

Solution: Quiz 5

1. Find the solution set for

$$\frac{x - x^2}{5 - x} \leq \left| \frac{3}{5 - x} - \left| \frac{3}{x - 5} \right| \right|.$$

Solution:

Notice that $\left| \frac{3}{x-9} \right| = \frac{3}{|x-9|}$ and $|x - 9| = \begin{cases} -(x - 5), & x - 5 < 0 \Leftrightarrow x < 5 \\ x - 5, & x - 5 \geq 0 \Leftrightarrow x \geq 5 \end{cases}$, We will consider 2 cases for x : (I) $x \geq 5$ and (II) $x < 5$.

(I) For $x \geq 5$, $|x - 5| = x - 5$ and $|5 - x| = -(5 - x) = x - 5$

$$\begin{aligned} \frac{x - x^2}{5 - x} &\leq \left| \frac{3}{5 - x} - \left| \frac{3}{x - 5} \right| \right| = \left| \frac{3}{5 - x} - \frac{3}{x - 5} \right| = \left| \frac{3}{5 - x} + \frac{3}{5 - x} \right| = \left| \frac{6}{5 - x} \right| = \frac{6}{x - 5} \\ \frac{x - x^2}{5 - x} &\leq \frac{6}{x - 5} \\ \frac{x^2 - x}{x - 5} - \frac{6}{x - 5} &\leq 0 \\ \frac{x^2 - x - 6}{x - 5} &\leq 0 \\ \frac{(x + 2)(x - 3)}{x - 5} &\leq 0 \end{aligned}$$

we consider $x = -2, 3, 5$ to subdivide the intervals.

	$x \in (-\infty, -2)$	$x \in (-2, 3)$	$x \in (3, 5)$	$(5, \infty)$
$\frac{(x2)(x-3)}{x-5}$	$\frac{(-)(-)}{(-)} = (-)$	$\frac{(+)(-)}{(-)} = (+)$	$\frac{(+)(+)}{(-)} = (-)$	$\frac{(+)(+)}{(+)} = (+)$

That is, the solution set in this case is $\{(-\infty, -2] \cup [3, 5)\} \cap (5, \infty) = \emptyset$.

(II) For $x < 5$, $|x - 5| = -(x - 5) = 5 - x$ and $|5 - x| = (5 - x)$

$$\begin{aligned} \frac{x - x^2}{5 - x} &\leq \left| \frac{3}{5 - x} - \left| \frac{3}{x - 5} \right| \right| = \left| \frac{3}{5 - x} - \frac{3}{5 - x} \right| = 0 \\ \frac{-x(x - 1)}{5 - x} &\leq 0 \Rightarrow \frac{x(x - 1)}{x - 5} \leq 0 \end{aligned}$$

we consider $x = 0, 1, 5$ to subdivide the intervals.

	$x \in (-\infty, 0)$	$x \in (0, 1)$	$x \in (1, 5)$	$(5, \infty)$
$\frac{(x)(x-1)}{x-5}$	$\frac{(-)(-)}{(-)} = (-)$	$\frac{(+)(-)}{(-)} = (+)$	$\frac{(+)(+)}{(-)} = (-)$	$\frac{(+)(+)}{(+)} = (+)$

That is, the solution set in this case is $\{(-\infty, 0] \cup [1, 5)\} \cap (-\infty, 5) = (-\infty, 0] \cup [1, 5)$.

From (I) and (II), the solution set is $\emptyset \cup (-\infty, 0] \cup [1, 5) = (-\infty, 0] \cup [1, 5)$.