EE/EC411: Microeconomic Analysis

Dominance Solvable Games

Asst. Prof. Pornthep Benyaapikul

Faculty of Economics, Thammasat University

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Solution Concepts

- The first solution concept we will introduce is that of deleting dominated strategies.
- Intuitively, we seek to delete from the set of strategies off every players those strategies that can never be beneficial for him regardless of the strategies selected by his opponents.
- Lets apply this solution concept to the standard prisoner's dilemma game and then we will define it more formally.

Dominated strategies in a PD game

- Prisoner's Dilemma. Let's first put ourselves in the shoes of player 1 (in rows)

		P_2	
		C	NC
P_1	C	-5,-5	0,-15
	NC	-15,0	-1,-1

- NC is strictly dominated for player 1, since

$$\begin{aligned} u_1(C, s_2) &> u_1(NC, s_2) \quad \text{for all } C \text{ or } NC \\ i.e., \ !5 &> \ !15 \text{ if } s_2 = C \\ 0 &> \ !1 \text{ if } s_2 = NC \end{aligned}$$

Dominated strategies in a PD game

- A similar argument applies to Player 2

		P_2	
		C	NC
P_1	C	-5,-5	0,-15
	NC	-15,0	-1,-1

- NC is strictly dominated for player 2, since

$$\begin{aligned} u_2(C, s_1) &> u_2(NC, s_1) \quad \text{for all } s_1\text{,either } C \text{ } or \text{ } NC \\ i.e., \;!\,5 &> \;!\,15 \text{ if } s_1 = C \\ 0 &> \;!\,1 \text{ if } s_2 = NC \end{aligned}$$

- Hence, the only undominated (remaining) strategy for both player 1 and player 2 is to confess.

Dominated strategies - Definition

- **Definition of strictly dominated strategy**:
  - A strategy s_i' is STRICTLY dominated by another strategy s_i^* if the latter does strictly better than s_i' against **every** strategy of the other players.

$$u_i\left(s_i^*, s_{-i}\right) > u_i\left(s_i', s_{-i}\right) \text{ for all } s_{-i} \in S_{-i}$$

- **Definition of weakly dominated strategy**:
  - A strategy s_i' is WEAKLY dominated by another strategy s_i^* if the latter does at least as well as s_i' against every strategy of the other players, and against some strategy it does strictly better.

$$\begin{aligned} u_i\left(s_i^*, s_{-i}\right) &> u_i\left(s_i', s_{-i}\right) \text{ for all } s_{-i} \in S_{-i} \\ u_i\left(s_i^*, s_{-i}\right) &> u_i\left(s_i', s_{-i}\right) \text{ for at least one } s_{-i} \in S_{-i} \end{aligned}$$

Dominated Strategies

- Note that this definition is quite demanding:
  - A player must find that one of his available strategies yields a larger payoff than other of his available strategies, *regardless* of what his opponents select.
- For instance, in the prisoner's dilemma game, one of the players could say:
  - "I don't care about what my opponent selects, my payoff is always higher with C than with NC."
- It makes sense to never use a strategy that is strictly dominated,: there must be another strategy that yields a larger payoff regardless of the strategy that my opponent selects.

Dominated Strategies

- For some games, we will be able to find strictly dominated strategies, and delete them, as they are never going to be used by rational players.
  - We will be able to identify strictly dominated strategies for each player, which allows us to delete them from the matrix...
    - Ultimately leaving us with a more reduced matrix.

- If after several rounds of deleting strictly dominated strategies, we identify that there are no more strictly dominated strategies that we can delete from the matrix, then the remaining cell (or cells) are our equilibrium prediction.
- Since in order to find this equilibrium (or equilibria) we iteratively delete those strategies that are strictly dominated, this procedure is referred as "Iterative Deletion of Strictly Dominated Strategies" (IDSDS).

Dominated Strategies

- Importantly, the application of IDSDS implies what we described the first day of class as "*Common Knowledge of rationality*":
  - Every player is rational:
    - A rational player would never use a strategy that yields a lower payoff than other available strategies, regardless of the strategy his opponent selects. That is, a rational player can actually delete those strategies that are strictly dominated from her set of available strategies.
  - Every player knows that every player is rational:
    - A rational player knows that he is competing against a rational player. Hence, player A can anticipate that player B would never be using strictly dominated strategies.
    - In other words, when player A considers which strategy to select, he does so by first deleting all strictly dominated strategies of player B from the matrix, since player B would never use them. This helps player A consider a reduced matrix with fewer cells to examine.

Dominated Strategies

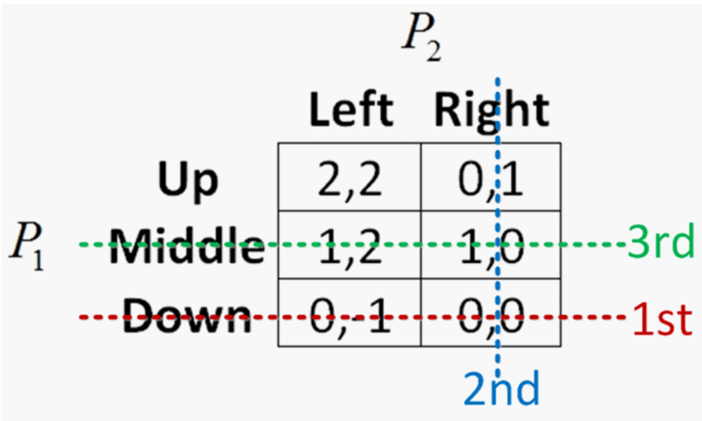
- "Common knowledge of rationality"(cont'd):
- Every player knows that every player knows that every player is rational:
  - By a similar argument, player A knows that player B has already deleted the strategies that are strictly dominated for player A, and that player B considers the reduced matrix once these strictly dominated strategies have been deleted.
- Every player knows that every player knows . . . ad infinitum.

Dominated Strategies

- For other games, however, we won't be able to find strictly dominated strategies.
  - The application of IDSDS doesn't have a bite, and all cells in the game are the "most precise" equilibrium prediction we can claim.
    - What a s. . . #%& (I mean imprecise) equilibrium prediction!
  - Don't despair:
    - We will discuss other solution concepts later on that will allow us to identify more precise equilibrium predictions.

