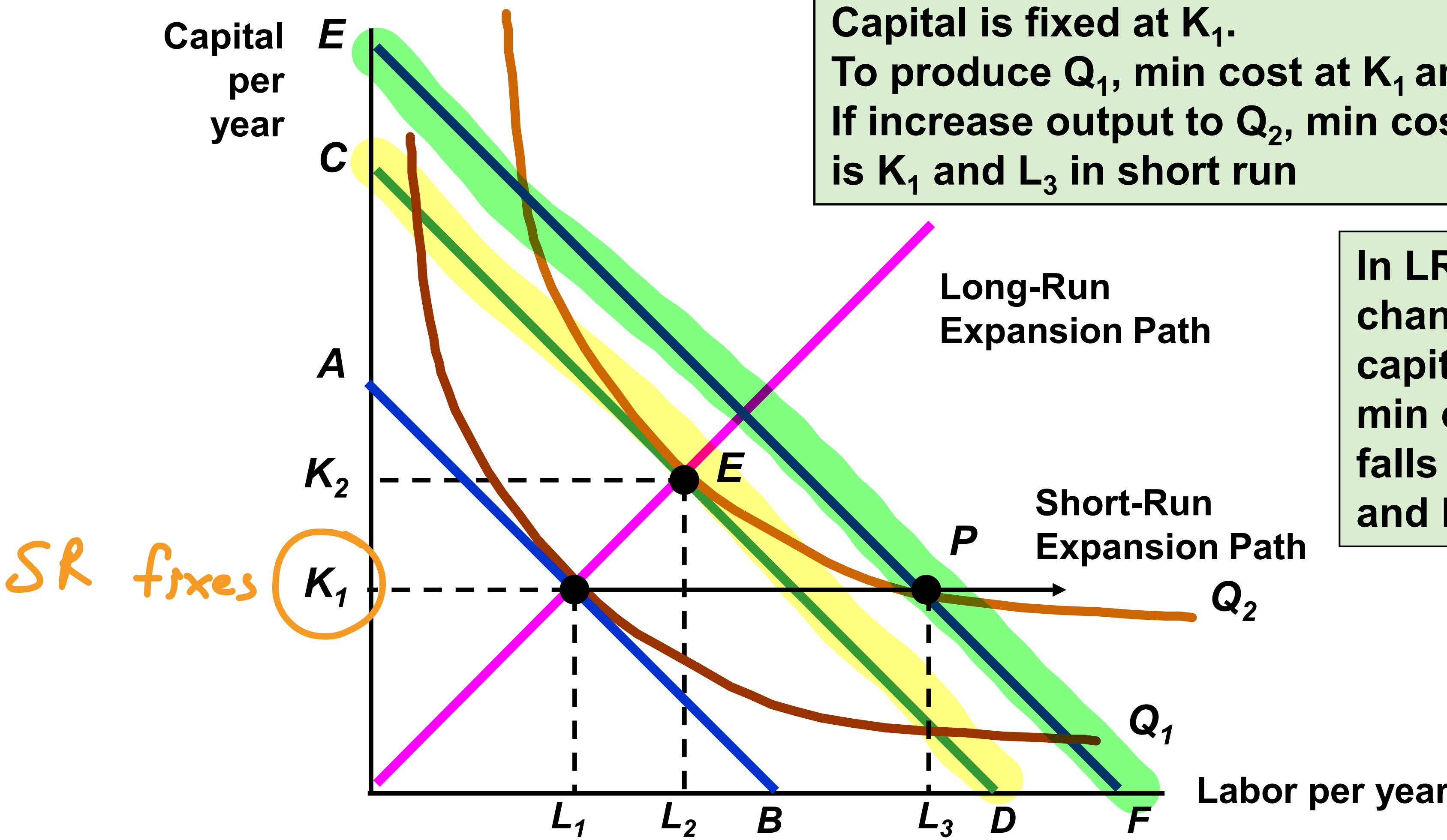
- In the short run some costs are fixed
- In the long run firm can change anything including plant size
  - Can produce at a lower average cost in long run than in short run
  - Capital and labor are both flexible
- We can show this by holding capital fixed in the short run and flexible in long run

