

ENDOGENOUS GROWTH MODELS

EE 462 Development Macroeconomics

Semester 1/2014

Topics

- Transitional Dynamics in Solow Model
- The AK Model
- Basic Human Capital Model
- The Lucas Model
- Growth Model with Externalities
- Romer's Model

Transitional Dynamics (Solow Model)

- Recall from Solow model, capital accumulation can be written as:

$$\Delta k = sy - (n+d)k$$

- Rewrite in terms of the rate of change in k :

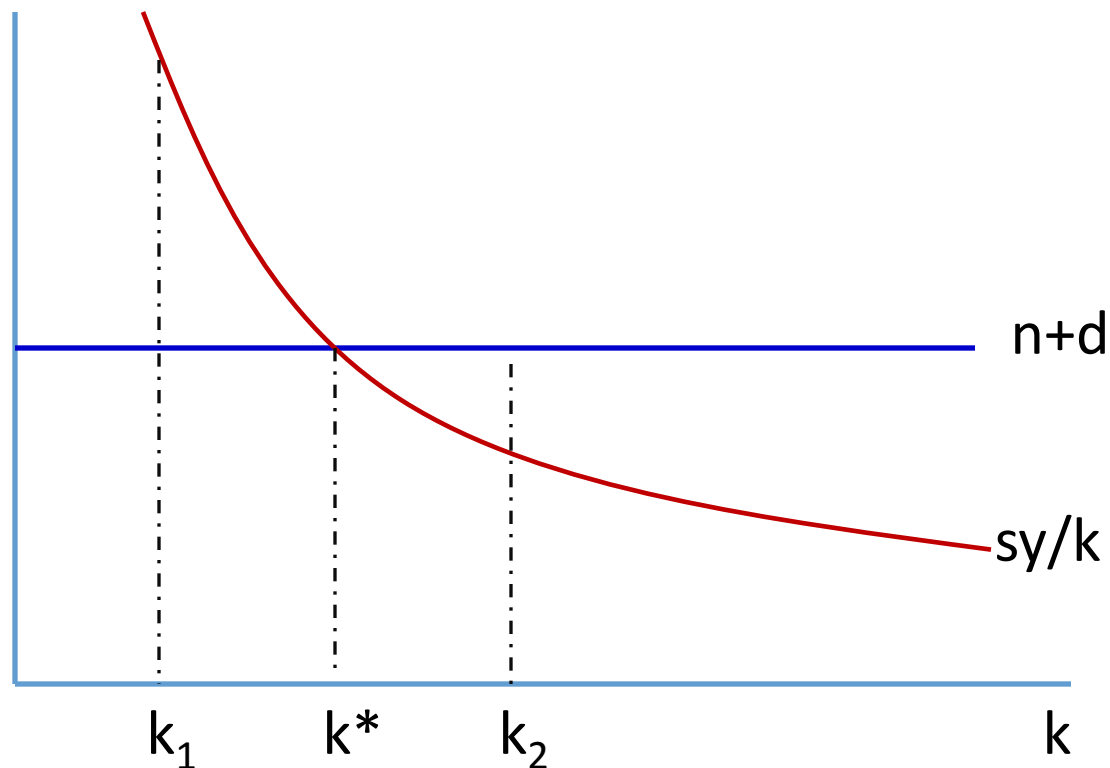
$$\Delta k/k = sy/k - (n+d)$$

- Given the production function: $y = f(k) = k^\alpha$. Then,

$$\rightarrow \Delta k/k = sk^{\alpha-1} - (n+d)$$

- Countries with high levels of k ($k > k_{ss}$) will have negative growth rates of k .
- Countries with low levels of k ($k < k_{ss}$) will have a positive growth rates of k .

Graph: Transitional Dynamics in Solow Model



The AK Model

- Assume a production function without diminishing returns:

$$Y = AK \quad \text{where } A > 0$$

- Output per capita can be written as:

$$y = Ak$$

where $y = Y/L$ and $k = K/L$

- The accumulation of capital per worker can be written as:

$$\Delta k/k = sA - (n+d)$$

$$\rightarrow \Delta k = sAk - (n+d)k$$

Graph: The AK Model

Transitional Dynamics: AK Model

Basic Human Capital Model (1)

- In this model, **human capital** is *deliberately accumulated* (i.e. not the outcome of population growth or exogenously specified technical progress).

- The production is given by:

$$y = k^\alpha h^{1-\alpha}$$

where h = human capital (unskilled labor is omitted).