Can we use IDSDS to solve a game?

- Consider the following 3%2 matrix:



- (Up,Left) is the only remaining strategy pair surviving IDSDS.
- These steps on the top of the same matrix can be confusing the first time we see them.
- Let's do one step at a time&

- More "step-by-step" presentation of the application of IDSDS to the previous example:

		P_2		
		Left	**Right**	
	Up	2,2	0,1	
P_1	**Middle**	1,2	1,0	
	~~**Down**~~	~~0,-1~~	~~0,0~~	1st step

- First, player 1's utility satisfies:
  - u_1 (Middle,s_2) $>$ u_1 (Down,s_2) for any strategy s_2 that player 2 selects.
  - Hence, "DOWN" is strictly dominated for player 1, and we can delete it since he will never use it.
- Next step &

- Hence, the remaining matrix after the first step of deleting a strictly dominated strategies is the following 2 % 2 matrix:

P_2

		Left	Right
P_1	Up	2,2	0,1
	Middle	1,2	1,0

2nd step

- Secondly, player 2's utility satisfies:
  - $u_2(\text{Left}, s_1) > u_2(\text{Right}, s_1)$ for any s_1 chosen by player 1.
  - Hence, "Right" is a strictly dominated strategy for player 2, and we can delete it since he will never select it
- Next step &

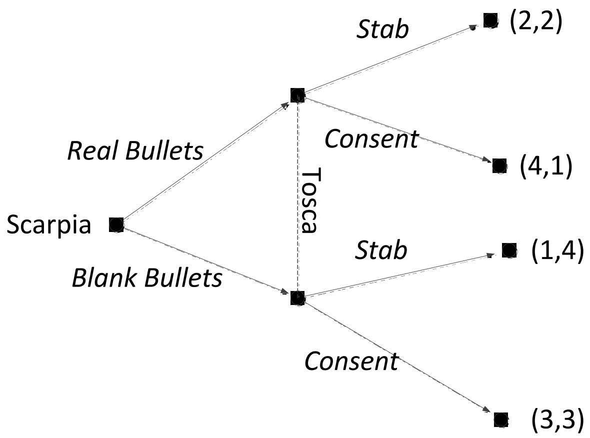
- The remaining matrix after two steps of applying IDSDS is:

		P_2
		Left
P_1	**Up**	2,2
3rd step	~~**Middle**~~	~~1,2~~

- In particular, player 1's utility satisfies:
  - $u_1\left(\text{Up},s_2\right) > u_1\left(\text{Middle},s_2\right)$, i.e., $2 > 1$, s_2: only "Left".
  - Hence, "Middle" is a strategy dominated strategy for player 1, and we can delete it.
- Therefore, the only cell surviving IDSDS is that corresponding to strategy profile (Up,Left) with corresponding payoff $(2, 2)$.

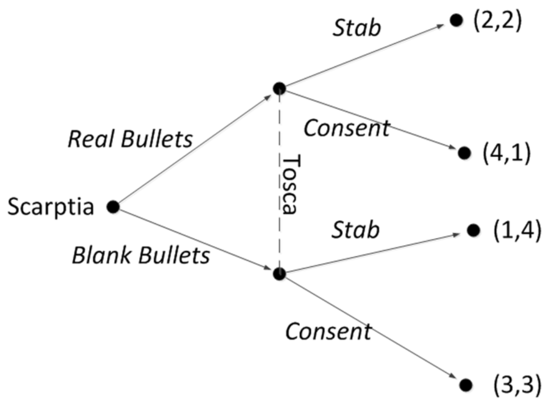
The "Tosca" game

- Based on the Puccini's opera
- Scarpia has kidnapped Mario (Tosca's boyfriend). Then Scarpia prepares a firing squad providing them either real or blank bullets.
- Without knowing whether Scarpia has given real or blank bullets to the firing squad, Tosca must choose to consent to Scarpia's sexual favors or stab him.



The "Tosca" game

- Since Tosca is uninformed when making her move, the previous sequential-move game can also be expressed using the following matrix form:



		Scarpia	
		Real	*Blanks*
Tosca	*Stab*	2,2	4,1
	Consent	1,4	3,3

- We can now check for the presence of strictly dominated strategies, i.e., we can apply IDSDS.

The "Tosca" game

- Let us apply IDSDS to the "Tosca" game:

		Scarpia	
		Real	*Blanks*
Tosca	*Stab*	2,2	4,1
	~~*Consent*~~	~~1,4~~	~~3,3~~

1^{st} step

- In particular, Tosca finds "Consent" to be strictly dominated
- since $u_{Tosca}(Stab, s_2) > u_{Tosca}(Consent, s_2)$ for any strategy s_2 that player 2 (Scarpia) selects, i.e., both for s_2=Real, and s_2=Blanks.

The "Tosca" game

- Therefore, the remaining matrix after Tosca deletes the strictly dominated strategy "Consent" is

		Scarpia	
		Real	Blanks
Tosca	Stab	2,2	4,1

2nd step

- We can now move to Scarpia, and note that "Blanks" is strictly dominated
- since $u_{Scarpia}(\text{Real}, s_1) > u_{Scarpia}(\text{Stab}, s_1)$, i.e., $2 > 1$, given that s_1: can only be Stab
- Therefore, the only strategy profile that survives IDSDS is $(\text{Stab}, \text{Real})$, with a bloody outcome!

The only strategy surviving IDSDS in the "Tosca" game. . .

Conflict between individual and group incentives

- Tosca finds “Stab” individually rational for her, since it strictly dominates “Consent”;
- Scarpia finds “Real bullets” individually rational for him, since it strictly dominates “Blanks”
- However, the group outcome would be better if they play (Consent, Blanks), yielding a payoff pair of (3,3), than if they play
  (Stab, Real), which only yields (2,2).
- This conflict between individual and group incentives is similar to that arising in the prisoner’s dilemma game, and many other games in economics and social sciences.

The "Team project" game

- Harrington pp. 68-69.
- Consider that two random students in a class are grouped together in a project.
- Each student must independently choose whether to exert a high, moderate or low effort.
- Student types conform to the usual stereotypes:
  - Jocks (reaching for a C),
  - Frat boys and sorority girls (reaching for a B+), and
  - Nerds (reaching for an A).
- As put by Harrington: Is there anyone we haven't offended?

