Heteroscedasticity

The nature of Heteroscedasticity

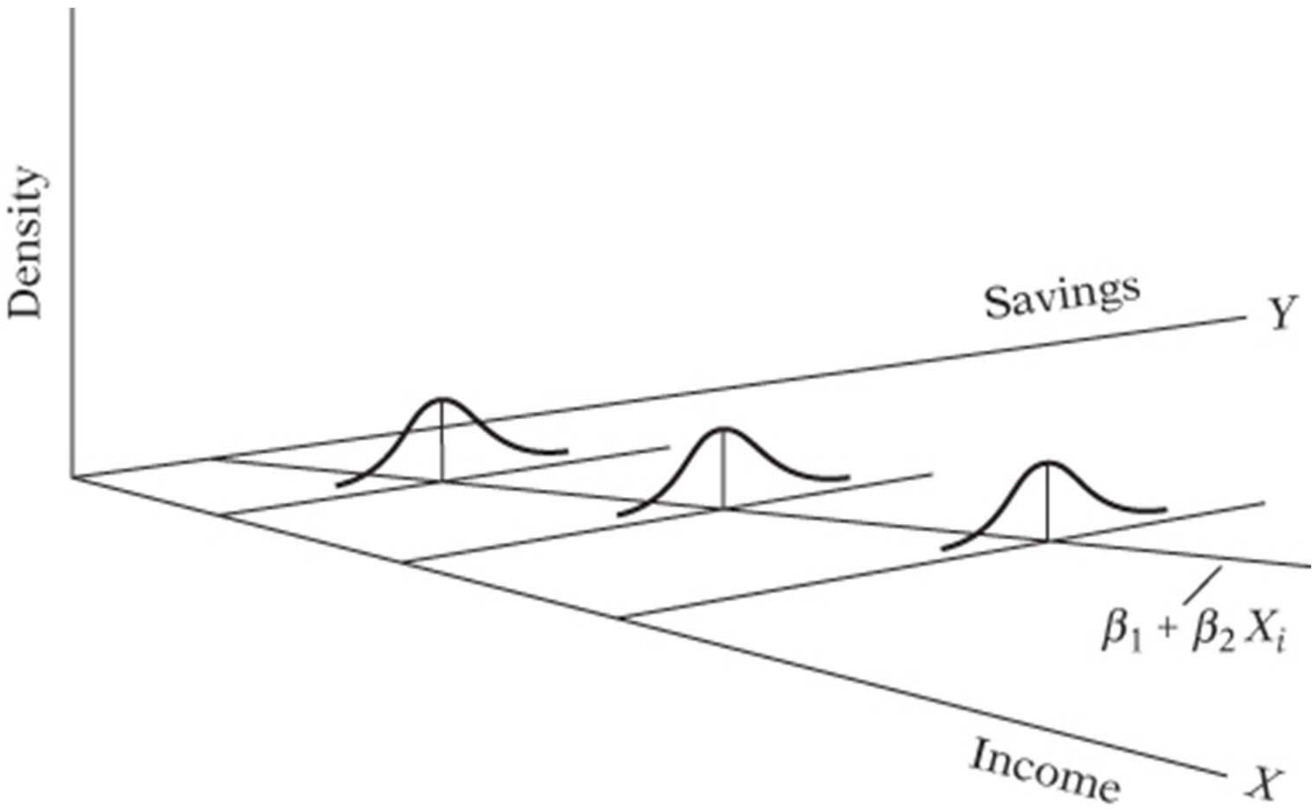
One of the important assumptions of CLRM is that the variance of each disturbance term u_i , conditional on the chosen values of the explanatory variables, is some constant number equal to σ^2 (Homoscedasticity)

$$E(u_i^2) = \sigma^2 \qquad i = 1, \ldots, n$$

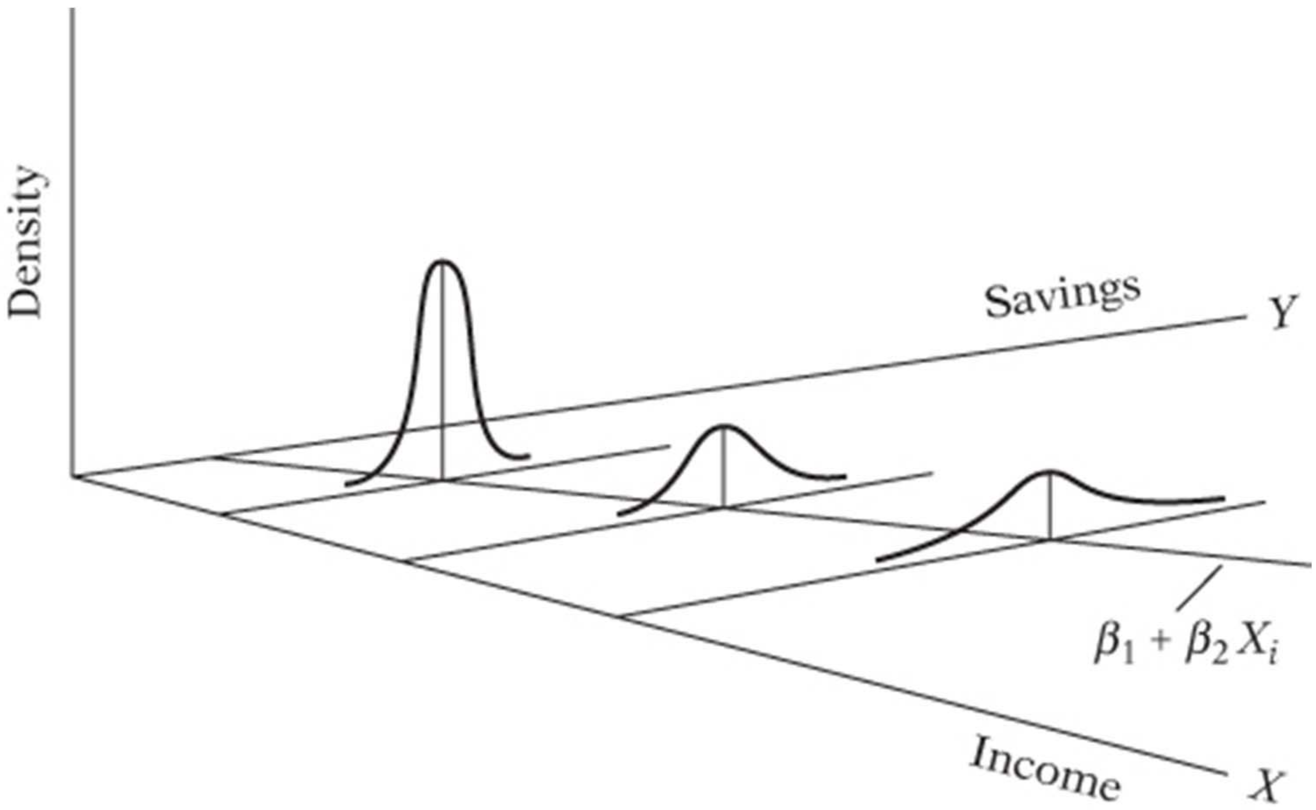
The conditional variance of Y_i increases as X increases. Here, the variances of Y_i are not the same. Here, there is heteroscedasticity.