The Inflexibility of Short-Run Production

SR cost will be higher

Capital is fixed at K_1.
To produce Q_1, min cost at K_1 and L_1
If increase output to Q_2, min cost is K_1 and L_3 in short run

In LR, can change capital and min costs falls to K_2 and L_2



To produce Q_1,

SR firm uses K_1 & L_1
LR firm uses K_1 & L_1

LR & SR costs are the same

To produce Q_2,

fixed in SR

SR firm uses K_1 & L_3
LR firm uses K_2 & L_2

SR cost > LR cost

CD is isocost for LR firm
EF is iso cost for SR firm

to produce Q_2

Long-Run Versus Short-Run Cost Curves

- Long-Run Average Cost (LAC)
  - Most important determinant of the shape of the LR AC and MC curves is relationship between scale of the firm's operation and inputs required to min cost
- Constant Returns to Scale
  - If input is doubled, output will double
  - AC cost is constant at all levels of output.

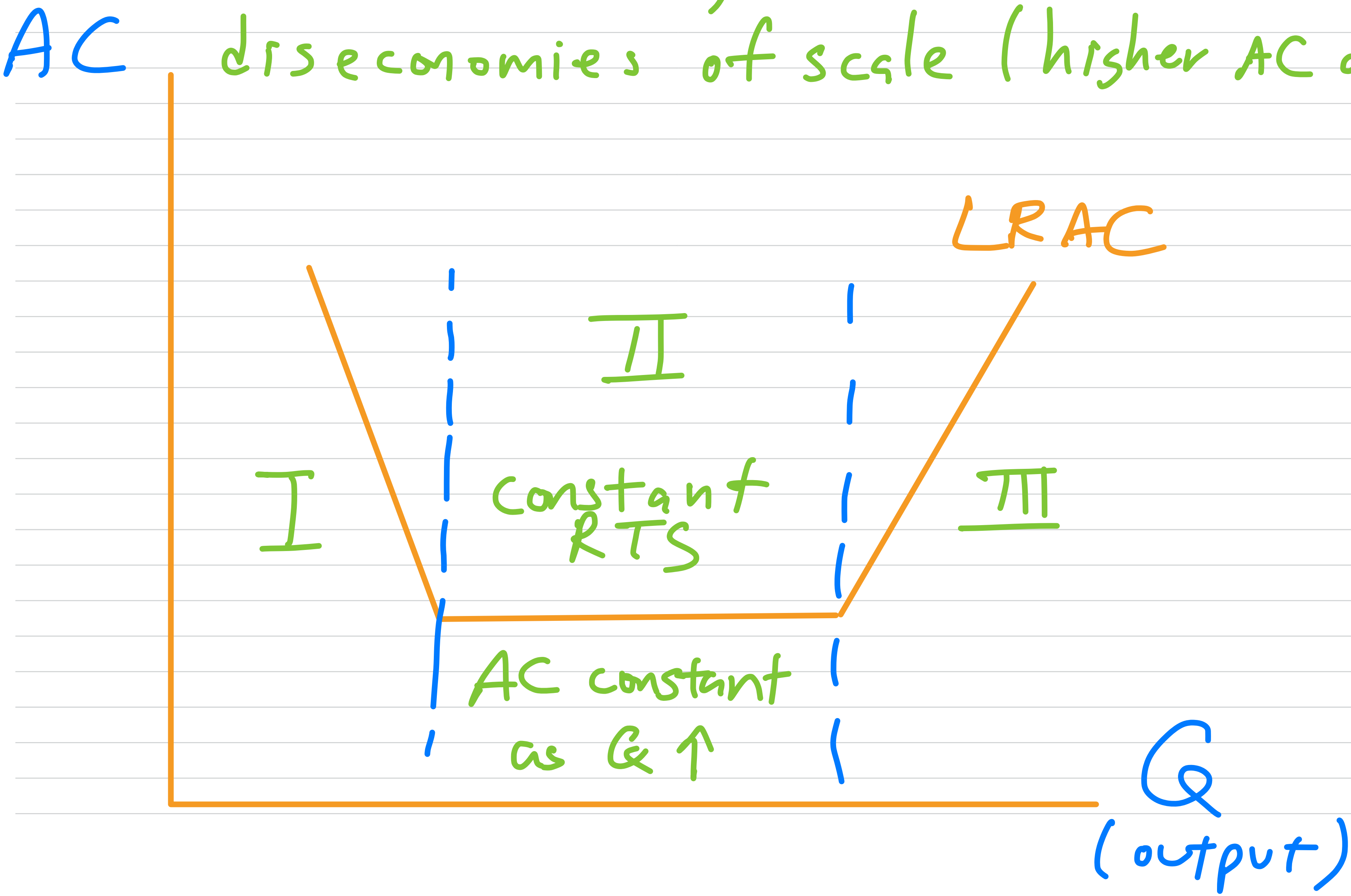
Long-Run Versus Short-Run Cost Curves

- Increasing Returns to Scale
  - If input is doubled, output will more than double

    very productive
  - AC decreases at all levels of output.
- Decreasing Returns to Scale
  - If input is doubled, output will less than double
  - AC increases at all levels of output

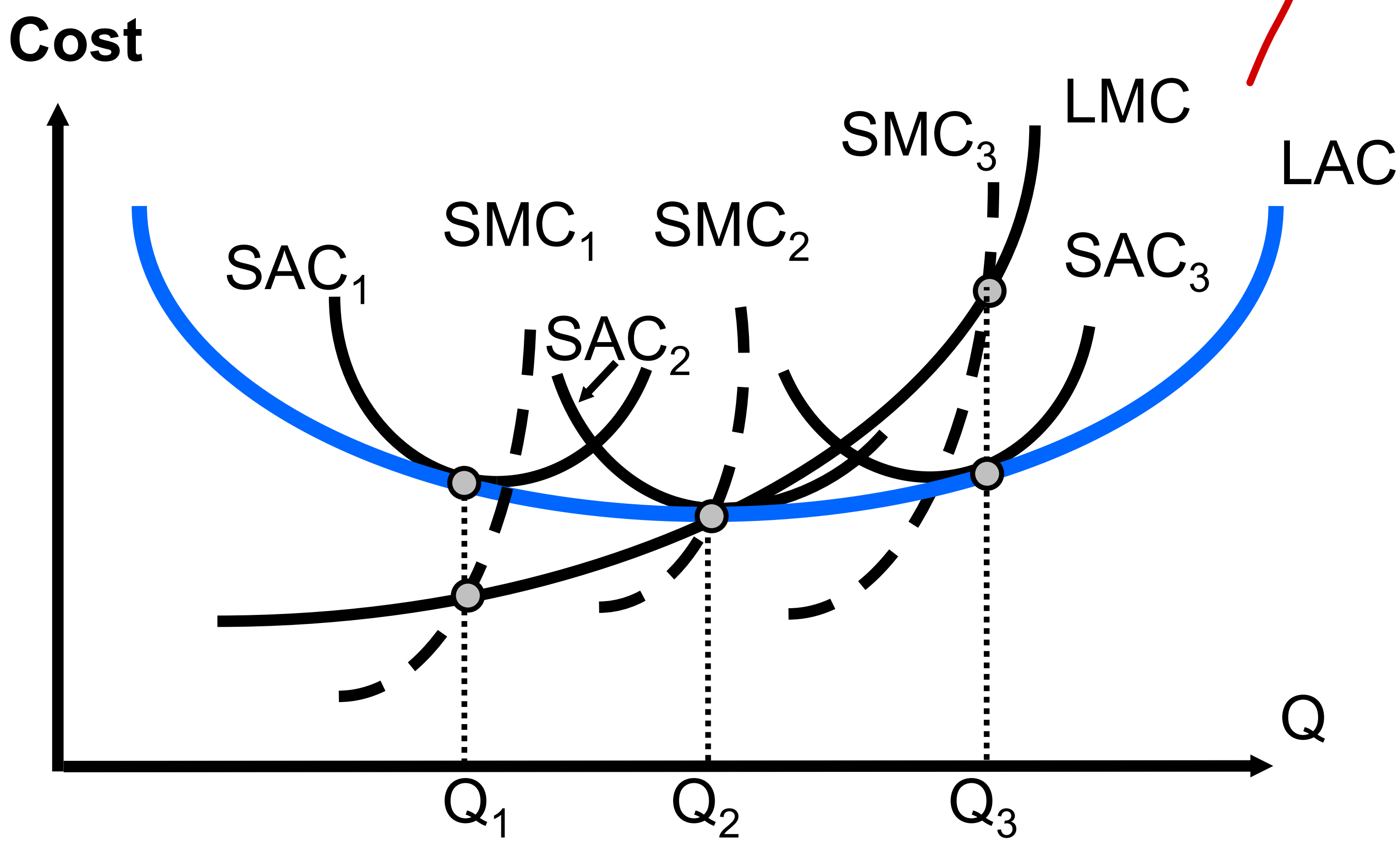
I: increasing RTS & economies of scale (lower AC as Q↑)