Team Project Between a Nerd and a Frat Boy

- We start applying IDSDS by putting ourselves in the shoes of the Nerd (column player): Low and Moderate effort are strictly dominated by High effort.

Frat boy \ Nerd	Low	Moderate	High
Low	0,1	2,2	6,3
Moderate	1,2	4,3	5,4
High	2,3	3,4	3,5

Strictly dominated for the Frat boy

Strictly dominated For the Nerd!

$u_{Nerd}(High, s_1) > u_{Nerd}(Low, s_1)$

$\Rightarrow u_{Nerd}(High, s_1) > u_{Nerd}(Moderate, s_1)$

For any s_1 selected by the Frat boy

Team Project Between a Nerd and a Frat Boy

- The Frat boy, anticipating that the Nerd will exert a High effort (his only surviving strategy) deletes Moderate and High effort as they are strictly dominated by Low effort.
- Hence, only (Low,High) survives the application of IDSDS.

Banning cigarette advertising

- Before 1971, you could find cigarette advertising such as these:

Banning cigarette advertising

- We don't see this type of ads either!

Banning cigarette advertising on TV

- Assuming an annual demand for cigarettes of 1,000,000,000, and considering the only two main firms in the 1970's (Philip Morris (PM) and RJ.Reynolds (RJR)), Philip Morris' profits are given by

$$\pi^{PM} = \underbrace{0.1}_{\substack{\text{profit per package} \\ \text{(It is 1970!)}}} \times \underbrace{1,000,000,000}_{\text{annual demand}} \times \left(\underbrace{\frac{ADV_{\text{pm}}}{ADV_{\text{PM}} + ADV_{\text{RJR}}}}_{\substack{\text{market share, which depends on how much} \\ \text{PM advertises relative to the total advertising}}} \right) - \underbrace{ADV_{\text{PM}}}_{\text{cost from advertising}}$$

Banning cigarette advertising on TV

- For simplicity, we assume that firms can only choose three levels of advertising: 5, 10 or 15 million.

Banning cigarette advertising on TV

- We then have a payoff matrix as follows

		R.J.Reynold		
		5	10	15
Philip Morris	5			
	10			
	15			

- How to construct the payoffs of every cell? &

Banning cigarette advertising on TV

- Let's do one example, where ADV_{PM}=5 and ADV_{PM}=15 (locate the cell aright hand corner of the top the matrix). In this strategy profile,

$$
\begin{aligned}
\pi^{\mathrm{PM}} &= 0.1 \times 1,000,000,000 \times \left(\frac{5}{5+15}\right) - 5 \\
&= 25 - 5 = 20 \text{ million} \\
\pi^{\mathrm{RJR}} &= 0.1 \times 1,000,000,000 \times \left(\frac{5}{5+15}\right) - 15 \\
&= 75 - 15 = 60 \text{ million}
\end{aligned}
$$

yielding the payoff pair (20,60) .

- Similarly for all other cells, entailing the following matrix &

Advertising cigarette game

- The resulting matrix is:

		R. J. Reynolds		
		Spend 5	*Spend 10*	*Spend 15*
Philip Morris	*Spend 5*	45,45	28,57	20,60
	Spend 10	57,28	40,40	30,45
	Spend 15	60,20	45,30	35,35

- Notice that when Philip Morris spends 5 and R.J.Reynolds spends 15, in the upper right-cell, the payoff pair is (20,60), as we found in the previous slide.

Banning Cigarette Advertising (cont'd)

		R. J. Reynolds			
		Spend 5	*Spend 10*	*Spend 15*	
	Spend 5	~~45,45~~	~~28,57~~	~~20,60~~	1st step
Philip Morris	*Spend 10*	57,28	40,40	30,45	
	Spend 15	60,20	45,30	35,35	

- We can start applying IDSDS:
  - First, note that "Spend 5" is strictly dominated for Philip Morris,
  - since $\pi^{PM}(15, s_2) > \pi^{PM}(5, s_2)$ for every strategy s_2 selected by R.J.Reynolds (any column).

Banning cigarette advertising on TV

- The remaining matrix is therefore:

		R.J.Reynolds		
		Spend 5	Spend 10	Spend 15
Philip	Spend 10	57,28	40,40	30,45
Morris	Spend 15	60,20	45,30	35,35

2^{nd} step

- We can now move to R.J, Reynolds, finding that "Spend 5" is also strictly dominated for R.J.Reynolds since

$$\pi^{\mathrm{RJR}}(15, s_1) > \pi^{\mathrm{RJR}}(5, s_1)$$

for every strategy s_1 selected by Philip Morris.(any row)

Banning cigarette advertising on TV

- The remaining matrix now becomes the following 2 % 2 matrix

		R.J.Reynolds	
		Spend 10	Spend 15
Philip	Spend 10	~~40,40~~	~~30,45~~
Morris	Spend 15	45,30	35,35

3rd step

- Moving back to Philip Moris, note that "Spend 10" is strictly dominated since

$$\pi^{PM}(15, s_2) > \pi^{PM}(10, s_2)$$

for every strategy s_2 chosen by R.J.Reynolds.(every column)

Banning cigarette advertising on TV

- Hence, the remaining matrix becomes the following (tiny) matrix:

		R.J.Reynolds	
		Spend 10	Spend 15
Philip Morris	Spend 15	45,30	35,35

4th step

- Finally, moving now to R.J. Reynolds,note that "Spend 10" is also strictly dominated.
- Therefore, the only strategy profile surviving IDSDS is (Spend 15, Spend 15) ,with both firms spending 15 million in advertising,which entails an equillibrium payoff of $35 millions for each firm.

Banning cigarette advertising on TV

- What if TV commercials were banned, making it unfeasible for firms to spend more than $10 million on advertising?
- Then, our payoff matrix reduces to the following 2%2 matrix.

		R. J. Reynolds	
		Spend 5	*Spend 10*
Philip Morris	*Spend 5*	45,45	28,57
	Spend 10	57,28	40,40

Banning cigarette advertising on TV

- Let us apply IDSDS on this matrix, to check how the presence of this ban on advertising modifies our equilibrium results.

