

Exercise 1

Part I: Compute the following limits.

1. $\lim_{x \rightarrow 25} \frac{5 - \sqrt{x}}{25 - x}$.
2. $\lim_{x \rightarrow 0} \frac{x^2}{\sqrt{x^2 + 12} - \sqrt{12}}$.
3. $\lim_{x \rightarrow 0} \frac{x \sin(x/2)}{|x|}$.
4. $\lim_{x \rightarrow -2} \frac{|x| - 2}{\sqrt{x^2 - 2x} - \sqrt{8}}$.
5. $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$.

Part II:

$$\text{Let } f(x) = \begin{cases} x, & \text{for } x < 0 \\ x^2, & \text{for } 0 \leq x < 2 \\ x, & \text{for } x > 2 \end{cases} .$$

1. Find each of the following limits (if exists).

- (a) $\lim_{x \rightarrow 0^-} f(x)$ and $\lim_{x \rightarrow 0^+} f(x)$.
- (b) $\lim_{x \rightarrow 0} f(x)$.
- (c) $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$.
- (d) $\lim_{x \rightarrow 2} f(x)$.

2. Find all the values of x at which the function $f(x)$ is discontinuous.

Limit and Continuity

1. Find each of the following limits (if it exists).

- (a) $\lim_{x \rightarrow 8} \frac{x^2 - 5x - 24}{x - 8}$
- (b) $\lim_{x \rightarrow 0} x^3(x^4 + 2x^3)^{-1}$
- (c) $\lim_{x \rightarrow 0} \left(\frac{x^3 + 3x - 1}{x} + \frac{1}{x} \right)$
- (d) $\lim_{x \rightarrow 2} \left(\frac{1}{x - 2} - \frac{6}{x^2 + 2x - 8} \right)$
- (e) $\lim_{x \rightarrow 5} \frac{\sqrt{x + 4} - 3}{x - 5}$
- (f) $\lim_{x \rightarrow 5} \frac{|x - 5|}{x - 5}$

$$(g) \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{5x^2-10x+5}}$$

2. Determine the number (if any) at which each given function is discontinuous.

$$(a) f(x) = x^3 + 2x - 1$$

$$(b) f(x) = \frac{x^2-1}{x^4-1}$$

$$(c) f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{for } x < 1 \\ 3, & \text{for } 1 \leq x < 2 \\ x^2 - 1, & \text{for } x \geq 2 \end{cases}$$

3. Find the values of a , b and c so that the given functions f and g are continuous.

$$(a) f(x) = \begin{cases} ax, & \text{for } x < 4 \\ x^2, & \text{for } x \geq 4 \end{cases}$$

$$(b) g(x) = \begin{cases} bx, & \text{for } x < 3 \\ c, & \text{for } x = 3 \\ -2x + 7, & \text{for } x > 3 \end{cases}$$

4. **Trigonometric functions:** Find the the following limits.

$$(a) \lim_{x \rightarrow 0} \frac{x}{\sin(\pi x)}$$

$$(b) \lim_{x \rightarrow 0} \tan(x)$$

$$(c) \lim_{x \rightarrow 0} \frac{\tan(x/2)}{x}$$

$$(d) \lim_{x \rightarrow 2} \frac{\sin(x-2)}{x^2+2x-8}$$

$$(e) \lim_{x \rightarrow 0} \frac{3x + \tan(x)}{x \cos(5x) + \sin(7x)}$$

$$(f) \lim_{x \rightarrow 0} x^4 \sin\left(\frac{1}{x}\right)$$

$$(g) \lim_{x \rightarrow \pi/4} \frac{\tan(x)-1}{\sin(x)-\cos(x)}$$

5. **Limit at infinity:** Find the the following limits.

$$(a) \lim_{x \rightarrow -\infty} \left(\frac{1}{5}x^7 - 5x^5 - x^2\right).$$

$$(b) \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2-8}}{1+2x} \text{ (Note: } \sqrt{x^2} = |x|)$$

$$(c) \lim_{x \rightarrow -\infty} \frac{2x-1}{\sqrt{3x^2-1}+2x}$$

$$(d) \lim_{x \rightarrow \infty} \frac{2x^4-x^2-8x}{1-3x^4}$$

(e) $\lim_{x \rightarrow -\infty} \frac{x^2 - 8}{1 - 2x}$

(f) $\lim_{x \rightarrow \infty} e^{1/x}$

(g) $\lim_{x \rightarrow \infty} \frac{3e^{5x} + 2e^{-2x}}{e^{-2x} - e^{5x}}$ and $\lim_{x \rightarrow -\infty} \frac{3e^{5x} + 2e^{-2x}}{e^{-2x} - e^{5x}}$

(h) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$