1. firm 1 ; $\pi_1 = TR_1 - TC_1$

$\pi_1 = (a - bq_1 - bq_2 - bq_3)q_1 - C_1$

$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 = 0$

$a - bq_2 - bq_3 = 2bq_1$

$q_1 = \frac{a - bq_2 - bq_3}{2b}$

substitute ② ▸ $q_1 = \frac{a - \left(\frac{a - q_3 b}{3b}\right)b - q_3 b}{2b}$

$q_1 = \frac{3a - a + q_3 b - 3q_3 b}{6b}$

$q_1 = \frac{a - q_3 b}{3b}$ — ①

Substitute ③ ▸ $q_1 = \frac{a - \left(\frac{a}{4b}\right)b}{3b}$

$q_1 = \frac{4a - a}{12b} = \frac{a}{4b}$

Firm 2 ; $\pi_2 = TR_2 - TC_2$

$\pi_2 = (a - bq_1 - bq_2 - bq_3)q_2 - C_2$

$\frac{\partial \pi_2}{\partial q_2} = a - bq_3 - 2bq_2 - bq_3 = 0$

$a - bq_1 - bq_3 = 2bq_2$

Substitute ① ▸ $2q_2 b = a - \left(\frac{a - q_2 b - q_3 b}{2b}\right)b - q_3 b$

$2q_2 b = \frac{2a - a + q_2 b + q_3 b - 2q_3 b}{2}$

$4q_2 b = a + q_2 b - q_3 b$

$3q_2 b = a - q_3 b$

$q_2 = \frac{a - q_3 b}{3b}$ — ②

Substitute ③ ▸ $q_2 = \frac{a - \left(\frac{a}{4b}\right)b}{3b}$

$q_2 = \frac{4a - a}{12b} = \frac{a}{4b}$

Firm 3 ; $\pi_3 = TR_3 - TC_3$

$\pi_3 = (a - bq_1 - bq_2 - bq_3)q_3 - C_3$

$\frac{\partial \pi_3}{\partial q_3} = a - bq_1 - bq_2 - 2bq_3 = 0$

$a - bq_1 - bq_2 = 2bq_3$

$q_3 = a - bq_1 - bq_2$

Substitute ① & ② ▸ $q_3 = \frac{a - b\left(\frac{a - q_3 b}{3b}\right) - b\left(\frac{a - q_3 b}{3b}\right)}{2b}$

$q_3 = \frac{3a - a + q_3 b - a + q_3 b}{6b}$

$q_3 = \frac{a + 2q_3 b}{6b}$

$6bq_3 = a + 2q_3 b$

$4bq_3 = a$

$q_3 = \frac{a}{4b}$

Equilibruim price ; $P = a - bQ$

$P = a - b(q_1 + q_2 + q_3)$

$P = a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$

$P = a - \left(\frac{3a}{4}\right)$

$P = \frac{a}{4}$

$P = 0.25a$

Firm 1 ; $\pi_1 = P \cdot Q - C_1$

$= 0.25a \cdot \frac{a}{4b} - C_1$

$$\pi_1 = \frac{a^2}{16b} - C_1$$

Firm 2 ; $\pi_2 = P \cdot q_2 - C_2$

$= 0.25a \cdot \frac{a}{4b} - C_2$

$$\pi_2 = \frac{a^2}{16b} - C_2$$

Firm 3 ; $\pi_3 = P \cdot q_2 - C_3$

$= 0.25a \cdot \frac{a}{4b} - C_3$

$$\pi_3 = \frac{a^2}{16b} - C_3$$

2. We assume that $A = q_1+q_2+q_3+q_n$

$$P = a-b(q_1+q_2+q_3+\dots+q_n)$$
$$P = a-bq_1-bq_2-\dots-bq_n$$

$$\pi_1 = (a-bq_1-bq_2-bq_3-\dots-bq_n)\cdot q_1 - C_1$$
$$\vdots$$
$$\pi_n = (a-bq_1-bq_2-bq_3-\dots-bq_n)\cdot q_n - C_n$$
$$\frac{\partial \pi_1}{\partial q_1} = a-2bq_1-bq_2-bq_3-\dots-bq_n = 0$$

$$q_1 = \frac{a}{2b} - 0.5(q_2+q_3+\dots+q_n)$$
$$q_n = \frac{a}{2b} - 0.5(q_1+q_2+q_3+\dots+q_{n-1})$$

$$q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1+q_2+q_3+\dots+q_n)$$
$$0.5q_1 = \frac{a}{2b} - 0.5A$$
$$q_1 = \frac{a}{b} - A \quad \text{——①}$$
$$q_2 = \frac{a}{b} - A$$
$$q_3 = \frac{a}{b} - A$$
$$\vdots$$
$$q_n = \frac{a}{b} - A$$

Since $A = q_1+q_2+q_3+q_n$ ▸ $A = n\left(\frac{a}{b} - A\right)$

$$A = ng - nA$$
$$A+nA = \frac{n(a)}{b}$$
$$A(1+n) = \frac{n(a)}{b}$$

substitute into ① $q_1 = \frac{a}{(n+1)b}$

$$q_i = \frac{a}{(n+1)b}$$

Equilibruim price ; $P = a-b(A)$

$$= a-b\left(\frac{na}{(n+1)b}\right)$$

$$= a-\left(\frac{n}{n+1}\right)a$$

$$= \frac{a(n+1)-na}{n+1}$$

$$= \frac{na+a-na}{n+1}$$

$$= \frac{a}{n+1}$$

$$\pi_i = P\cdot q_i - C_i$$
$$\pi_i = \frac{a}{n+1}\cdot\frac{a}{(n+1)b} - C_1$$

$$\pi_i = \frac{a^2}{(n+1)^2 b} - C_1$$

3. if $n = \infty$ ▸ if we make $q_i = \frac{a}{(n+1)b}$ ⟶ nearly zero and each firm will set a nearly zero unit

▸ if we make $A = nqi$ ⟶ nearly zero. Q of every firm combined will be nearly ▸ ∞ unit

▸ if we make $P = \frac{A}{n+1}$ ⟶ nearly zero, when supply increase, price will decrease ▸ nearly zero

▸ If we make $\pi_i = \frac{a^2}{(n+1,b)} - C_i$ ▸ $-C_i$ ▸ Each firm has loss profit.

if $n = 1$ ▸ it make $q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$. Since $Q = \frac{a}{2b} < Q = \frac{na}{(n-1)b}$, monopoly will set less quantity.

▸ if we make $A = nqi \times Q$. the firm will be a monopoly.

▸ if we make $P = \frac{a}{n+1} = \frac{a}{2}$. Since $\frac{a}{2} > P = \frac{a}{n+1}$, Monopoly will set higher price.

▸ if we make $\pi_i = \frac{a^2}{(n+\pi^2 b)} - Ci = \frac{a^2}{4b} - C_i$, which higher than $\pi_i = \frac{a^2}{(n+1)^2 b}$. the monopoly will get higher profit.

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