		R. J. Reynolds		
		Spend 5	*Spend 10*	
Philip Morris	*Spend 5*	~~45,45~~	~~28,57~~	1st step
	Spend 10	57,28	40,40	

- First, "Spend 5" is strictly dominated for Philip Morris since

$$\pi^{PM}(10, s_2) > \pi^{PM}(5, s_2)$$

- for every strategy s_2 selected by R.J.Reynolds (every column), i.e., both for s_2=Spend 5 and s_2=Spend 10.

Banning cigarette advertising on TV

- Hence, our matrix reduces to

		R.J.Reynolds	
		Spend 10	Spend 15
Philip Morris	Spend 10	57,28	40,40

2^{nd} step

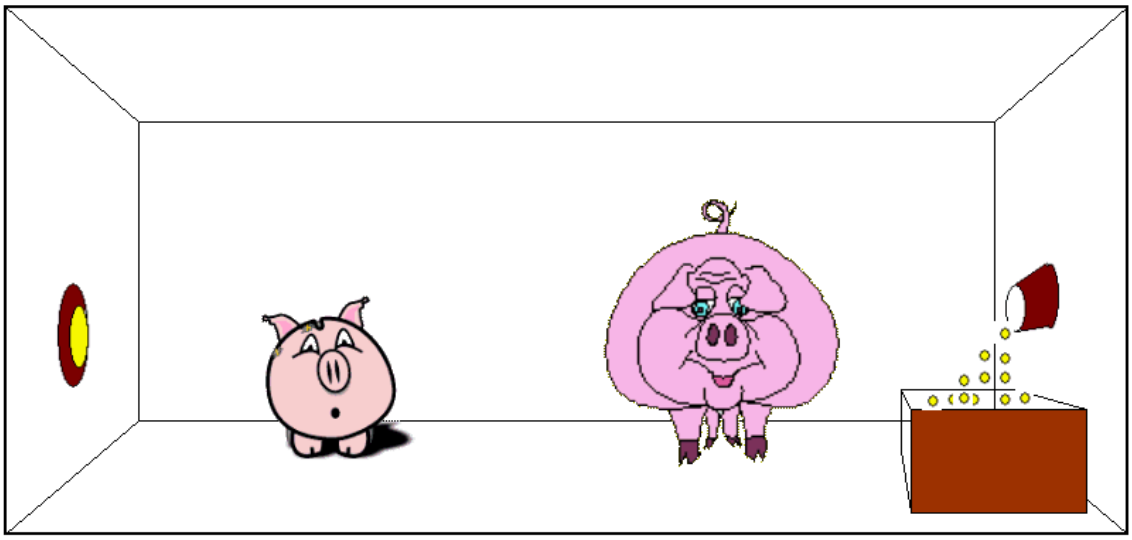
- We can now move to R.J. Reynlods and note that "Spend 5" is also strictly dominated.
- Therefore, the only strategy profile surviving IDSDS is (Spend 10, Spend 10), yielding equillibrium profits of $40 million for each firm.
- The ban actually helped firms increase their profits, from $35 to $40 million!!

Natural question at this point: Does the order of deletion matter?

- No!
- That's great news: in your application of IDSDS, it does not matter which strategy you start deleting first,(whether you start identifying strictly dominated strategies for the row or the column player) you will end up finding the same strategy profile (or profiles).
- Check it with your classmate: start applying IDSDS in a large matrix (3%3 for instance), you will end up with the same equilibrium prediction.

Boxed Pigs game

Boxed Pigs Example



Cost to press
button = 2 units

When button is pressed,
food given = 10 units

The "Boxed-Pigs" game

- Harrington, pp. 71-72.
- Explanation of payoffs:
  - If the large pig waits at the dispenser while the small pig presses the button, the large pig gets 9 out of 10 units of food. The small pig only gets 1, yielding a net payoff of 1-2=-1 once we subtract the 2 units he spends from pressing the lever.
    - This explains the (-1,9) payoff pair.

		Large pig	
		Press lever	*Wait at dispenser*
Small pig	*Press lever*	1,5	−1,9
	Wait at dispenser	4,4	0,0

The "Boxed-Pigs" game

- Explanation of payoffs:
  - If the small pig waits at the dispenser while the large pig presses the button, the small pig gets 4 out of 10 units of food. The large pig gets the remaining 6, yielding a net payoff of 6-2=4 once we subtract the 2 units he spends from pressing the lever.
    - This explains the (4,4) payoff pair.

		Large pig	
		Press lever	*Wait at dispenser*
Small pig	*Press lever*	1,5	−1,9
	Wait at dispenser	4,4	0,0

The "Boxed-Pigs" game

- Explanation of payoffs:
  - If both pigs press the button, then the small pig gets 3 out of 10 units of food, yielding a net payoff of 3-2=1. The large pig gets the remaining 7 units, yielding a net payoff of 7-2=5.
    - This explains the (1,5) payoff pair.
  - If neither pig presses the button then they starve, yielding the (0,0) payoff pair.

		Large pig	
		Press lever	*Wait at dispenser*
Small pig	*Press lever*	1,5	−1,9
	Wait at dispenser	4,4	0,0

The "Boxed-Pigs" game

- Let us check which strategy pair can be deleted using IDSDS:

		Large pig		
		Press lever	*Wait at dispenser*	
Small pig	*Press lever*	~~1,5~~	~~−1,9~~	1st step
	Wait at dispenser	4,4	0,0	

- First, the small pig's payoffs satisfy

$$u_{\text{small pig}}\left(\text{Wait}, s_2\right) > u_{\text{small pig}}\left(\text{Press}, s_2\right)$$

- for every strategy s_2 selected by player 2 (the large pig), i.e., both for s_2=Press and for s_2=Wait.