$$E(u_i^2) = \sigma_i^2 \qquad i = 1, \ldots, n$$

Homoscedastic disturbances



Heteroscedastic disturbances

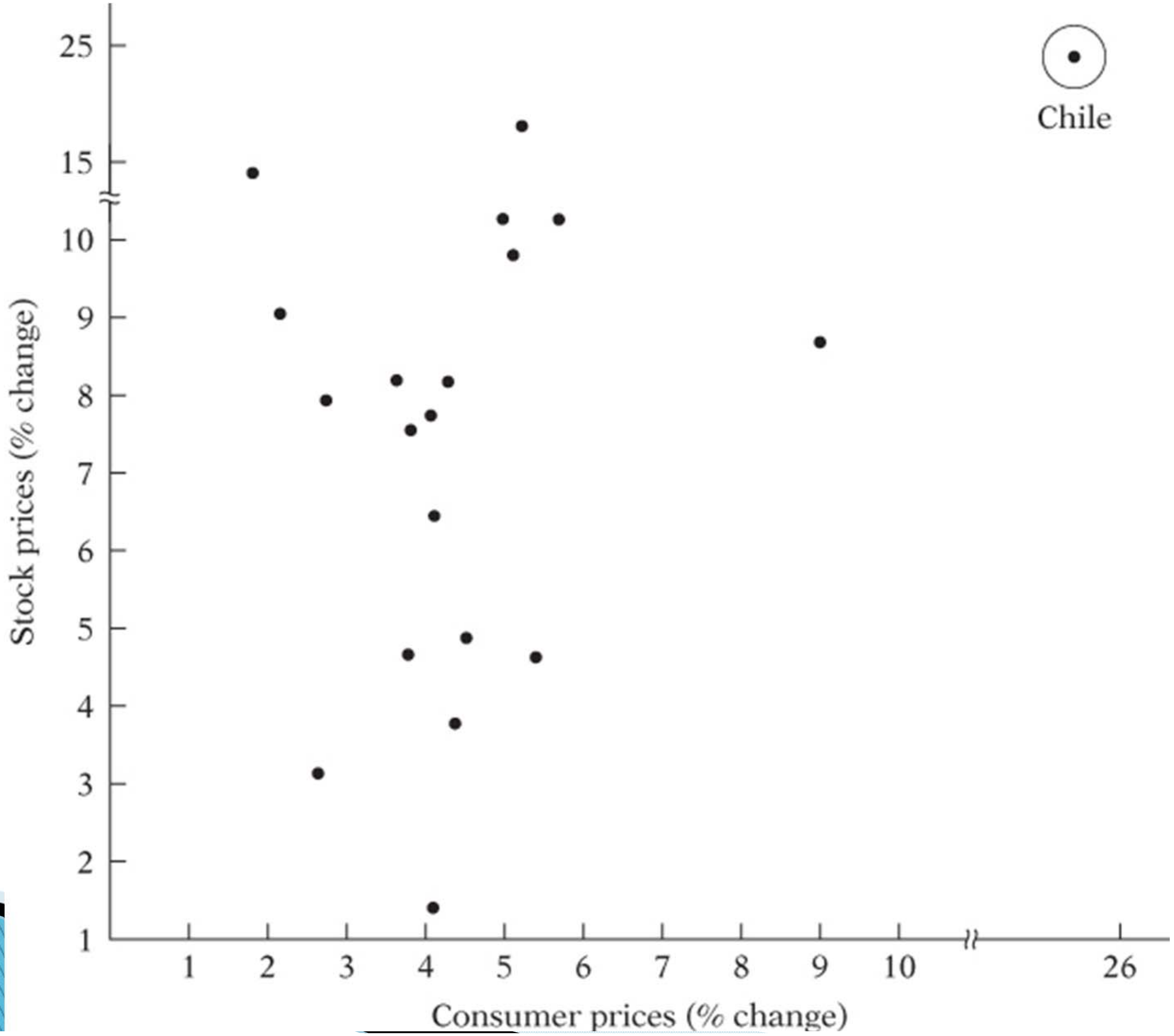


Several reasons why the variances of u_i may be variable

- As incomes grow, people have more discretionary income and hence more scope for choice about the disposition of their income. Hence σ_i^2 is likely to increase with income.

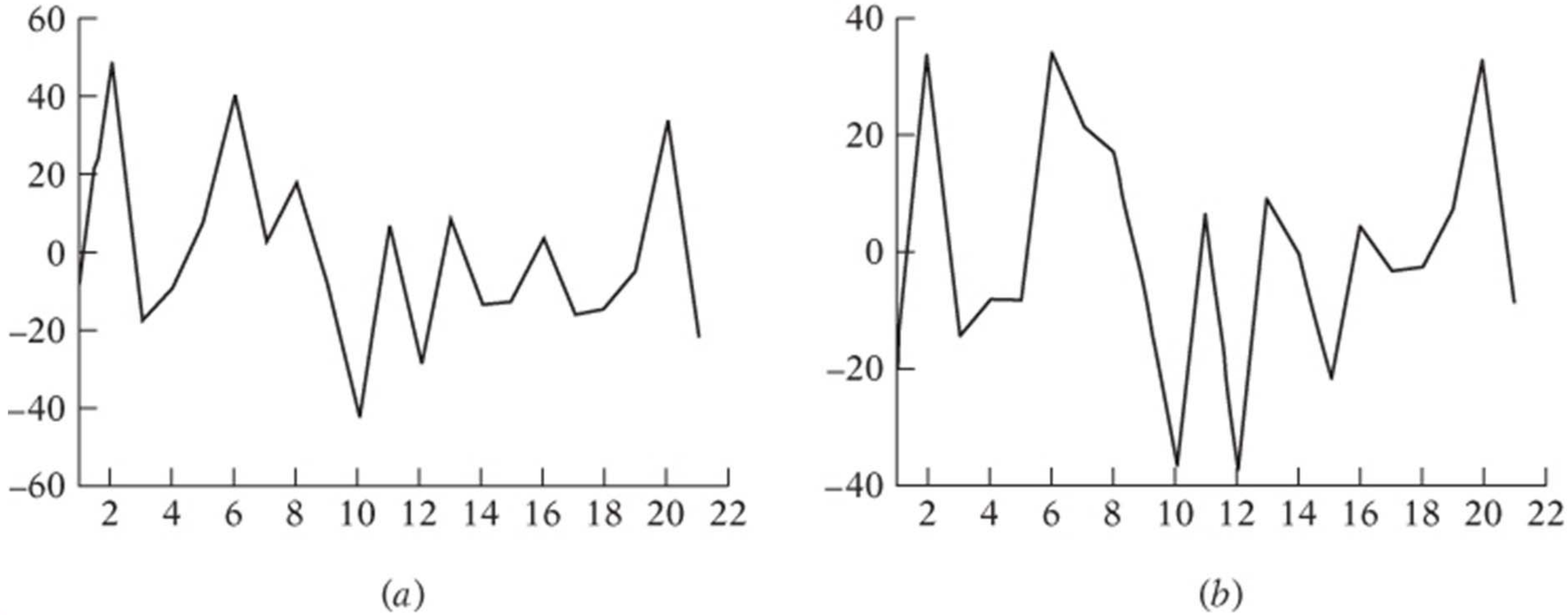
- As data collecting techniques improve, σ_i^2 is likely to decrease

