# Chapter 7

## Hypothesis Testing

# Steps in Hypothesis Testing – Traditional Method

# Steps in Hypothesis Testing

The goal of testing hypotheses is to determine whether a claim or conjecture about some feature of the population, a parameter, is strongly supported by the information obtained from the sample data.

# Steps in Hypothesis Testing (Step 1)

Step 1: State the null and alternative hypotheses.

The null hypothesis, symbolized by $H_0$, is a statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.

The alternative hypothesis, symbolized by $H_1$, is a statistical hypothesis that states the existence of a difference between a parameter and specific value, or states that there is a difference between two parameters. ← The statement to be proved

# Steps in Hypothesis Testing (Step 1) (Cont.)

The null and alternative hypotheses are stated together, and the null hypothesis contains the equals sign, as shown (where *k* represents a specified number).

| Two-tailed test | Right-tailed test | Left-tailed test |
|---|---|---|
| $H_0: \mu = k$ | $H_0: \mu \leq k$ | $H_0: \mu \geq k$ |
| $H_1: \mu \neq k$ | $H_1: \mu > k$ | $H_1: \mu < k$ |

# Steps in Hypothesis Testing (Step 1) (Cont.)

Example 1 : For each conjecture, state the null and alternative hypotheses.

a. The average age of taxi drivers in NYC is 36.3 years.
b. The average income of nurses is $36,250.
c. The average age of disk jockeys is greater than 27.6 years.
d. The average pulse rate of female joggers is less than 72 beats per minute.
e. The average bowling score of people who enrolled in a basic bowling class is less than 100.
f. The average cost of a DVD recorder is $297.75.

# Steps in Hypothesis Testing (Step 2)

Step 2: Select a level of significance (α).

In the hypothesis-testing situation, there are four possible outcomes. In reality, the null hypothesis may or may not be true, and a decision is made to reject or not reject it on the basis of the data obtained from a sample.

| | $H_0$ is true | $H_0$ is false |
|---|---|---|
| $H_0$ is rejected | Type I error | Correct decision |
| $H_0$ is not rejected | Correct decision | Type II error |

# Steps in Hypothesis Testing (Step 2) (Cont.)

The level of significance is the maximum probability of committing a type I error. This probability is symbolized by $\alpha$. That is, P( type I error ) = $\alpha$.

The probability of a type II error is symbolized by $\beta$. That is, P( type II error ) = $\beta$.

In most hypothesis-testing situations, $\beta$ cannot easily be computed; however, $\alpha$ and $\beta$ are related in that decreasing one increases the other.

# Steps in Hypothesis Testing (Step 3)

Step 3: Determine the statistical test.

A statistical test uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.

The numerical value obtained from a statistical test is called the test value.

To determine the test statistic, choose between z test or t test.

- When $\sigma$ is known or when $n \geq 30$ => use z test
- When $\sigma$ is unknown and when $n < 30$ => use t test

# Steps in Hypothesis Testing (Step 4) (Cont.)

Step 4: Define the rejection or critical region.

The critical or rejection region is the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected.

The critical value separates the critical region from the non critical region. The symbol for critical value is C.V.

# Steps in Hypothesis Testing (Step 4) (Cont.)

Consider a normal curve for the rejection region. The curve is divided into two parts

1. Rejection region has area under the curve in the tail equal to $\alpha$ for one-tailed test or $\alpha/2$ for two tailed test.
2. Acceptance region is the rest of the area.

<u>Eg.</u>

# Steps in Hypothesis Testing (Step 5 and Step 6)

Step 5: Compute the test value.

Compute z value or t value.

Step 6: Make a decision to reject or not reject the null hypothesis.

- Is your computed value falls in the rejection region?

  If yes, reject $H_0$.

  If no, fail to reject $H_0$.
- State the conclusion.

# P-Value Method for Hypothesis Testing ( or Step 7)

Step 7: Compute p-value.

The P-value (or probability value) is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.

# P-Value Method for Hypothesis Testing ( or Step 6) (Cont.)

Methods to find P-Value:

- For $H_1$: $\mu < k$ :

  P-value = $P(z \leq z_{obs})$

- For $H_1$: $\mu < k$ :

  P-value = $P(z \geq z_{obs})$

- For $H_1$: $\mu \neq k$ :

  P-value = $P(z \leq -|z_{obs}| \text{ or } z \geq |z_{obs}|)$

  $= P(z \leq -|z_{obs}|) + P(z \geq |z_{obs}|)$

  $= 2P(z \geq |z_{obs}|)$

If P-value $\leq \alpha$, reject $H_0$.