The "Boxed-Pigs" game

- After the first step of IDSDS the remaining matrix is:

		Large pig	
		Press lever	Wait at dispenser
Small pig	Wait at dispenser	4,4	0,0

2^{nd} step

- We can now move to the large pig, and note that his payoffs satisfy

$$u_{\text{Large pig}}\left(\text{Press}, s_1\right) > u_{\text{Large pig}}\left(\text{Wait}, s_1\right), \text{ i.e., } 4 > 0,$$

since the only surving strategy s_1, we need to consider for the small pig is $s_1 = $Wait

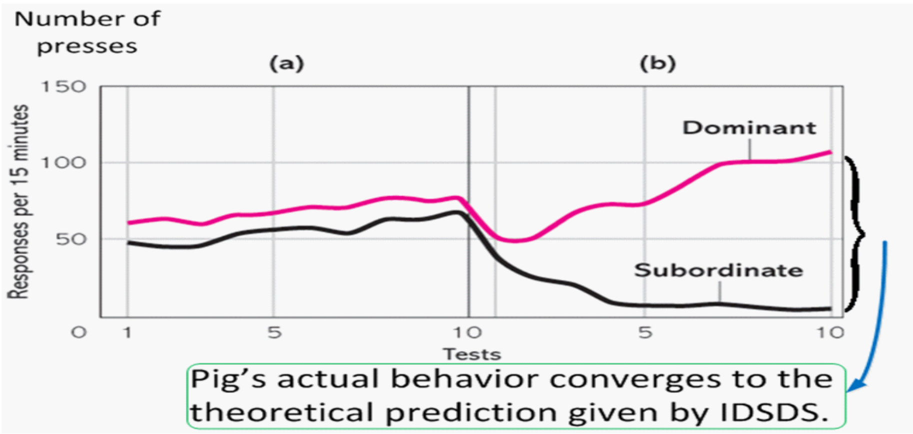
The "Boxed-Pigs" game

- Hence, the strategy profile (Wait, Press), in which only the large pig presses the lever, is the only one surviving the application of IDSDS.

The "Boxed-Pigs" game

- One second. . .
  - The use of IDSDS implies that every pig is rational and understands that the other pig is rational. . . Come on!!!
- They might not operate using different levels of reasoning as (most) humans do, but instead operate by trial and error.

- Let us see one example about how pigs actually behaved in a controlled experiment.



The Doping Game

- Harrington, pp. 73-75
- So far we analyzed strictly dominated strategies with only two players.
- What if we have **three players**?

Ben chooses *steroids*

Maurice \ Carl	*Steroids*	*No steroids*
Steroids	2,3,3	3,1,5
No steroids	1,4,5	5,2,6

Ben chooses *no steroids*

Maurice \ Carl	*Steroids*	*No steroids*
Steroids	3,4,1	4,2,2
No steroids	5,5,2	6,6,4

The Doping Game

- First, we check if Ben(the matrix player) has some strictly dominated strategy.

Ben chooses *steroids*

Maurice \ Carl	*Steroids*	*No steroids*
Steroids	2,3,3	3,1,5
No steroids	1,4,5	5,2,6

Ben chooses *no steroids*

Maurice \ Carl	*Steroids*	*No steroids*
Steroids	3,4,1	4,2,2
No steroids	5,5,2	6,6,4

- We compare $u_3(\text{steroids}, s_1, s_2)$ against $u_3(\text{No steroids}, s_1, s_2)$ where s_1 and s_3 are fixed across matrices.
- No steroids is a strictly dominated strategy for Ben, as it yields a lower payoff than steroids, for every profile (s_1, s_3) of the other two athletes.
- We can then delete "No steroids" from Ben by deleting the

The Doping Game

- We are hence left with the left-hand matrix:
- We can now move to Carl (column player) and search whether he has strictly dominated strategies.

Carl 2^{nd} step

Maurice	Steroids	No steroids
Steroids	2, 3, 3	3, 1, 5
No steroids	1, 4, 5	5, 2, 6

- Similarlly as for Ben, Carl finds "No steroids" to be strictly dominated by "steroids", and hence we delete two columns from the matrix.

The Doping Game

- Hence, the above matrix reduces to the following 2%1 matrix:

		Steroids
Maurice	Steroids	2,3,3
	No steroids	~~1,4,5~~

3rd step

- Moving now to Maurice (row player), we note that "No steroids" is strictly dominated by "steroids".
- Hence, the only strategy profile surviving IDSDS is

(Steroids,Steroids,Steroids)

Another example:

	X	Y	Z	
A	~~3,3~~	~~0,5~~	~~0,4~~	2^{nd}
B	~~0,0~~	~~3,1~~	1,2	(B,Z) survives IDSDS
	1^{st}	3^{rd}		

- Hence the only strategy profile surviving IDSDS is (B,Z).
- Great! But can we apply IDSDS to **all** kinds of games and obtain sharp predictions (a unique strategy profile)? No!

Imprecise equillibrium predictions

- Harrington, pp. 76-78

		Player 2			
		w	*x*	~~*y*~~	*z*
Player 1	*a*	3,2	4,1	~~2,3~~	0,4
	~~*b*~~	~~4,4~~	~~2,5~~	~~1,2~~	~~0,4~~
	c	1,3	3,1	~~3,1~~	4,2
	d	5,1	3,1	~~2,3~~	1,4

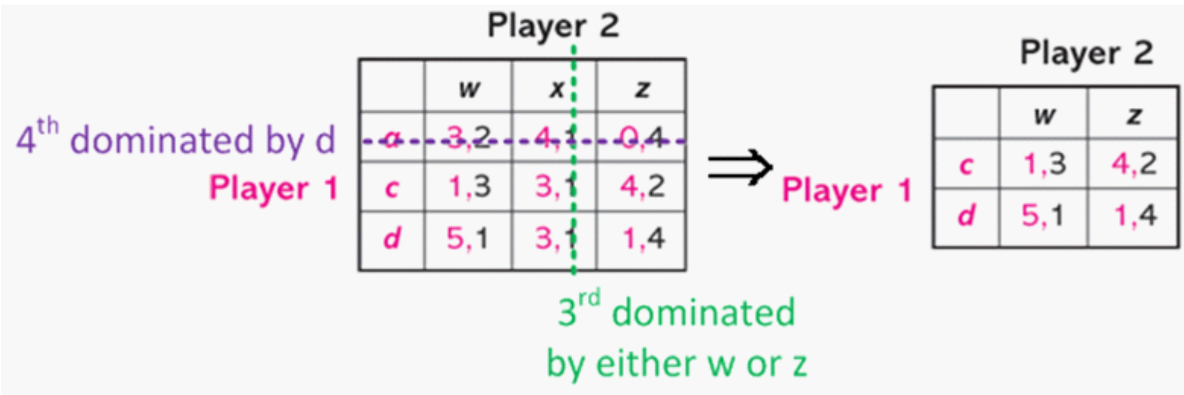
1st (dominated by d)

2nd (dominated by z)

- Now what? Too much information?
- Let's clarify things, by rewriting the matrix after deleting the strictly dominated strategies b and y.