Heteroscedasticity can also arise as a result of the presence of outliers



Specification error

- Some important variables are omitted from the model



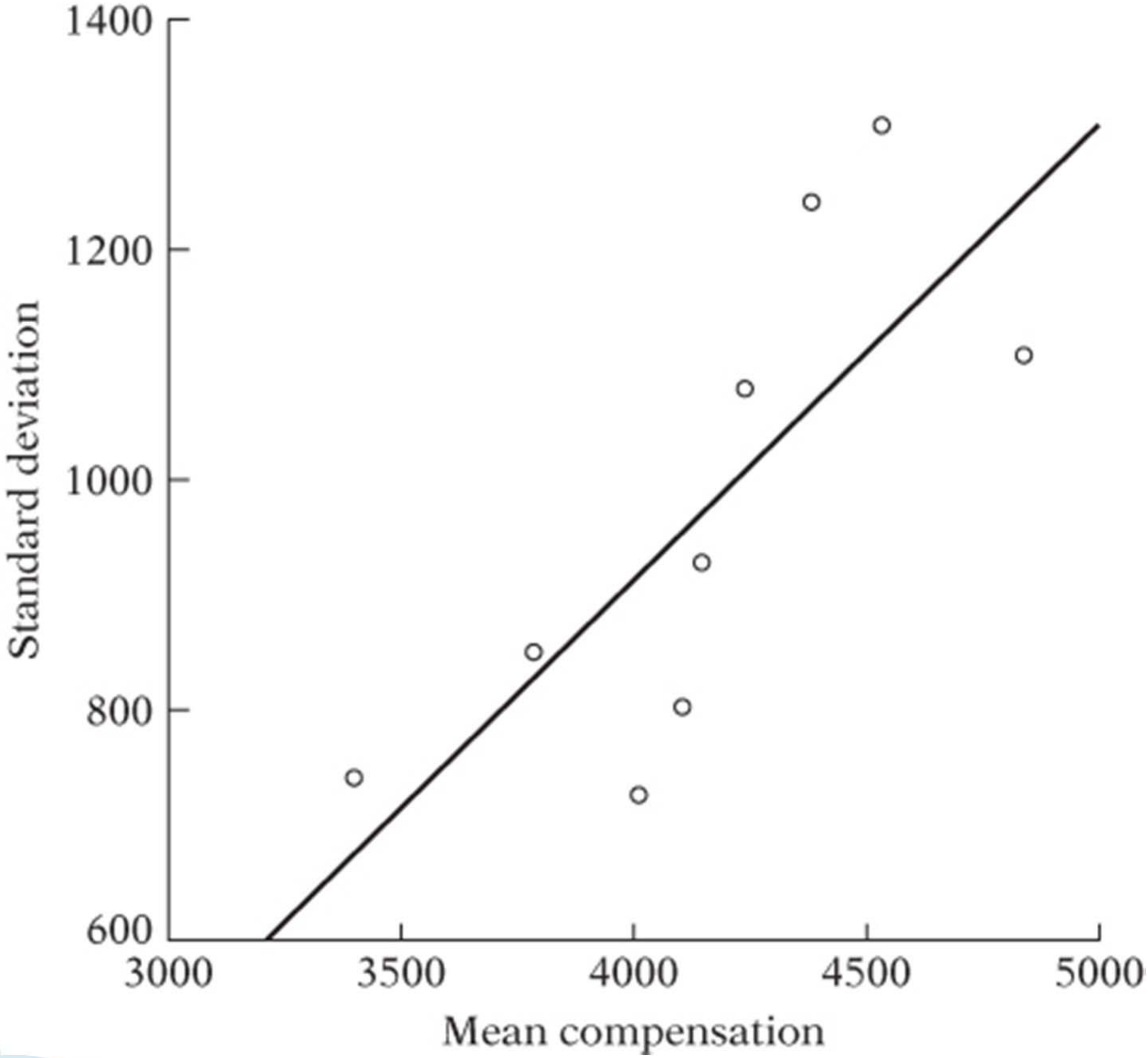
- Incorrect data transformation
- Incorrect functional form

- Heteroscedasticity is likely to be more common in cross-sectional than in time series data
- Example
  - Compensation per employee in 10 nondurable goods manufacturing industries, classified by the employment size of the firm or the establishment for the year 1958
  - Table shows that on the average large firms pay more than small firms
  - There is considerable variability in earnings among various employment classes as indicated by the estimated standard deviations of earnings
  - On average, the standard deviation of compensation increases with the average value of compensation

IND	A	B	C	D	E	F	G	H	I
1	2994	3295	3565	3907	4189	4486	4676	4968	5342
2	1721	2057	3336	3320	2980	2848	3072	2969	3822
3	3600	3657	3674	3437	3340	3334	3225	3163	3168
4	3494	3787	3533	3215	3030	2834	2750	2967	3453
5	3498	3847	3913	4135	4445	4885	5132	5342	5326
6	3611	4206	4695	5083	5301	5269	5182	5395	5552
7	3875	4660	4930	5005	5114	5248	5630	5870	5876
8	4616	5181	5317	5337	5421	5710	6316	6455	6347
9	3538	3984	4014	4287	4221	4539	4721	4905	5481
10	3016	3196	3149	3317	3414	3254	3177	3346	4067
11	3396	3787	4013	4014	4146	4241	4387	4538	4843
12	743.7	851.4	727.8	805.06	929.9	1080.6	1243.2	1307.7	1112.5
13	9355	8584	7962	8275	8389	9418	9795	10281.00 1	1750