III: decreasing RTS & diseconomies of scale (higher AC as Q↑)



Short and Long-run Average Cost Curves

LR is a collection of many SR



Long-Run Versus Short-Run Cost Curves

- Long-run marginal cost leads long-run average cost:

  Avg vs Marginal

  - If LMC < LAC, LAC will fall
  - If LMC > LAC, LAC will rise
  - Therefore, LMC = LAC at the minimum of LAC
- In special case where LAC if constant, LAC and LMC are equal

Long Run Costs

- **Economies of Scale**: a situation when LAC declines with a larger output due to

  Cost advantages of large firms

  - increasing returns to scale
  - On a larger scale, workers can better specialize
  - Firm can use more efficient machine
  - Lumpiness in investment
  - Scale can provide flexibility – managers can organize production more effectively
  - Firm may be able to get inputs at lower cost if can get quantity discounts. Lower prices might lead to different input mix

    – quantity discount

Long Run Costs

- **Diseconomies of scale**: a situation when LAC increases with a larger output due to cost disadvantages
  - Factory space and machinery may make it more difficult for workers to do their job efficiently
  - Managing a larger firm may become more complex and inefficient as the number of tasks increase
  - Bulk discounts can no longer be utilized. Limited availability of inputs may cause price to rise

Long Run Costs

output elasticity of total cost

- Economies of scale are measured in terms of cost-output elasticity, E_C
- E_C is the percentage change in the cost of production resulting from a 1-percent increase in output

$$E_C = \frac{\Delta LTC / LTC}{\Delta Q / Q} = \frac{\Delta LTC / \Delta Q}{LTC / Q} = \frac{LMC}{LAC}$$

$$\frac{\% \Delta TC}{\% \Delta Q}$$

e.g.

$$\frac{\% \Delta TC}{\% \Delta Q} = 2$$

$Q \uparrow$ by 10% $\longrightarrow$ $TC \uparrow$ by 20%

$\longrightarrow$ decreasing RTS

& diseconomies of scale

Long Run Costs

- E_C is equal to 1, LMC = LAC

Constant RTS

  - Costs increase proportionately with output
  - Neither economies nor diseconomies of scale
- $E_C < 1$ when LMC < LAC
  - Economies of scale

increasing RTS

  - LAC are declining
- $E_C > 1$ when LMC > LAC
  - Diseconomies of scale
  - Both LMC and LAC are rising

Production with Two Outputs: Economies of Scope

- Many firms produce more than one product and those product are closely linked
- Examples:
  - Chicken farm--poultry and eggs
  - Automobile company--cars and trucks
  - University--Teaching and research
  - Nation Group
  - Choke Chai Farm

Production with Two Outputs: Economies of Scope

Advantages

- Both use capital and labor.
- The firms share management resources.
- Both use the same labor skills and type of machinery.

Economies of Scope means that the production of one good reduces the cost of producing another related good.

Eco. of Scale : cost saving
when firm produces more
of the same product

Eco. of Scope : cost saving
when firm produces
related products.

Production with Two Outputs: Economies of Scope

- There is no direct relationship between economies of scope and economies of scale.
  - May experience economies of scope and diseconomies of scale
  - May have economies of scale and not have economies of scope

Dynamic Changes in Costs: The Learning Curve

- Firms may lower their costs not only due to economies of scope, but also due to managers and workers become more experienced at their jobs
- As management and labor gain experience with production, the firm's marginal and average costs may fall

