B.E. International Program

Faculty of Economics, Thammasat University

Final Examination: 1/2013

Subject: MA 217 Calculus for Social Sciences 2

Date: Saturday 7 December 2013 Time: 13.30 – 16.30 hrs.

Seat No…………………..

ID.No……………………

INSTRUCTIONS

DO NOT TURN OVER UNTIL TOLD THAT YOU MAY DO SO.

This paper has ***9*** *questions. Students are to answer* ***ALL*** *questions. Show all steps in answering the questions. Simplify your answers where possible. Necessary formulas are on the last page. You may detach the last page for your convenience. The mark for each question is given next to the problem –* ***use your time wisely****. Full mark of* **50** *can be obtained without completing all. (TOTAL:* **55** *marks)*

Students:

1. Non-graphic scientific calculators are allowed. Textbooks, lecture notes or any reading materials are **NOT** allowed in the examination room. If you are caught doing so you will automatically receive an **"F"** for the course and be suspended for one academic year.

2. All communication equipment (mobile phones, pagers, etc.) **must be switched off**.

3. Write in **black or blue ink only**. Any writing in pencil will **NOT** be marked except for curve sketching.

4. **All of the Thammasat University examination rules are applied.**

1.

(a) For a three-variable function $f(x,y,z)=2x+6y+10z$ subjected to $x^2+y^2+z^2=35$, use Lagrange Multiplier Method to determine the absolute minimum. Is it possible to use matrix to solve for critical points and why?

(b) For a three-variable function $f(x,y,z)=2x+6y+10z$ subjected to $x^2+y^2+z^2\le 35$, utilise information from (a) to determine the absolute maximum.

(7 marks)

2. For the following linear system

$$\begin{aligned} 3x_1+4x_2+x_4&=33+x_3 \\ x_1+x_3+3x_4+2&=x_2 \\ 2x_2+3x_3+5x_4&=3+x_1 \end{aligned},$$

find the solution. If it has an infinite number of solutions, give the answer in vector form.

(7 marks)

3. Let $\underline{\mathbf{A}}=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 1\\ 3 & 0 & a\end{bmatrix}$

(a) If a = 3, is it possible for a linear system $\underline{\mathbf{A}}\underline{\mathbf{x}}=\underline{\mathbf{b}}$ to have a unique solution, an infinite number of solution or no solution? Answer with reasons.

(b) If a = -1 and $\underline{\mathbf{B}}=\begin{bmatrix}9\\8\\3\end{bmatrix}$, determine the solution $\underline{\mathbf{x}}$. (Note: the solution can be determined without calculating $\underline{\mathbf{A}}^{-1}$.)

(7 marks)

4.

(a) If $\underline{\mathbf{A}}$ is a $m\times n$ matrix, is it possible for a linear system $\underline{\mathbf{A}}\underline{\mathbf{x}}=\underline{0}$ to have a unique solution, an infinite number of solution or no solution? Answer all three cases with reasons.

(b) If $\underline{\mathbf{A}}$ is a $m\times n$ matrix and the rank of $\underline{\mathbf{A}}\le n$, is it possible for a linear system $\underline{\mathbf{A}}\underline{\mathbf{x}}=\underline{\mathbf{b}}$ to have a unique solution, an infinite number of solution or no solution? Answer all three cases with reasons.

(6 marks)

5. Let $\underline{\mathbf{A}} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$,

(a) for what values of *a*, *b*, *c* and *d* will $\underline{\mathbf{A}}^{-1}$ exist?

(b) If $\underline{\mathbf{A}}^{-1}$ exists, then determine it and show that your answer is correct.

(4 marks)

6. Given $\underline{\mathbf{A}} = \begin{bmatrix} x^2 & y^2 & z^2 & 1 \\ 3 & 9 & 8 & 1 \\ 3 & 4 & 5 & 1 \\ 3 & 7 & 5 & 1 \end{bmatrix}$, $\underline{\mathbf{B}} = \begin{bmatrix} 0 & 0 & 1 & -13 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 4 & 0 & 0 & 3 \end{bmatrix}$ and $\underline{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 \\ \frac{1}{6} & 0 & -4 & 0 \\ -17 & \frac{1}{8} & -7 & -\frac{1}{4} \end{bmatrix}$

a) If $y = 4$ and $z = 13$, determine all possible values of *x* such that $\det \underline{\mathbf{A}} = \det(\underline{\mathbf{B}}\underline{\mathbf{C}}^2)$.

b) If $x = 3$, $y = 1$ and $z = 2$, determine $\det \underline{\mathbf{A}}$, $\det(-2\underline{\mathbf{A}}^{\mathrm{T}})$ and $\det \underline{\mathbf{D}}$, provided that $\underline{\mathbf{B}} = 9\underline{\mathbf{A}}\underline{\mathbf{D}}^3\underline{\mathbf{C}}^{-1}$.

c) If $x = 3$, $y = 1$ and $z = 2$, use Cramer's rule to solve for **ONLY** x_4 if $\underline{\mathbf{A}}\underline{\mathbf{x}} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$.

(13 marks)

7. Determine this integral $I = \int_0^\infty \int_0^\infty xye^{-x^2-y^2}\,dxdy$.

(3 marks)

8. Determine this integral $I = \int_0^1 \int_0^{2-2x} \int_0^{2-2x-y} y\,dzdydx$.

(5 marks)

9. At a certain factory, output *Q* is related to inputs *x* and *y* by the expression $Q = 2x^2 + 3x^2y + y^3$. If the input is varied as $0 \le x \le 5$ and $0 \le y \le 7$, what is the average output of the factory?

(3 marks)

Formulas

Differentiation

We assume that u is a differentiable function of x.

$\frac{d}{dx}(c) = 0$	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
$\frac{d}{dx}\left(x^n\right) = n\,x^{n-1}$	$\frac{d}{dx}\left(u^n\right) = n\,u^{n-1}\frac{du}{dx}$
$\frac{d}{dx}\left[c\ f(x)\right] = c\,f'(x)$	$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$
$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$	$\frac{d}{dx}\left(e^u\right) = e^u\frac{du}{dx}$
$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(\log_b u) = \frac{1}{(\ln b)u} \cdot \frac{du}{dx}$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$	$\frac{d}{dx}\left(a^u\right) = a^u(\ln a)\frac{du}{dx}$

Integration

$\int k\,dx = kx + C$	$\int\left[f(x) \pm g(x)\right]dx = \int f(x)\,dx \pm \int g(x)\,dx$
$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$	$\int [u(x)]^n du = \frac{u^{n+1}}{n+1} + C$
$\int e^x dx = e^x + C$	$\int e^u du = e^u + C$
$\int kf(x)dx = k\int f(x)dx$	$\int \frac{1}{u} du = \ln\lvert u\rvert + C,\ u \neq 0$

Integration by parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$