EE 320 Introductory Mathematical Economics

Homework 2 – Suggested Answers

1. a) det = 1(3x3 - 2x2) -1(-2x3 - 2x1) + 2(-2x2 - 3x1) = -1
   b) det = 1(-1x2 - 1x4) - 2(2x2 - 1x2) + 1(2x4 - -1x2) = 0
   c) det = 2(3x4 - 0x7) - 0(5x4 - 0x6) + 0(5x7 - 3x6) = 24
   d) det = 2(10x4 - 15x1) - 3(5x4 - 15x2) + 1(5x1 - 10x2) = 65

2. a) Yes

   b) $A^2 = (I - X(X'X)^{-1}X')\,(I - X(X'X)^{-1}X')$

   $= II - X(X'X)^{-1}X' - X(X'X)^{-1}X' + X(X'X)^{-1}X'X(X'X)^{-1}X'$

   $= I - X(X'X)^{-1}X' - X(X'X)^{-1}X' + X\cdot I\cdot(X'X)^{-1}X'$

   $= I - X(X'X)^{-1}X' - \cancel{X(X'X)^{-1}X'} + \cancel{X\,(X'X)^{-1}X'}$

   $= I - X(X'X)^{-1}X' \quad = A$ *Q.E.D.*

   a) Must $(X'X)$ be square? Must X be square?
   b) Show that matrix A is idempotent – that is $A = A^2$.

Note: This property has useful implications in econometric applications.

3.

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} Q'_d \\ Q_s \\ P \end{bmatrix} = \begin{bmatrix} 140 \\ 80 \\ 0 \end{bmatrix}$$

$Q^* = 104$, $P^*_s = 12$, $P^*_d = 7$

4. $$\begin{bmatrix} 1-b+m & a \\ m & -h \end{bmatrix} \begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} -bT + I_0 + G_0 + X_0 \\ M_0 \end{bmatrix}$$

$$Y^* = \frac{\begin{vmatrix} -bT+I_0+G_0+X_0 & a \\ M_0 & -h \end{vmatrix}}{\begin{vmatrix} 1-b+m & a \\ m & -h \end{vmatrix}} = \frac{h(-bT+I_0+G_0+X_0)+aM_0}{am+h(1-b+m)}$$

$$r^* = \frac{\begin{vmatrix} 1-b+m & -bT+I_0+G_0+X_0 \\ m & M_0 \end{vmatrix}}{\begin{vmatrix} 1-b+m & a \\ m & -h \end{vmatrix}} = \frac{m(-bT+I_0+G_0+X_0)-(1-b+m)M_0}{am+h(1-b+m)}$$

5. $x_1 = 150$, $x_2 = 200$, and $x_3 = 100$.