If P-value $> \alpha$, fail to reject $H_0$.

# $z$ Test for a Mean

# $z$ Test for a Mean

The z test is a statistical test for the mean of a population.

- When $\sigma$ is known, the formula for the z test is

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

- When $\sigma$ is unknown but $n \geq 30$, the formula for the z test is

$$z = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

# $z$ Test for a Mean

Example 2: A certain city wants to conclude that the mean level of carbon monoxide is less then 4.9 ppm. Over the last 30 days the observed average CO level is 4.6. Does this constitute evidence to support the claim with $\sigma = 0.8$ and significance level of $\alpha = 0.025$. Find P-value for this test.

# $z$ Test for a Mean (Cont.)

Example 3: A random sample of 100 recorded deaths in Canada during the past year showed an aver life span of 71.8 years, with a standard deviation of 8.9 years. Does this seem to indicate that the average life span today is greater than 70 years? Use a significance level of $\alpha = 0.05$ (i.e. 5% level of significance). Find P-value for this test.

# $z$ Test for a Mean (Cont.)

Example 4: It is claimed that the average height of a student is 65.4 inches. To test this claim a random sample of 100 heights among students revealed a sample mean of 64.2 and sample standard deviation of 4.1. Is this sufficient evidence to reject the claim? Use $\alpha = 0.05$. Find P-value for this test.

# *t* Test for a Mean

# $t$ Test for a Mean

The *t* test is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed $\sigma$ is unknown and $n < 30$.

The formula for the $t$ test is

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

where the degrees of freedom are d.f. = $n - 1$.

# $t$ Test for a Mean (Cont.)

Example 5: An engineering student examined the minimum bursting pressure of a certain type and size of PVC irrigation pipe. The manufacturer claims a mean bursting pressure of more than 350 psi. A sample of ten such pipes was experimentally determined to have the following bursting pressure:

| | | | | |
|---|---|---|---|---|
| 401 | 359 | 383 | 427 | 414 |
| 415 | 389 | 463 | 394 | 428 |

Test to see if the manufacturer's claim appears true. Use a level of significance of 0.05. Find P-value for this test.

# *t* Test for a Mean (Cont.)

Example 6 : From past experience, a teacher believes that the average score on a real estate exam is 75. A sample of 20 students' exam scores is as follows:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 80 | 68 | 72 | 73 | 76 | 81 | 71 | 71 |
| 65 | 50 | 63 | 71 | 70 | 70 | 76 | 75 |
| 69 | 70 | 72 | 74 | | | | |

Test the claim that the students' average is still 75. Use $\alpha = 0.01$. Use the P-value method.

# $z$ Test for Proportion

# *z* Test for Proportion

A hypothesis test involving a population proportion can be considered as a binomial experiment when there are only two outcomes and the probability of a success does not change from trial to trail. Since the normal distribution can be used to approximate the binomial distribution when $np \geq 5$ and $nq \geq 5$, the standard normal distribution can be used to test hypotheses for proportions.

Formula for the z Test for Proportions:

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

# $z$ Test for Proportion (Cont.)

Example 7: A university official believes that the percentage of the students that currently hold part time job has increased from 23% from 4 years ago. To test this belief, a new study was performed and it was found that 200 out of a random sample of 800 students hold part time hob. Is there enough evidence to support the belief? Use a level of significance of 0.05.

# *z* Test for Proportion (Cont.)

Example 8: Of families 48.8% have stock holdings. A random sample of 250 families indicated that 142 owned some type of stock. At what level of significance would you conclude that this was a significant difference?

# Confidence Intervals
# and
# Hypothesis Testing

# Confidence Interval and Hypothesis Testing

Relationship Between Confidence Intervals and Hypothesis Testing:

When the confidence interval contains the hypothesized mean, do not reject the null hypothesis.

When the confidence interval does not contain the hypothesized mean, reject the null hypothesis.

# Confidence Interval and Hypothesis Testing (Cont.)

Example 9 : Charter bus records show that in past years, the buses carried an average of 42 people per trip to Niagara Falls. The standard deviation of the population in the past was found to be 8. This year, the average of 10 trips showed a mean of 48 people booked. Can one reject the claim, at $\alpha = 0.10$, that the average is still the same? Find the 90% confidence interval of the mean. Dows the confidence interval interpretation agree with the hypothesis-testing results? Explain. Assume that the variable is normally distributed.