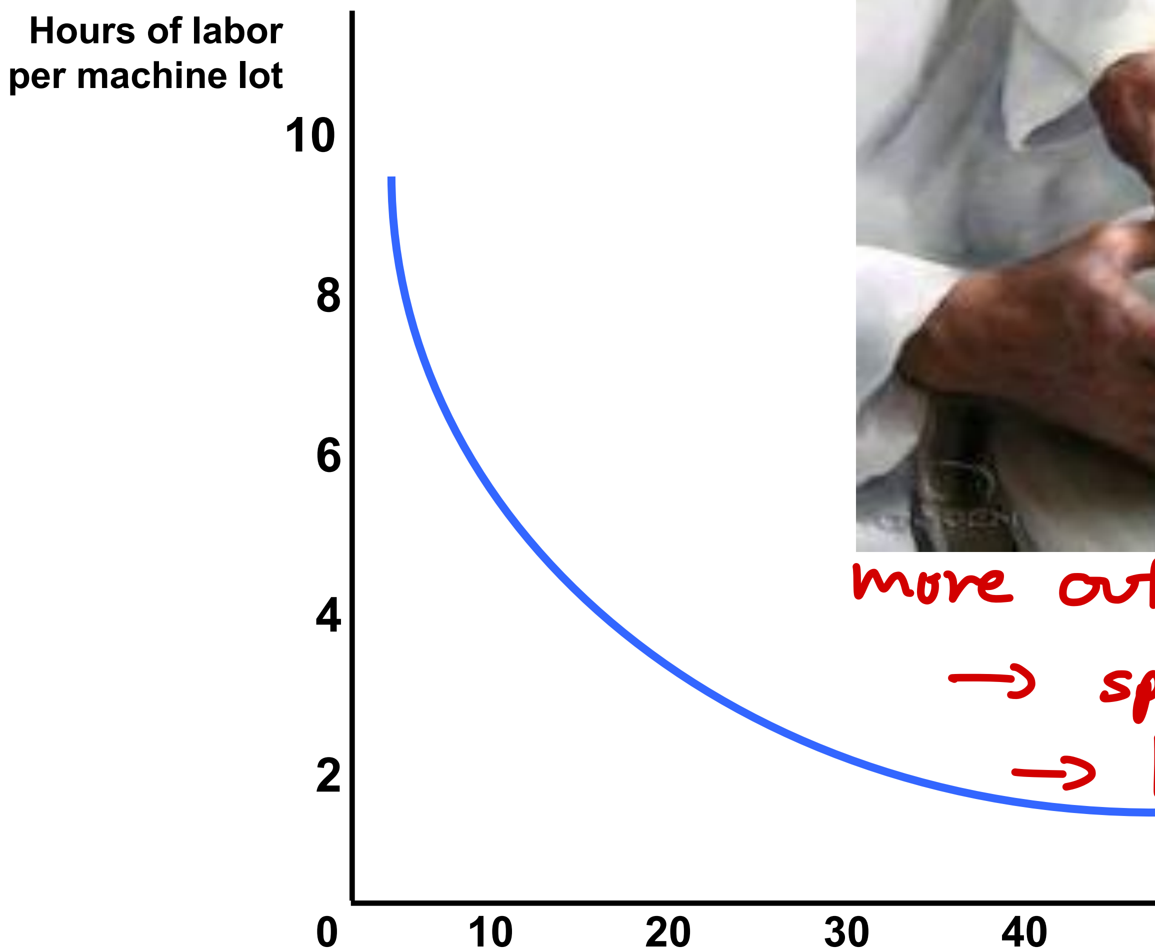
Dynamic Changes in Costs: The Learning Curve

- Reasons
  - Speed of work increases with experience
  - Managers learn to schedule production processes more efficiently
  - More flexibility is allowed with experience. May include more specialized tools and plant organization
  - Suppliers become more efficient passing savings to company

Dynamic Changes in Costs: The Learning Curve

- The learning curve measures the impact of worker's experience on the costs of production.
- It describes the relationship between a firm's cumulative output and amount of inputs needed to produce a unit of output.

The Learning Curve



more output
→ specialisation
→ higher productivity

Dynamic Changes in Costs: The Learning Curve

- Observations
  - New firms may experience a learning curve, not economies of scale.
    - Should increase production of many lots regardless of individual lot size
  - Older firms have relatively small gains from learning.
    - Should produce its machines in very large lots to take advantage of lower costs associated with size

Economies of Scale Versus Learning