IND = Industry
where: 1=Food and Kindred Products
2=Tobacco Products
3=Textile Mill Products
4=Apparel and Related Products
5=Paper and Allied Products
6=Printing and Publishing
7=Chemicals and Allied Products
8=Petroleum and Coal Products
9=Rubber and Plastic Products
10=Leather and Leather Products
11=Average Compensation
12=Standard Deviation
13=Average Productivity

EMPLOYMENT SIZE = Average Number of Employees
where: A=1 to 4
B=5 to 9
C=10 to 19
D=20 to 49
E=50 to 99
F=100 to 249
G=250 to 499
H=500 to 999
I=1000 to 2499



1400
1200
1000
800
600
Standard deviation
3000
3500
4000
4500
5000
Mean compensation

OLS Estimation in the Presence of Heteroscedasticity

$$\hat{\beta}_2 \ is\ best\ linear\ unbiased\ estimator(BLUE)$$

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

$$\hat{\beta}_2 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$\operatorname{var}(\hat{\beta}_2) = \frac{\sum x_i^2 \sigma_i^2}{(\sum x_i^2)^2}$$

$$\operatorname{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2} \quad Homoscedasticity$$

Is $\hat{\beta}_2$ still BLUE when we drop only homoscedasticity assumption and replace it with the assumption of heteroscedasticity?

$\hat{\beta}_2$ is **no longer best and the minimum variance** given by

$$\text{var}(\hat{\beta}_2) = \frac{\sum x_i^2 \sigma_i^2}{(\sum x_i^2)^2}$$

The Method of Generalized Least Squares (GLS)

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

$$Y_i = \beta_1 X_{0i} + \beta_2 X_i + u_i \quad where \quad X_{0i} = 1 \quad for \quad each \quad i$$

Now assume that the heteroscedastic variance σ_i^2 are known

$$\frac{Y_i}{\sigma_i} = \beta_1 \left(\frac{X_{0i}}{\sigma_i} \right) + \beta_2 \left(\frac{X_i}{\sigma_i} \right) + \left(\frac{u_i}{\sigma_i} \right)$$

$$Y_i^* = \beta_1^* X_{0i}^* + \beta_2^* X_i^* + u_i^*$$

$$\operatorname{var}(u_i^*) = E(u_i^*)^2 = E\left(\frac{u_i}{\sigma_i}\right)^2 \quad Since\ E(u_i^*) = 0$$

$$= \frac{1}{\sigma_i^2} E(u_i^2) \quad Since\ \sigma_i^2\ is\ known$$

$$= \frac{1}{\sigma_i^2} (\sigma_i^2) Since\ E(u_i^2) = \sigma_i^2$$

$$= 1$$

- Since we are still retaining the other assumptions of the classical model, the finding that it is u_i^* that is homoscedastic suggests that if we apply OLS to the transformed

$$\frac{Y_i}{\sigma_i} = \beta_1\left(\frac{X_{0i}}{\sigma_i}\right) + \beta_2\left(\frac{X_i}{\sigma_i}\right) + \left(\frac{u_i}{\sigma_i}\right)$$

it will produce estimators that are BLUE. In short, the estimated $\beta_1^* \text{ and } \beta_2^*$ are now BLUE and not the OLS estimators $\hat{\beta}_1 \text{ and } \hat{\beta}_2$

GLS is OLS on the transformed variables that satisfy the standard least-squares assumptions

The estimators thus obtained are known as GLS estimators, and it is these estimators that are BLUE

$$\frac{Y_i}{\sigma_i} = \hat{\beta}_1^* \left(\frac{X_{0i}}{\sigma_i} \right) + \hat{\beta}_2^* \left(\frac{X_i}{\sigma_i} \right) + \left(\frac{\hat{u}_i}{\sigma_i} \right)$$

$$Y_i^* = \hat{\beta}_1^* X_{0i}^* + \hat{\beta}_2^* X_i^* + \hat{u}_i^*$$

$$\sum \hat{u}_i^{2*} = \sum (Y_i^* - \hat{\beta}_1^* X_{0i}^* - \hat{\beta}_2^* X_i^*)^2$$

$$\sum \left(\frac{\hat{u}_i}{\sigma_i} \right)^2 = \sum \left[\left(\frac{Y_i}{\sigma_i} \right) - \hat{\beta}_1^* \left(\frac{X_{0i}}{\sigma_i} \right) - \hat{\beta}_2^* \left(\frac{X_i}{\sigma_i} \right) \right]^2$$

the GLS estimator of β_2^*

$$\hat{\beta}_2^* = \frac{(\sum w_i)(\sum w_i X_i Y_i) - (\sum w_i X_i)(\sum w_i Y_i)}{(\sum w_i)(\sum w_i X_i^2) - (\sum w_i X_i)^2}$$

$$\operatorname{var}(\hat{\beta}_2^*) = \frac{\sum w_i}{(\sum w_i)(\sum w_i X_i^2) - (\sum w_i X_i)^2}$$

