

1.2.2) Comparative Static analysis in Math framework

Having solved for the equilibrium solution, what economists usually ask is what would happen to the equilibrium if something, previously assumed to be fixed, has changed.

Example 1.C (cont.): National income model

- From the example 1.B, it is straightforward to solve for all the endogenous equilibrium solutions, Y^* , C^* , Y_d^* .

$$\begin{aligned}
 Y &= C + G + I \\
 Y &= [a + b(Y - T)] + G + I \\
 Y &= [a + b(Y - T)] + G + I \\
 Y - bY &= a - bT + G + I \\
 Y &= \frac{a - bT + G + I}{1 - b}
 \end{aligned}$$

- Numerically, if $a = 1$, $T_0 = \$0$, $I_0 = \$1$, $G_0 = \$1$ and $b = 0.5$, this yields us,

$$\begin{aligned}
 Y &= \frac{a - bT + G + I}{1 - b} \\
 Y &= \frac{1 - 0.5(0) + 1 + 1}{1 - 0.5} \\
 Y &= \frac{3}{0.5} = \underline{\underline{6}}
 \end{aligned}$$

Exercise 2.A:

HW submit w/ HW 1

2.A.1) Given a demand function by $p = a - bQ$, derive the formula for the elasticity of demand, and show that the third property holds

2.A.2) Given the market supply $p = c + dQ$ where $d \geq 0$, show that

- (i) elasticity of supply is always greater than 1 if $c > 0$,
- (ii) elasticity of supply is always equal to 1 if $c = 0$,
- (iii) elasticity of supply is always less than 1 if $c < 0$.

2 A 1) $PED = \frac{1}{-b} \times \frac{P_0}{Q_0}$

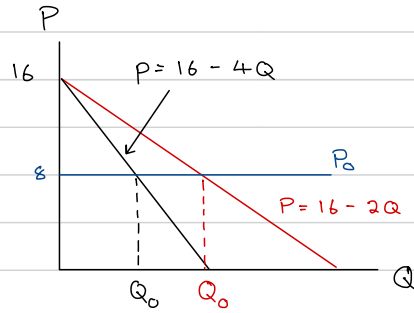
Note that elasticity can also be expressed as $E_d = \left(\frac{\text{Price}}{\text{Quantity demanded}} \right) \left(\frac{1}{\text{Slope of demand}} \right)$.

Proof of 3rd property:

if PEDs are equal

$$\frac{1}{-4} \times \frac{8}{2} = \frac{1}{-2} \times \frac{8}{4}$$

$$-1 = -1 \quad \checkmark$$



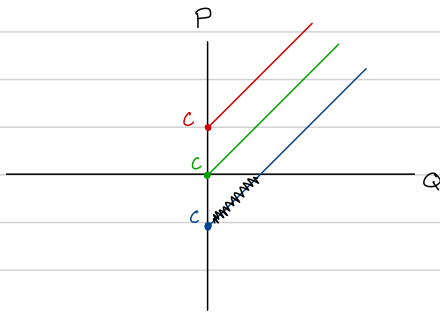
$$Q_0 \rightarrow 8 = 16 - 4Q_0$$

$$Q_0 = 2$$

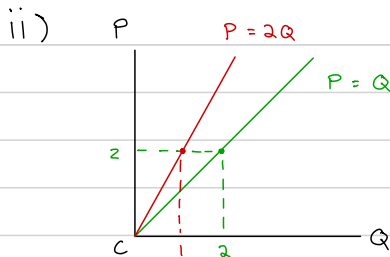
$$Q_0 \rightarrow 8 = 16 - 2Q_0$$

$$Q_0 = 4$$

2A2)



i) $PED = \frac{1}{b} \times \frac{P_0}{Q_0} > 0$ bc b, P_0, Q_0 are positive
and $P_0 \geq Q_0$ always



$$PED = \frac{1}{2} \times \frac{2}{1} = 1$$

$$PED = \frac{1}{1} \times \frac{1}{1} = 1$$

iii) $Q_0 > P_0$ always, so $\frac{P_0}{Q_0} \in [0, 1)$ can't be 1 bc then $b = \infty$

also $\frac{1}{b} \in (0, 1]$, multiply each other and the result < 1

Example 2.I: A monopolist firm faces the market demand given by $P = 10 - Q$. Consider the following questions if the cost function $C(Q) = 4Q$.

- What is the revenue-maximizing level of output?

revenue function $TR(Q) = P(Q) \times Q$

$P = 10 - Q \rightarrow TR(Q) = 10Q - Q^2$

max TR $\rightarrow \frac{dTR}{dQ} = 10 - 2Q = 0$

$10 = 2Q$

$Q = 5$

At $Q = 5$, $TR = 25$

Slope of TR = marginal revenue

- What is the break-even output?

$\pi = 0 \rightarrow TR - TC = 0 \rightarrow TR = TC$

$10Q - Q^2 = 4Q$

$6Q = Q^2$

$Q = 0, 6$

- What is the profit-maximizing level of output?

$C(Q) = 4Q$

$MC = \frac{dC}{dQ} = 4$

$MR = 10 - 2Q$

$10 - 2Q = 4$

$6 = 2Q$

$Q = 3$

① $MR = MC$

② $\pi = TR - TC$

$\frac{d\pi}{dQ} = MR - MC = 0$

$MR = MC$

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Exercise 2B. Consider a function that relates tax revenues R , in billions of dollars, to the average tax rate t such that $R = 350t - 500t^2$.

(a) What tax rate(s) is consistent with raising tax revenues equal to \$60 billion?

(b) What tax rate(s) is consistent with raising tax revenues equal to \$61.25 billion?

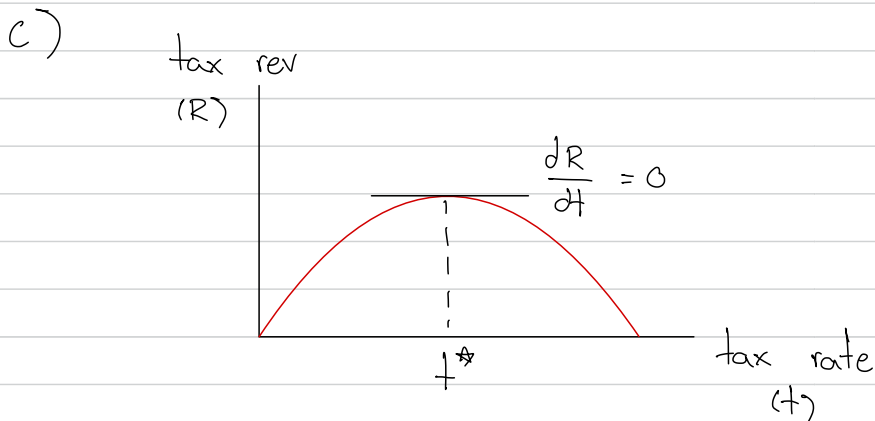
(c) What tax rate is consistent with the maximum tax revenue?



$$60 = 350t - 500t^2$$
$$500t^2 - 350t + 60 = 0$$
$$t = 0.3, 0.4 \quad \text{Ans: } 30\%, 40\%$$

b)

$$500t^2 - 350t + 61.25 = 0$$
$$t = 0.35 \quad \text{Ans: } 35\%$$



$$\frac{dR}{dt} = 350 - 1000t = 0$$
$$350 = 1000t$$
$$t = 0.35 \quad \text{Ans: } 35\%$$