Imprecise equillibrium predictions

- We can now detect that x is strictly dominated for player 2, and ...
- that a is strictly dominated for player 1, which yields the following 2%1 matrix:



- However,at this point, there are no more strictly dominated strategies for player 1 or player 2.

Imprecise equillibrium predictions

- Hence, the only prediction that we can make using IDSDS is that any of the following set of four strategy pairs surviving IDSDS can be ultimately played:
  - (c,W),(c,Z),(d,W),(d,Z).
- That's not a very precise prediction about how the game will be played!!

Battle of the Sexes game

- Another example where applying IDSDS doesn't allow us to identify a unique outcome:

		Wife	
		F	O
Husband	F	3,1	0,0
	O	1,3	1,3

- There are no strictly dominated strategies for the Husband.
- There are no strictly dominated strategies for the Wife.
- Hence, all four strategy profiles (F.F).(F,O),(O,F),(O,O), are the most precise equillibrium prediction we can provide applying IDSDS.

Battle of the Sexes game

- Is this imprecise equillibrium prediction only happening for this particular example?.. No!

Matching Pennies game

- Yet, another example in which the application of IDSDS yields imprecise equillibrium predictions:

		P_2	
		Head	Tails
P_1	Head	1,-1	-1,1
	Tails	-1,1	1,-1

- There are no undominated strategies for Player 1 or Player 2.
  - No predictive power! (Inexistence of a single strategy part that perfectly describes how the game will be played).

Allowing for randomizations in IDSDS

- So far we applied IDSDS by checking if a player's play of a particular strategy (with 100% probability, also referred as a pure strategy) strictly dominates another pure strategy.
- What if we check if a mixed strategy (where the player randomizes among two or more strategies) strictly dominates another strategy?
  - Let's check this in the following example.

Allowing for randomizations in IDSDS

- Watson, page 73

		Player 2		
		F	*C*	*B*
	F	0,5	2,3	2,3
Player 1	*C*	2,3	0,5	3,2
	B	5,0	3,2	2,3

- There are no strictly dominated strategies (using pure strategies) for player 1 nor for player 2.

Allowing for randomizations in IDSDS

		Player 2: *F*	Player 2: *C*	Player 2: *B*
Player 1	*F*	0,5	2,3	2,3
Player 1	*C: (1-q)*	2,3	0,5	3,2
Player 1	*B: (q)*	5,0	3,2	2,3

- **Mixing:** If player 1 mixes between B (with probability q) and C (with probability $1 ! q$) he obtains an expected utility that exceeds the utility from selecting F, regardless of what strategy player 2 chooses
  - In order to show this results, we must separately consider the case in which player 2 chooses F (left column), C (middle column), and B (right column).

Allowing for randomizations in IDSDS

		Player 2		
		F	*C*	*B*
	F	0,5	2,3	2,3
Player 1	*C*	2,3	0,5	3,2
	B	5,0	3,2	2,3

- **If player 2 chooses F** (in the left column), then player 1's EU from mixing is $5q + 2(1 \; ! \; q)$, which exceeds his utility from F (zero), if

$$5q + 2(1 \; ! \; q) > 0 \quad ' \quad 5 \; ! \; 2q + 2 > 0 \quad ' \quad q > \; ! \frac{2}{3}$$

which holds by assumption given that $q \; \$ \; [0, 1]$.

Allowing for randomizations in IDSDS

		Player 2		
		F	↓ *C*	*B*
	F	0,5	2,3	2,3
Player 1	*C*	2,3	0,5	3,2
	B	5,0	3,2	2,3

- **If player 2 chooses C** (in the middle column), then player 1's EU from mixing is $3q + 0\,(1 - q)$, which exceeds his utility from F (2), if

$$3q + 0\,(1 - q) > 2 \iff 3q > 2 \iff q > \frac{2}{3}$$

- This is a necessary condition for the mixing to yield a larger EU than F. (We will keep it in mind.)

Allowing for randomizations in IDSDS

		Player 2: F	Player 2: C	Player 2: B
	F	0,5	2,3	2,3
Player 1	C: (1-q)	2,3	0,5	3,2
	B: (q)	5,0	3,2	2,3

- **If player 2 chooses B** (in the right column), then player 1's EU from mixing is $2q + 3(1 - q)$, which exceeds his utility from F (2), if

$$2q + 3(1 - q) > 2 \iff 2q + 3 - 3q > 2 \iff 1 > q$$

which holds by assumption given that $q \in [0, 1]$.

Allowing for randomizations in IDSDS

- Hence, as long as player 1 mixes between B and C and assigns to B a probability $q > \frac{2}{3}$, he obtains an expected utility that exceeds the payoff he obtains from selecting F, regardless of the strategy player 2 selects.
- We can therefore delete strategy F from the matrix, since it is strictly dominated by a randomization between B and C, and thus player 1 would never use it.
- Deleting F (upper row), we obtain the following reduced matrix:

Player 2

		F	*C*	*B*
Player 1	*C*	2,3	0,5	3,2
	B	5,0	3,2	2,3

Allowing for randomizations in IDSDS

- Given the above reduced matrix, we can now move to player 2.
- Can we identify a strictly dominated strategy for player 2?

		Player 2		
		F	C	B
Player 1	C	2,3	0,5	3,2
	B	5,0	3,2	2,3

- Yes, F is strictly dominated by C for player 2.

Allowing for randomizations in IDSDS

- We can hence delete the column corresponding to strategy F for player 2 (since it is strictly dominated), yielding the following reduced 2%2matrix:

		Player 2: C	Player 2: B
Player 1	C	0,5	3,2
	B	3,2	2,3

Allowing for randomizations in IDSDS

- However, for the remaining matrix does not have any further strictly dominated strategies that we can delete for player 1:

		Player 2	
		C	B
Player 1	C	0,5	3,2
	B	3,2	2,3

since

$$\underbrace{u_1(C,C)}_{0} < \underbrace{u_1(B,C)}_{3} \text{ if player 2 plays } C$$

$$\text{But } \underbrace{u_1(C,B)}_{3} < \underbrace{u_1(B,B)}_{0} \text{ if player 2 plays } B$$

- Hence, neither C is strictly dominated by B, nor B is strictly

Allowing for randomizations in IDSDS

- Nor for player 2...