$$where \;\; w_i = 1/\sigma^2$$

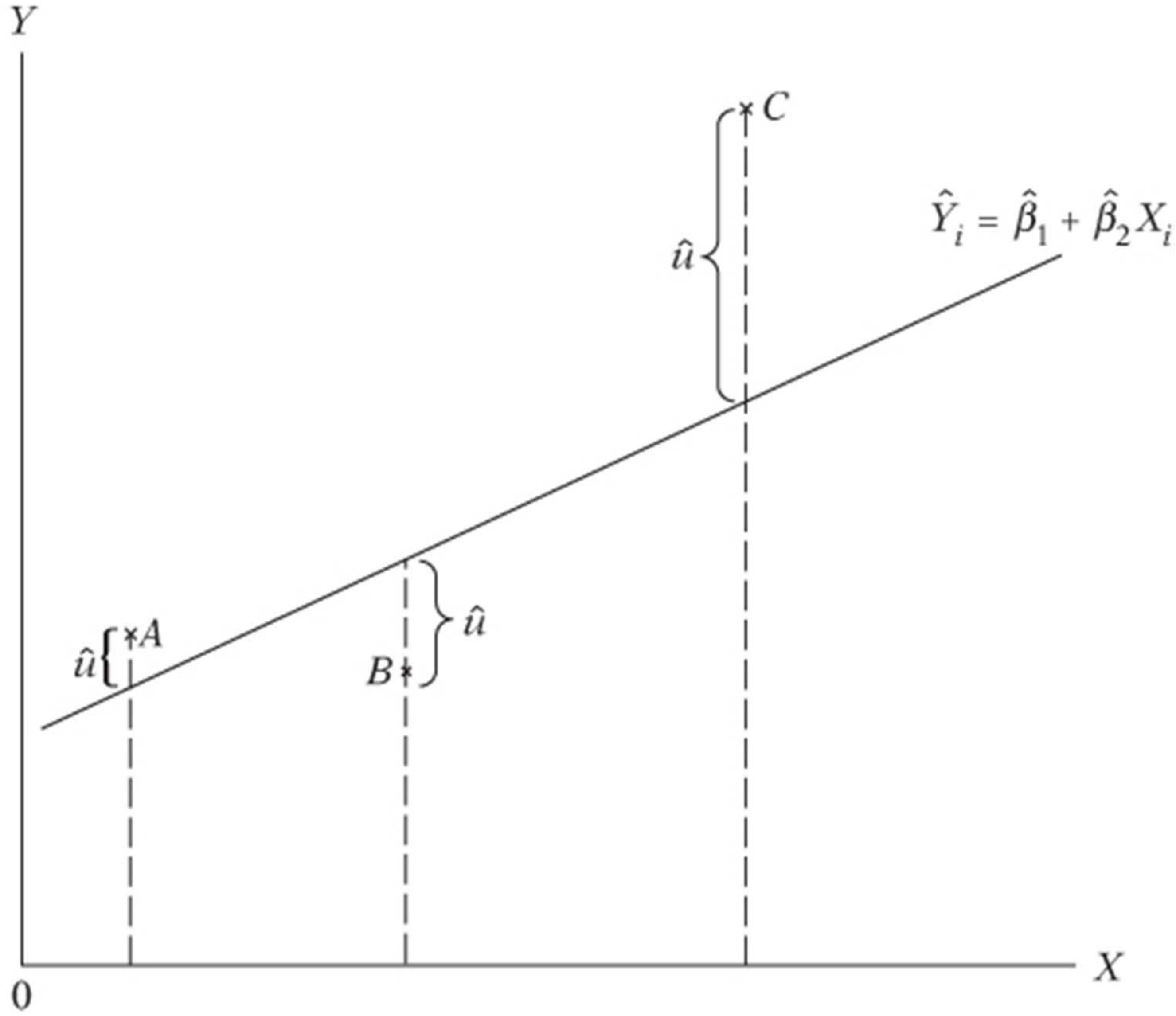
Difference between OLS and GLS

OLS we minimize

$$\sum \hat{u}_i^2 = \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2$$

GLS we minimize

$$\sum w_i \hat{u}_i^2 = \sum w_i (Y_i - \hat{\beta}_1^* X_{0i} - \hat{\beta}_2^* X_i)^2, w_i = 1/\sigma_i^2$$



Y
C
$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$
$\hat{u}$
$\hat{u}$
A
$\hat{u}$
B
0
X

Consequences of using Ols in the presence of heteroscedasticity

- OLS Estimation allowing for heteroscedasticity

- OLS Estimation disregarding heteroscedasticity

We refer to a Monte Carlo study conducted by Davidson and MacKinnon

They consider the following simple model, which in their notation is

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

They assume that $\beta_1 = 1, \beta_2 = 1, and\ u_i \sim N(0, {X_i}^{\alpha})$

They assume that the error variance is heteroscedastic and related to the value of the regressor X with power α

Based on 20,000 replications and allowing for various values for α, they obtain the standard errors of the two regression coefficients using OLS, OLS allowing for heteroscedasticity, and GLS

Example P 374-375

	Standard error of $\hat{\beta}_1$			Standard error of $\hat{\beta}_2$		
Value of α	OLS	OLS_{het}	GLS	OLS	OLS_{het}	GLS
0.5	0.164	0.134	0.110	0.285	0.277	0.243
1.0	0.142	0.101	0.048	0.246	0.247	0.173
2.0	0.116	0.074	0.0073	0.200	0.220	0.109
3.0	0.100	0.064	0.0013	0.173	0.206	0.056
4.0	0.089	0.059	0.0003	0.154	0.195	0.017

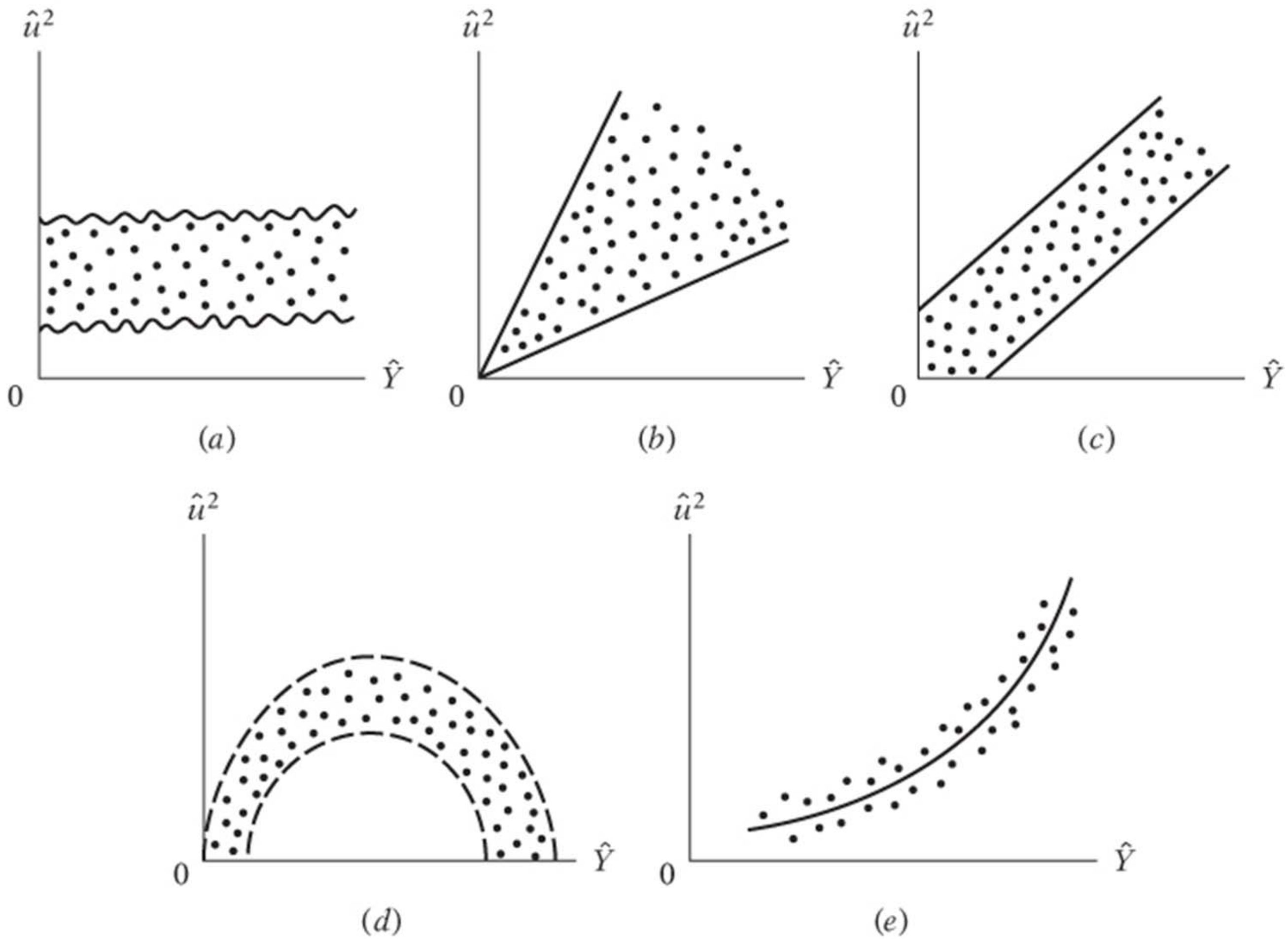
The most striking feature of these results is that OLS, with or without correction for heteroscedasticity, consistently overestimates the true standard error obtained by the (correct) GLS procedure, especially for large values of α , thus establishing the superiority of GLS

