EE 320 Introductory Mathematical Economics

Semester 1/2017

Homework 1

There are five questions in total. Each of them is worth equally.

1. Suppose that total cost function of Good Z is given by $C(Q) = 270 + 24Q$, where Q is the output level.

   a. (4 points) If the price of Good Z is \$30 per unit. Derive the profit function of Good Z, and determine the break-even quantity, and illustrate by graph.

   Ans. $\pi(Q) = 6Q - 270$

   $Q_{BE} = 45$ units.

   b. (2 points) Suppose that the producer of Good Z requires a minimum profit of \$2130. How many units of Good Z should the producer sell in the market?

   Ans. Q = 400 units.

   c. (4 points) Suppose now that the producer of Good Z becomes a monopolist and faces a given demand curve: $P = 61 - Q$. Derive the new total revenue and profit functions. What is the range of output should the producer produce in order to gain positive profits?

   Ans. $TR(Q) = 61Q - Q^2$; $\pi(Q) = 37Q - Q^2 - 270$

   For positive profit, 10 < Q < 27.

2. General market equilibrium.

a. Consider the following system of equations:

$$Q_{d1} = a_0 + a_1 P_1 + a_2 P_2 + a_3 P_3 \; ; \qquad Q_{s1} = \alpha_0 + \alpha_1 P_1$$
$$Q_{d2} = b_0 + b_1 P_1 + b_2 P_2 \; ; \qquad Q_{s2} = \beta_0 + \beta_2 P_2$$
$$Q_{d3} = c_0 + c_1 P_1 + c_3 P_3 \; ; \qquad Q_{s3} = \gamma_0 + \gamma_3 P_3$$

where $a_i, b_i, c_i \; (for \; i = 1, 2, 3)$ and $\alpha_0, \alpha_1, \beta_0, \beta_2, \gamma_0, \gamma_3$ are parameters. If $a_2 > 0$ and $a_3 < 0$, what would be the signs of $b_1, b_2, c_1,$ and c_3? What are the relationships among the three goods?

Ans. Since $a_2 > 0$ and $a_3 < 0$, $b_1 > 0$ and $c_1 < 0$. Also, by the law of demand, both b_2 and c_3 are negative. We can tell that good 1 and good 2 are substitutes, and good 1 and good 3 are complements. However, we cannot tell the relationship between good 2 and good 3.

b. Consider the following system of equations:

$$Q_{d1} = 20 - P_1 + 2P_2 \; ; \qquad Q_{s1} = 2P_1 - 2$$
$$Q_{d2} = 18 + 3P_1 - 2P_2 \; ; \qquad Q_{s2} = 2 + 4P_2$$

Find the equilibrium price and quantity for the two goods.

$P_1 = 41/3$; $P_2 = 19/2$

$Q_1 = 76/3$; $Q_2 = 40$

3. Suppose the market demand function for an eco car for a group of consumers is given by: $Q = 30 + 0.1Y - 2P$, where P is the price of an eco car, and Y is the average income of these consumer (in \$1,000). Suppose also that the market supply function is: $P = 0.5Q + 15$.

a. (2 points) Is the car considered as normal or inferior goods to this group of consumers. Explain.

Ans. Normal goods.

b. (4 points) Derive the income elasticity of demand and the price elasticity of demand for this eco car.

Ans. $\varepsilon_d = -2P/Q$ and $\varepsilon_Y = 0.1Y/Q$

c. (4 points) Suppose that the average income is \$15,000. Determine the equilibrium price and equilibrium quantity in this market.

Ans. (Q*,P*) = (750, 390)

4. Consider the following IS-LM model:

Commodity market:

$$Y = C + I + G_0$$
$$C = C_0 + bY, \qquad (C_0 > 0,\ 0 < b < 1)$$
$$I = I_0 - ar + iY, \qquad (I_0 > 0,\ a > 0,\ i > 0)$$

Money market:

$$M_S = M_0$$
$$M_D = mY - hr, \qquad (m > 0,\ h > 0)$$

a. (4 points) Write out the explicit IS-LM system of equations, and determine the equilibrium national income (Y) and equilibrium interest rate (r).

Ans. IS: $Y = \frac{C_o+I_0+G_0-ar}{(1-b+i)}$

LM: $Y = \frac{M_0+hr}{m}$

$Y^* = \frac{h(C_o+I_0+G_0)+aM_0}{am+(1-b+i)h}$ and $r^* = \frac{m(C_o+I_0+G_0)-(1-b+i)M_0}{am+(1-b+i)h}$

b. (2 points) Find the impact of an exogenous increase in money supply (M_0) on the equilibrium national income found in part (a). Assume everything else remains constant.

Ans. $\frac{\Delta Y^*}{\Delta M_0} = \frac{a}{am+(1-b+i)h}$

c. (2 points) Suppose that $C_0 = 150$, $I_0 = 100$, $G_0 = 50$, $b = 0.6$, $a = 400$, $h = 800$, $M_0 = 100$, $i = 0.1$ and $m = 0.4$. Find the equilibrium national income and interest rate.

Ans. $Y^* = 560$ and $r^* = 0.05$

d. (2 points) Based on the information in part (c), if the autonomous consumption (C_0) change by the amount λ ($\lambda > 0$), all else constant, what is the *change* in the equilibrium national income?

Ans. $\frac{\Delta Y}{\Delta C_0} = \frac{h}{am+(1-b+i)h} = 1.6$

If $\Delta C_0 = \lambda$, then $\Delta Y^* = 1.6\lambda$.

5. The demand and supply curves in the market for wine are given by the following equations:

Demand: $P = 40 - 0.25Q_d$

Supply: $Q_s = 2P + 4$

where Q is the quantity of wine in bottles and P is the price per bottle of wine.

Answer the following questions:

a. (1 point) Find the pre-tax equilibrium price and quantity, i.e. (P*,Q*).

Ans. (P*,Q*) = (26, 56)

b. (3 point) Calculate the price elasticities of demand and supply at the equilibrium. Based on the values of calculated elasticities, what would the total revenue change if the market price can be increased by 1%?

Ans. Elasticity of demand = (-4)*(26/56) = -13/7 = -1.86

Elasticity of supply = 2*(26/56) = 13/14 = 0.93

If the price increased by 1%, the quantity demanded will *decrease* by 1.86%. Since TR = PxQ, the 1% increase in price and the 1.86% decrease in quantity will result in a reduction in total revenue.

Now suppose that the government in this economy has levied an excise tax of $6 on the *producers* of wine.

c. (2 points) Find the post-tax equilibrium prices and quantity.

Ans. Set Ps = Pd-6 ⇒ 160-4Pd = 2(Pd-6) + 4.

⇒ After-tax equilibrium quantity = 48 units; after-tax equilibrium price for consumer = $28; after-tax equilibrium price for producer = $22.

d. (2 point) How much total tax revenue can government collect?

Ans. Tax revenue =t*Q = $6*48 = $288

e. (2 point) In terms of economic incidence, what percentages of the tax are borne by consumers and producers?

Ans.

Per-unit tax burden for consumer = (28-26)/6 = ⅓ or 33.33%

Per-unit tax burden for producer = (26-22)/6 = ⅔ or 66.67%

Since producer's supply is relatively more inelastic than consumer's demand, producer would bear more tax burden.