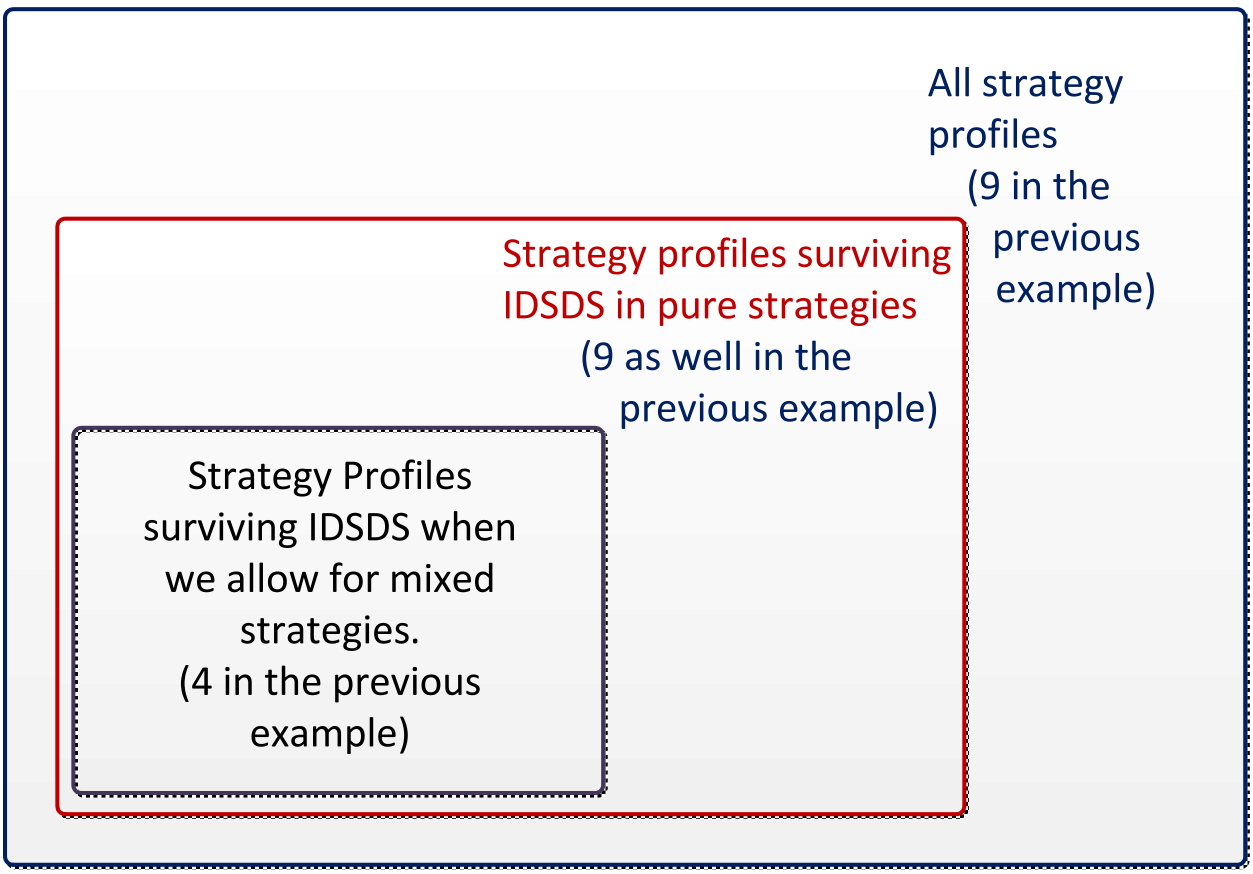
		Player 2	
		C	*B*
Player 1	*C*	0,5	3,2
	B	3,2	2,3

$u_2\,(C, C)\,\square u_2\,(C, B)$ if player 1 plays C (top row)

But $u_2\,(B, C)\,\square u_2\,(B, B)$ if player 1 plays B (bottom row)

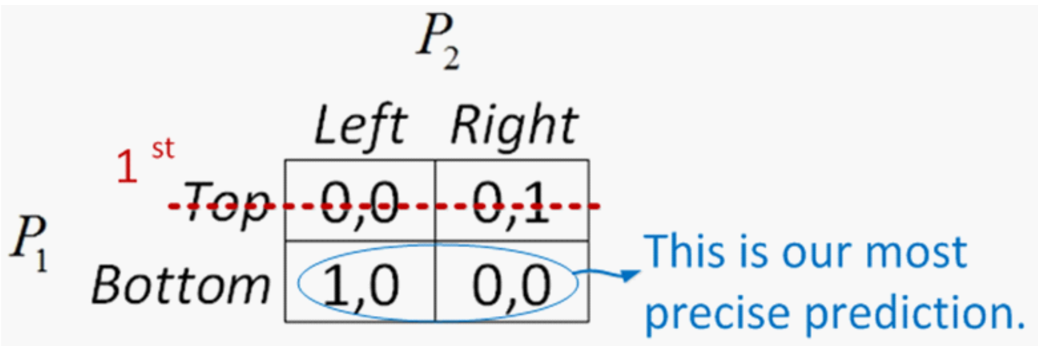
- Check signs ($>$ or $<$) as a practice.
- Hence, we cannot further eliminate strictly dominated strategies, and our (very imprecise) equilibrium prediction is $((C, C), (C, B), (B, C), (B, B))$.