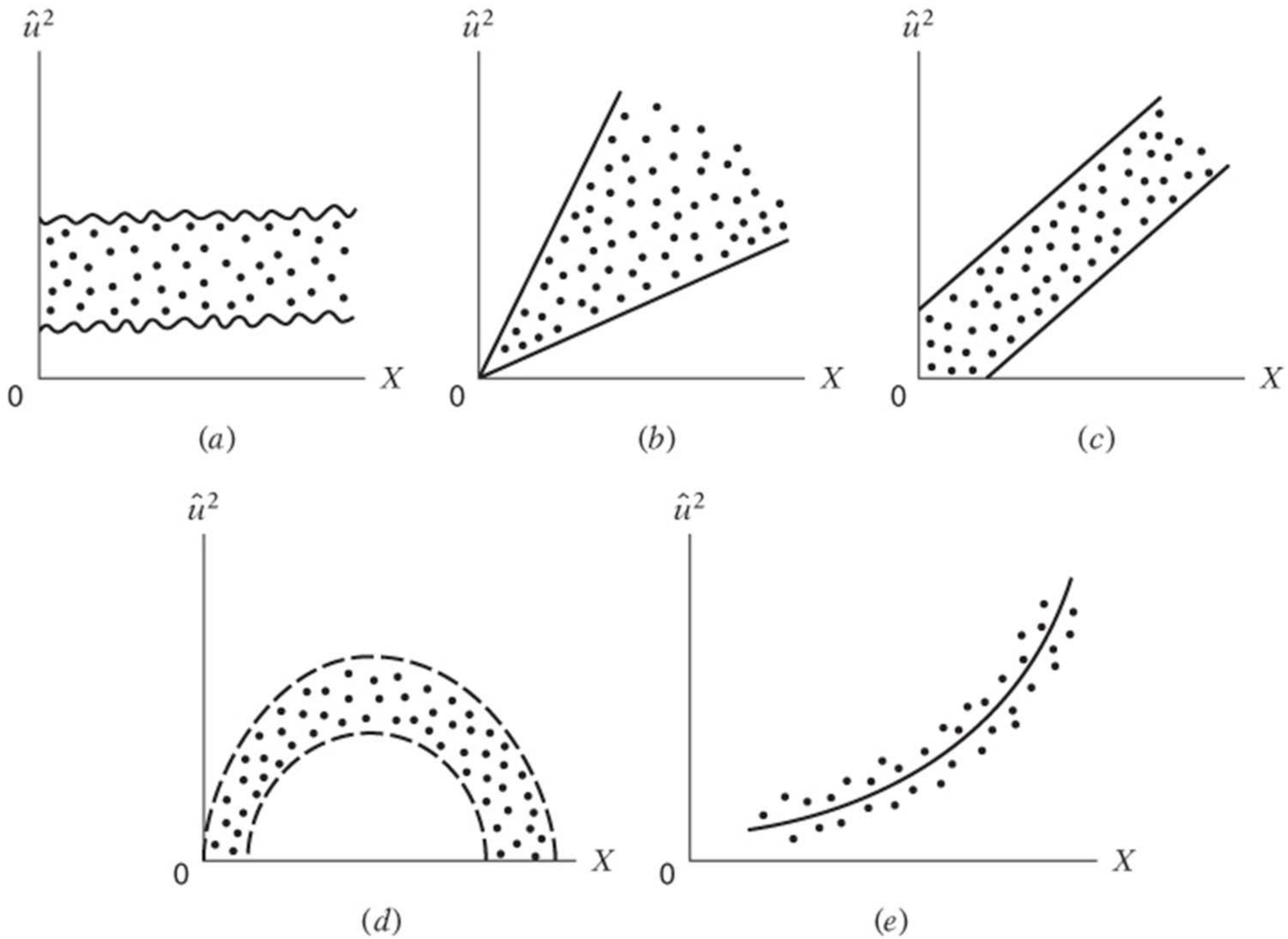
Detection of Heteroscedasticity

Detection of Heteroscedasticity

- Informal method
  - Graphical method
- Formal methods
  - Park Test
  - Breusch-Pagan Test
  - White's General Heteroscedasticity Test

Graphical method





$\hat{u}^2$
X
0
(a)
$\hat{u}^2$
X
0
(b)
$\hat{u}^2$
X
0
(c)
$\hat{u}^2$
X
0
(d)
$\hat{u}^2$
X
0
(e)

Park Test

Park formalizes the graphical method by suggesting that σ_i^2 is some function of the explanatory variable X_i. The functional form he suggests is

$$\sigma_i^2 = \sigma^2 X_i^{\beta} e^{\nu i}$$

$$or$$

$$\ln \sigma_i^2 = \ln \sigma^2 + \beta \ln X_i + \nu_i$$

Where ν_i is the stochastic disturbance term

Since σ_i^2 is generally not known. Park suggests using $\hat{u}_i^2$ as a proxy and running the following regression:

$$\ln \hat{u}_i^2 = \ln \sigma^2 + \beta \ln X_i + v_i$$

$$= \alpha + \beta \ln X_i + v_i$$

If β turns out to be statistically significant, it would suggest that heteroscedasticity is present in the data

Example

Table 11.1 Relationship between compensation and productivity

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

Y = average compensation in thousands of dollars

X = average productivity in thousands of dollars

i = ith employment size of the establishment

Step 1

Run the OLS regression disregarding the heteroscedasticity question

$$\hat{Y}_i = 1992.3452 + 0.2329 X_i$$

$$se = (936.4791) \quad (0.0998)$$

$$t = (2.1275) \quad (2.333)$$

$$R^2 = 0.4375$$

Step 2

We obtain $\hat{u}_i$ from this regression, and then in the second stage we run the regression

$$\ln \hat{u}_i^2 = \ln \sigma^2 + \beta \ln X_i + v_i$$
$$= \alpha + \beta \ln X_i + v_i$$

$$\ln \hat{u}_i^2 = 35.817 - 2.8099 \ln X_i$$
$$se = (38.319) \quad (4.216)$$
$$t = (0.934) \quad (-0.667)$$
$$R^2 = 0.0595$$

Breusch-Pagan Test (LM Test)

Consider the k-variables linear regression model

$$Y_i = \beta_1 + \beta_2 X_{2i} + \ldots + \beta_k X_{ki} + u_i$$

we assume that

$$E(u \mid x_1, x_2, \ldots, x_k) = 0$$

, so that OLS is unbiased and consistent.