- A fraction s of output is saved in order to accumulate capital:

$$\Delta k = sy + d_k$$

- Another fraction q of output is saved to augment the quality of human capital:

$$\Delta h = qy + d_h$$

Basic Human Capital Model (2)

- Let r denote the ratio of human to physical capital in the long run: $r = h/k$
- The growth rate of physical capital can be written as:

$$\Delta k/k = sr^{1-\alpha}$$

- The growth rate of human capital can be written as:

$$\Delta h/h = qr^{-\alpha}$$

- Assume $d_k = d_h$. Since the two growth rates of h and k are the same in the long run, we have:

$$r = q/s$$

- The larger the ratio of saving in h relative to that of k , the larger is the long-run ratio of h to k .

Basic Human Capital Model (3)

- In the long run, all variables must growth at the same rate.
- The growth rate of physical capital is:

$$\Delta k/k = sr^{1-\alpha} = s^\alpha q^{1-\alpha}$$

- Implications of the model:
 - It is possible to have diminishing returns to k, and yet there is no converge in output per capita.
 - The rate of savings and the rate of investment in human capital affect the growth rate. That is, the growth rate is determined within the model.
 - Here, physical and human capital *together* exhibits constant returns. However, if a third factor (e.g. unskilled labor) is introduced, physical and human capital together would exhibit diminishing returns, and results should be similar to Solow model.

The Lucas (1988) Model

- Lucas (JME, 1988) defines human capital as the skill embodied in workers.
- The production function is given by:

$$Y = AK^\alpha(uhN)^{1-\alpha} = AK^\alpha(L)^{1-\alpha}$$

where N = total workers

h = stock of human capital (of each worker)

u = proportion of total labor spent on producing output

$L = uhN$ = effective worker

- Human capital grows at a constant rate:

$$\Delta h/h = a(1-u)$$

Where a reflects quality or efficiency of education

Lucas Model (cont'd)

- Physical capital accumulation is defined as usual: $\Delta K/K = sY$.
- Rewrite the production function in terms of per effective worker:

$$y_e = k_e^\alpha$$

- The growth rate of physical capital per effective worker is:



- At the equilibrium in the long run,
 - Capital per effective worker is:
 - Output per effective worker is:
- Note: This simplified version of Lucas model is similar to the Solow Model with augmented labor technology.

Lucas Model (cont'd)

- In the long run, there is no change in physical capital per effective worker. Thus, the growth rate of physical capital per worker can be written as:
 - Main driver for growth is the growth rate of human capital.
 - Policies that promote knowledge accumulate will result in higher growth rate of output.
 - $1-u$ can be thought of as the production of knowledge.

Growth Model with Externalities (1)

- Suppose there are several firm within the economy.
- Let the production function of each firm be:

$$Y = EK^\alpha P^{1-\alpha}$$

where Y = output, K = capital, P = labor employed, E = a measure of overall *external productivity*.

- Now, E is a positive externality generated by the joint capital accumulation of all firms in the economy:

$$E = a(K^*)^\beta$$

where $a > 0$ and $\beta > 0$.

- Substitute E in the production function, and we get:

$$Y = a(K^*)^\beta K^\alpha P^{1-\alpha}$$

Growth Model with Externalities (2)

- Suppose that all the firms are owned by a *benevolent planner* (who values the capital investment of one firm on the overall productivity of other firms).
 - *Positive externalities* created by capital accumulation give a *macroeconomic* production function that may exhibit *increasing returns*.
- Assume all the firms are identical. The “social” production function is:

$$Y = aK^{\alpha+\beta}P^{1-\alpha}$$

- Per capita economic growth is not only positive, also tends to accelerate over time.

Romer's Model (Knowledge Externalities)

- Romer (1990) and Jones (1995) use a model of firms that invest in R&D.
- A firm's R&D raises its profits, but also create a positive externality on other firm (lead to increasing returns overall).
- The production function is given by:

$$Y = K^\alpha (AL_Y)^{1-\alpha}$$

where A = stock of ideas/knowledge

L_Y = labor used in the production sector

L = total labor: $L = L_Y + L_A$ (L_A = labor used in R&D)

- Capital accumulation equation is given by:

$$\Delta K/K = s_K Y - dK$$

Romer's Model (cont'd)

- Assume the production function for ideas:

$$\Delta A = \delta A^\phi (L_A)^\lambda$$

- Suppose the population growth is n , and that $L_A/L = s_R$.

- $L_Y/L = 1 - s_R$

- Question: What determine the growth rate of A (i.e. growth rate of ideas)?

- It depends on the parameters ϕ , λ and growth of L_A .

- Define the *growth rate of ideas* as:

- $g_A = \Delta A/A =$

Romer's Model (cont'd)

- Case 1: assumes $0 < \lambda < 1$ and $\phi < 1$ (Jones, 1995).



- Case 2: assumes: $\lambda = 1$ and $\phi = 1$. (Romer, 1990).