Allowing for randomizations in IDSDS



Problems with dominance

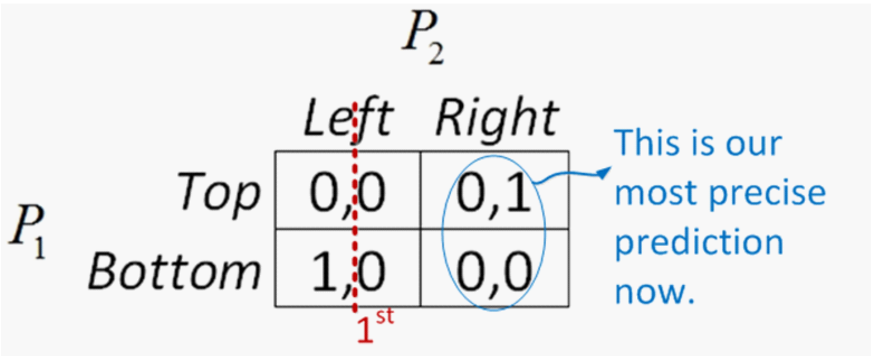
1. **1st problem**: Lack of predictive power in some games (see previous examples).
2. **2nd problem**: *Order of elimination matters*: only if we eliminate *weakly*(rather than strictly) dominated strategies.



- First, we eliminate Top as being weakly dominated by Bottom
- No further deletions for player 2 since he is indifferent between *Left* and *Right*.

Problems with dominance

- But what if we start by eliminating Left from Player 2 (it is a weakly dominated strategy for him).



- No further dominated strategies to delete since player 1 is indifferent between *Top* and *Bottom*.
- Bottom line: the set of strategies surviving IDWDS (NOT for IDSDS) depends on the order of deletion.

Problems with dominance:

3. **3rd problem**: Layers of Rationality:
   - The application of IDSDS assumes the iterative thinking, sometimes requiring many layers (many steps).
   - In the prisoner's dilemma game it is reasonable to assume that my opponent will never use NC since it such strategy is strictly dominated. But we only require 2 steps of IDSDS in order to find a unique equilibrium prediction.
   - What about 3%3 matrices requiring many levels of IDSDS.
   - More importantly, what about 3%3 matrices for which we can only find strictly dominated strategies if we allow players to randomize?
     - Let's do one example (as a practice, and to confirm how cognitively demanding this process can be).

Problems with dominance

- Layers of rationality (Example of 3x3 matrix)

Player 2

Player 1		L	C : (p)	R : (1-p)
	U	5, 1	0, 4	1, 0
	M	3, 1	0, 0	1, 5
	D	3, 3	4, 4.5	2, 5

Problems with dominance

- As a practice, let's check if L is strictly dominated by a mixed strategy that puts p probability on C, and $1 ! p$ probability on R.
- Such mixed strategy yields a expected payoff for player 2 that exceeds his payoff from L:
  - If player 1 plays U (top row),

$$p * 4 + (1 ! p) * 0 = 4p > 1(\text{payoff from } L) \text{ if } p > \frac{1}{4}$$

  - If player 1 plays M (middle row),

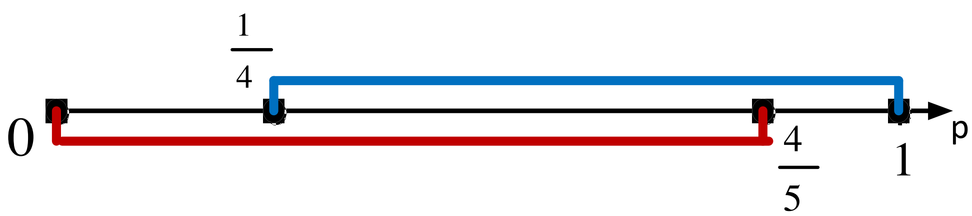
$$p * 0 + (1 ! p) * 5 = 5 ! 5p > 1\,(\text{from } L) \;\text{ if } 4 > 5p \;\; ' \;\; \frac{4}{5} > p$$

  - If player 1 plays D (bottom row),

$$\begin{aligned} p * 4.5 + (1 ! p) * 5 &= 4.5p + 5 ! 5p \\ &= 5 ! 0.5p > 3\,(\text{from } L) \;\text{ if } 2 > 0.5p \\ &' \;\; 4 > p \end{aligned}$$

Problems with dominance

- Hence, for all $p \in \left(\frac{1}{4}, \frac{4}{5}\right)$, the mixed strategy between C and R strictly dominates L, and we can delete L because of being strictly dominated.



Problems with dominance

- An example of the previous mixed strategy is one that assigns $p = \frac{1}{2}$, since $p = \frac{1}{2}$ indeed satisfies $p \; \$ \; \frac{1}{4}, \frac{4}{5}$, which yields the following expected payoff for player 2:
  - If player 1 plays U,

$$\frac{1}{2} * 4 + \frac{1}{2} * 0 = 2 > 1$$

  - If player 1 plays M,

$$\frac{1}{2} * 0 + \frac{1}{2} * 5 = \frac{5}{2} > 1$$

  - If player 1 plays D,

$$\frac{1}{2} * 4.5 + \frac{1}{2} * 5 = \frac{9.5}{2} = 4.75 > 3$$

- And, hence, strategy L is strictly dominated by the mixed strategy that puts probability $p = \frac{1}{2}$ to C and $1 \; ! \; p = \frac{1}{2}$ to R.

Problems with dominance

- And the remaining matrix after deleting strategy L for player 2, is

		Player 2	
		C	R
	U	0, 4	1, 0
Player 1	M	0, 0	1, 5
	D	4, 4.5	2, 5

Problems with dominance

- In the remaining matrix (after deleting strategy L), we can move to player 1, noting that U and M are strictly dominated by D.

Player 2

		C	R	
2nd Step	~~U~~	~~0, 4~~	~~1, 0~~	
Player 1	~~M~~	~~0, 0~~	~~1, 5~~	2nd Step
	D	4, 4.5	2, 5	

Problems with dominance

- Hence, the remaining matrix is

		Player 2 C	Player 2 R
Player 1	D	4, 4.5	2, 5

3^{rd} Step

- Moving now to player 2, note that C is strictly dominated by R.
- Hence, (D, R) survives IDSDS, with associated payoffs $(2, 5)$.
- Can people go over 3 steps of IDSDS
  - (specifically when the first involves mixing)?

Problems with dominance

- **3^{rd} problem(cont'd)**:Layers of rationality
  - While in some games the layers of rationality might be demanding (as in the game we just analyzed)...
  - We can assume that, if the stakes are sufficiently high (millions of dollars), individuals or firms would spend as much time as necessary in order to carefully analyze these players (it took us only a few minutes!) in order to maximize their payoffs as much as possible.