☺ Test procedure ☺

Step 1 Estimate Equation

$$Y_i = \beta_1 + \beta_2 X_{2i} + \ldots + \beta_k X_{ki} + u_i$$

by OLS and obtain the squared OLS residuals $\hat{u}^2$

Step 2 Run the regression

$$\hat{u}_i^2 = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \alpha_4 X_{4i} + \ldots + \alpha_k X_{ki} + \nu_i$$

Keep the R-Squared from this regression, $R^2_{\hat{u}^2}$

Step 3 Form either the F statistics or the LM statistic

$$F = \frac{R^2_{\hat{u}^2} / k}{(1 - R^2_{\hat{u}^2}) / (n - k - 1)}$$

Where k is the number of regressors in step 2

The LM statistic for Heteroscedasticity is

$$LM = n \cdot R^2_{\hat{u}^2}$$

Under the null hypothesis, LM is distributed asymptotically as χ^2_k

The Breusch-Pagan test is **an asymptotic, or large sample, test** and in the present example 30 observations may not constitute a large sample. It should also be pointed out that in small samples the test is sensitive to the assumption that the disturbances u_i are normally distributed.

White’s General Heteroscedasticity Test

Consider the following three-variable regression model

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

The White test proceeds as follows:

Step 1 Given the data, we estimate

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

And obtain the residuals $\hat{u}_i$

Step 2 We then run the following (auxiliary) regression

$$\hat{u}_i^2 = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \alpha_4 X_{2i}^2 + \alpha_5 X_{3i}^2 + \alpha_6 X_{2i} X_{3i} + \nu_i$$

Obtain the R-Squared from this (auxiliary) regression

Step 3 Under the null hypothesis that there is no heteroscedasticity, it can be shown that sample size (n) times the R-squared obtained from the auxiliary regression asymptotically follows the chi-square distribution with df equal to the number of regressors (excluding the constant term) in the auxiliary regression. That is,

$$n \cdot R^2 \underset{asy}{\sim} \chi^2_{df}$$

Step 4 If the chi-square value obtained in

$$n \cdot R^2 \underset{asy}{\sim} \chi^2_{df}$$

Exceeds the critical chi-square value at the chosen level of significance, the conclusion is that there is heteroscedasticity. It is does not exceed the critical chi-square value, there is no heteroscedasticity, which is to say that in the auxiliary regression

$$\hat{u}_i^2 = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \alpha_4 X_{2i}^2 + \alpha_5 X_{3i}^2 + \alpha_6 X_{2i} X_{3i} + \nu_i$$

,

$$\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0$$

Example P.387-388

From Cross-sectional data on 41 countries

$$\ln Y_i = \beta_1 + \beta_2 \ln X_{2i} + \beta_3 \ln X_{3i} + u_i$$

$$Y = ratio\ of\ Trade\ taxes\ to\ total\ Government\ revenue$$

$$X_2 = ratio\ of\ the\ sum\ of\ Exports\ plus\ imports\ to\ GNP$$

$$X_3 = GNP\ per\ capita$$

- By applying White's heteroscedasticity test to the residuals obtained from regression, the following results were obtained.

$$\hat{u}_i^2 = -5.8417 + 2.5629 \ln Trade_i + 0.6918 \ln GNP_i$$
$$-0.4081(\ln Trade_i)^2 - 0.0491(\ln GNP_i)^2$$
$$+0.0015(\ln Trade_i)(\ln GNP_i)$$
$$R^2 = 0.1148$$

$$n \cdot R^2 = 41(0.1148) = 4.7068$$

The 5 percent critical chi-square value for 5 df is 11.0705.

4.7068<11.0705 On the basis of the White test, that there is no heteroscedasticity

Remedial Measures

Remedial Measures

- When σ_i^2 is known: The Method of Weighted Least Squares

- When σ_i^2 is not known

When σ_i^2 is known

If σ_i^2 is known, the most straightforward method of correcting heteroscedasticity is by means of weighted least squares, for the estimators thus obtained are BLUE.

Example

TABLE 11.4
Illustration of Weighted Least-Squares Regression

Source: Data on Y and σ_i (standard deviation of compensation) are from Table 11.1. Employment size: 1 = 1–4 employees, 2 = 5–9 employees, etc. The latter data are also from Table 11.1.

Compensation, Y	Employment Size, X	σ_i	Y_i/σ_i	X_i/σ_i
3,396	1	742.2	4.5664	0.0013
3,787	2	851.4	4.4480	0.0023
4,013	3	727.8	5.5139	0.0041
4,104	4	805.06	5.0978	0.0050
4,146	5	929.9	4.4585	0.0054
4,241	6	1,080.6	3.9247	0.0055
4,387	7	1,241.2	3.5288	0.0056
4,538	8	1,307.7	3.4702	0.0061
4,843	9	1,110.7	4.3532	0.0081

Note: In regression (11.6.2), the dependent variable is (Y_i/σ_i) and the independent variables are $(1/\sigma_i)$ and (X_i/σ_i).

$$\hat{Y}_i = 3417.833 + 148.767 X_i$$

Source	SS	df	MS
Model	1327891.27	1	1327891.27
Residual	87312.7333	7	12473.2476
Total	1415204	8	176900.5

Number of obs	=	9
F(1, 7)	=	106.46
Prob > F	=	0.0000
R-squared	=	0.9383
Adj R-squared	=	0.9295
Root MSE	=	111.68

Y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
X	148.7667	14.4183	10.32	0.000	114.6728	182.8605
_cons	3417.833	81.13632	42.12	0.000	3225.976	3609.69

$$\left(\frac{\hat{Y}_i}{\sigma_i}\right) = 3406.639\left(\frac{1}{\sigma_i}\right) + 154.153\left(\frac{X_i}{\sigma_i}\right)$$

Source	SS	df	MS
Model	175.811214	2	87.905607
Residual	.128115078	7	.018302154
Total	175.939329	9	19.5488143

Number of obs	= 9
F(2, 7)	= 4803.02
Prob > F	= 0.0000
R-squared	= 0.9993
Adj R-squared	= 0.9991
Root MSE	= .13529

Ysigma	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Xsigma	154.2118	16.95407	9.10	0.000	114.1218	194.3018
consigma	3406.277	80.96623	42.07	0.000	3214.822	3597.731

Example

Heteroscedasticity

R&D Expenditure, Sales, and Profits in 14 Industry Groupings in the United States, 2005 (all figures in millions of dollars)

Since the cross-sectional data presented in this table are quite heterogeneous, in a regression of R&D on sales, heteroscedasticity is likely

TABLE 11.5
Sales and Employment for Companies Performing Industrial R&D in the United States, by Industry, 2005 (values are in millions of dollars)

Source: National Science Foundation, Division of Science Resources Statistics, Survey of Industrial Research and Development: 2005 and the U.S. Census Bureau Annual Survey of Manufacturers, 2005.

	Industry	Sales	R&D	Profits
1	Food	374,342	2,716	234,662
2	Textiles, apparel, and leather	51,639	816	53,510
3	Basic chemicals	109,899	2,277	75,168
4	Resin, synthetic rubber, fibers, and filament	132,934	2,294	34,645
5	Pharmaceuticals and medicines	273,377	34,839	127,639
6	Plastics and rubber products	90,176	1,760	96,162
7	Fabricated metal products	174,165	1,375	155,801
8	Machinery	230,941	8,531	143,472
9	Computers and peripheral equipment	91,010	4,955	34,004
10	Semiconductor and other electronic components	176,054	18,724	81,317
11	Navigational, measuring, electromedical, and control instruments	118,648	15,204	73,258
12	Electrical equipment, appliances, and components	101,398	2,424	54,742
13	Aerospace products and parts	227,271	15,005	72,090
14	Medical equipment and supplies	56,661	4,374	52,443

```
      Source |       SS       df       MS              Number of obs =      14
-------------+------------------------------           F(  1,    12) =    2.48
       Model |   208733442     1   208733442           Prob > F      =  0.1410
    Residual |  1.0083e+09    12  84021567.1           R-squared     =  0.1715
-------------+------------------------------           Adj R-squared =  0.1025
       Total |  1.2170e+09    13  93614788.2           Root MSE      =  9166.3

------------------------------------------------------------------------------
          rd |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       sales |   .0437234   .0277404     1.58   0.141    -.0167178    .1041646
       _cons |   1337.874   5015.141     0.27   0.794     -9589.18    12264.93
------------------------------------------------------------------------------
```

$$\widehat{R\&D_i} = 1338 + 0.0437 Sale_i$$
$$se = (5015)(0.0277)$$
$$t = (0.27) \quad (1.58)$$

$$r^2 = 0.172$$

There is a positive relationship between R&D and sales, although it is not statistically significant at the traditional levels

White Test

Source	SS	df	MS
Model	9.2405e+16	2	4.6203e+16
Residual	1.2022e+17	11	1.0929e+16
Total	2.1263e+17	13	1.6356e+16

Number of obs	=	14
F(2, 11)	=	4.23
Prob > F	=	0.0435
R-squared	=	0.4346
Adj R-squared	=	0.3318
Root MSE	=	1.0e+08

muhat2	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
sales	577.6563	1307.934	0.44	0.667	-2301.087	3456.4
sales2	.0008456	.0031711	0.27	0.795	-.006134	.0078253
_cons	-4.67e+07	1.12e+08	-0.42	0.685	-2.94e+08	2.00e+08

$$\hat{u}_i^2 = -46,746,325 + 578Sales_i + 0.000846Sales_i^2$$

$$se = (112,224,348) \quad (1308) \quad (0.003171)$$

$$t = (-0.42) \quad (0.44) \quad (0.27)$$

$$R^2 = 0.435$$

Using the R^2 value and n=14, we obtain $nR^2 = 6.090$
Under the null hypothesis of no heteroscedasticity, this should follow a chi-square distribution with 2 df (because there are two regressors). The p-value of obtaining a chi-square value of as much as 6.090 or greater is about 0.0476. Since this is a low value, the White test also suggests that there is heteroscedsticity.

The true error variance is unknown, we cannot use the method of weighted least squares to obtain heteroscedasticity-corrected standard errors and t-values.

Therefore, we would have to make some educated guesses about the nature of the error variance.

White's heteroscedasticity-consistent standard errors

$$\widehat{R\&D}_i = 1337.87 + 0.0437 Sale_i$$
$$se = (4892.447)(0.0411)$$
$$t = (0.27) \qquad (1.06)$$

$$r^2 = 0.172$$

We see that the parameter estimates have not changed, the standard error of the intercept coefficient has decreased slightly, and the standard error of the slope coefficient has increased slightly. But remember that the White procedure is strictly a large-sample procedure, where as we have only 14 observations

STATA

Source	SS	df	MS		
Model	208733442	1	208733442	Number of obs =	14
Residual	1.0083e+09	12	84021567.1	F(1, 12) =	2.48
Total	1.2170e+09	13	93614788.2	Prob > F =	0.1410
				R-squared =	0.1715
				Adj R-squared =	0.1025
				Root MSE =	9166.3

RD	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Sales	.0437234	.0277404	1.58	0.141	-.0167178	.1041646
_cons	1337.874	5015.141	0.27	0.794	-9589.18	12264.93

```
. whitetst

White's general test statistic :    6.0842  Chi-sq( 2)  P-value =  .0477
```

$$H_0 : Homoscedasticity$$

$$H_1 : Otherwise$$

White's general test statistic is 6.0842.

Degree of freedom =2

Critical value of Chi-square at 5 percent significance level is 5.99147

6.0842 > 5.99147 Reject the null hypothesis

The White test also suggests that there is heteroscedsticity.

Breusch-Pagan test

Source	SS	df	MS
Model	208733442	1	208733442
Residual	1.0083e+09	12	84021567.1
Total	1.2170e+09	13	93614788.2

Number of obs	=	14
F(1, 12)	=	2.48
Prob > F	=	0.1410
R-squared	=	0.1715
Adj R-squared	=	0.1025
Root MSE	=	9166.3

RD	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Sales	.0437234	.0277404	1.58	0.141	-.0167178	.1041646
_cons	1337.874	5015.141	0.27	0.794	-9589.18	12264.93

```
. estat hettest

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of RD

         chi2(1)      =     8.83
         Prob > chi2  =   0.0030
```

$H_0 : Homoscedasticity$

$H_1 : Otherwise$

Breusch-Pagan test’s general test statistic is 8.83.

Degree of freedom =1

Critical value of Chi-square at 5 percent significance level is 8.83 > 3.84146

Reject the null hypothesis

The Breusch-Pagan test also suggests that there is heteroscedsticity.

Source

Gujarati, D.N. (2009) Basic Econometrics. 5th ed. Singapore, McGraw-Hill.