Tuesday, January 17, 2012
11:12 AM

WEEK 2 (17 JAN 2012)

STEPS IN ECONOMETRIC ANALYSIS

ECONOMIC THEORY ⇒ $C = f(Y)$ $\quad \frac{\partial C}{\partial Y} > 0$

↓

$0 < MPC < 1$

MATHEMATICAL MODEL OF THEORY ⇒ $C = a + bY, \quad 0 < b < 1$

↓

ECONOMETRIC MODEL OF THEORY ⇒ $C_i = a + bY_i + \varepsilon_i$

↓

DATA

↓

ESTIMATION OF ECONOMETRIC MODEL

$\hat{C} = -184.08 + 0.7064\,Y$ ($\hat{a}$, $\hat{b}$)

↓

HYPOTHESIS TESTING

① ECONOMIC MEANING TEST: INVESTIGATE SIGN(S), MAGNITUDE

② GOODNESS OF FIT: HOW WELL RHS VARIABLE(S) COULD EXPLAIN THE BEHAVIOR OF LHS VARIABLE

INDICATOR: R^2 OR ADJ. R^2

$R = 0.9$

SIGNIFICANCE OF THE ESTIMATED COEFFICIENT(S)

(t-test, z-score, chi square test)

③ ECONOMETRIC TEST ⇒ FOR VALIDITY AND RELIABILITY OF THE MODEL

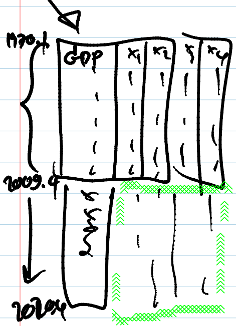
- MULTI COLLINEARITY
- HETEROSCEDASTICITY
- AUTO CORRELATION

↓

FORECASTING OR PREDICTION

↓

USING THE MODEL FOR CONTROL OR POLICY RESPONSES



REVIEW OF SOME STATISTICAL CONCEPTS

RANDOM VARIABLES (X)

A VARIABLE THAT TAKES ON ALTERNATIVE VALUES, EACH W/ A PROBABILITY LESS THAN OR EQUAL TO ONE.

RANDOM VARIABLES
- DISCRETE – ASSUME ONLY A FINITE NUMBER OF VALUES
- CONTINUOUS – ASSUME A NONDENUMERABLE NUMBER OF VALUES

WE CAN DESCRIBE A RDV. BY EXAMINING THE PROCESS WHICH GENERATES ITS VALUES

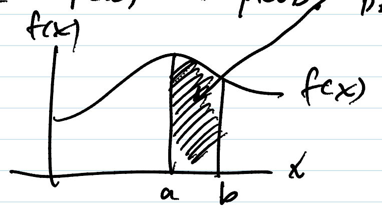
THIS PROCESS IS CALLED "PROBABILITY DISTRIBUTION"

PROBABILITY: A CHANCE / LIKELIHOOD OF OCCURENCE OF EVENT.

IF A = AN EVENT $\{a < x < b\}$

$$P\{A\} = P\{a < x < b\} = \int_a^b f(x)\,dx$$

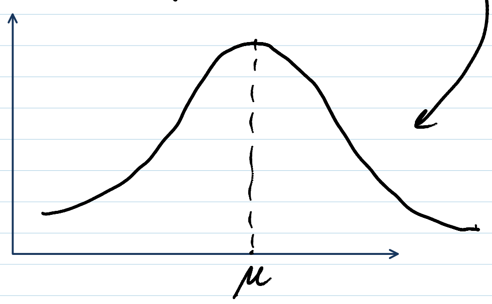
WHERE $f(x)$ = PROB. DENSITY FUNCTION.



EXPECTED VALUES ≡ CENTER OF THE DISTRIBUTION

EXAMPLE:

- IF X IS A RDV. WITH MEAN = μ AND VARIANCE = σ_x^2
- $X \sim N(\mu, \sigma_x^2)$
- $f(x_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x_i - \mu)^2}$



VARIANCE ≡ DISPERSION OR SPREAD (AROUND ITS' MEAN)

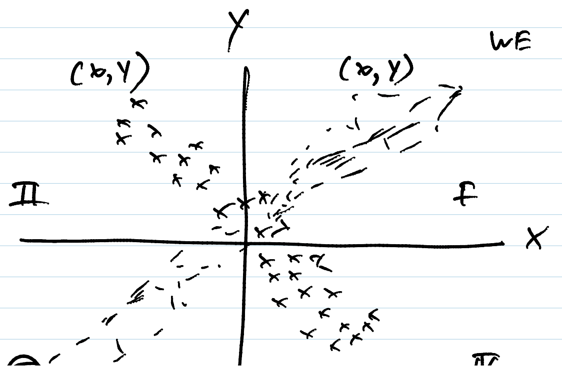
COVARIANCE: IF X, Y ARE INDEPENDENT ⟹

$$\mathrm{COV}(X, Y) = 0$$

CORRELATION: MEASURE OF ASSOCIATION BETWEEN ANY TWO VARIABLES.

IN ECONOMETRIC MODEL (REGRESSION MODEL)

$$\underbrace{Y_i = a + bx_i}_{\text{ASSOCIATION}} + u_i$$

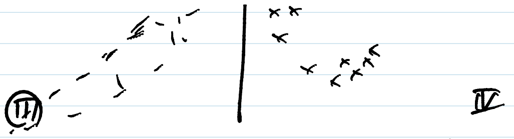


WE ASSUME THAT X_i POSTULATES' Y_i

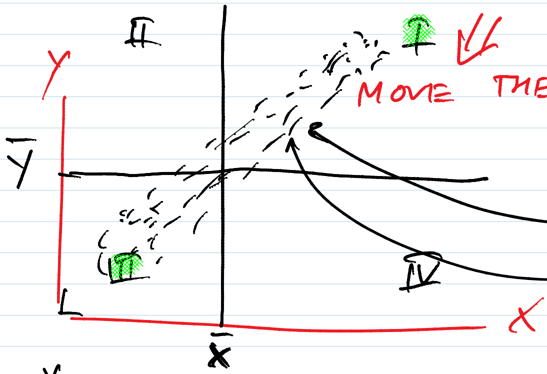
PRODUCT OF $x, y = x \cdot y$

CONSIDER $\sum_{i=1}^{n} x_i y_i$ = SUMMATION OF PRODUCTS OF X AND Y

BUT! NOT ALWAYS GOOD TO



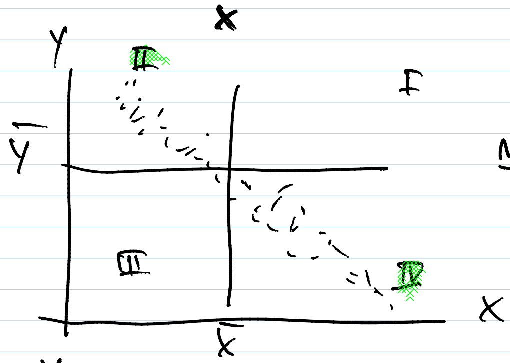
BUT! NOT ALWAYS GOOD TO ANALYSE THE ABSOLUTE.



MOVE THE ORIGIN TO

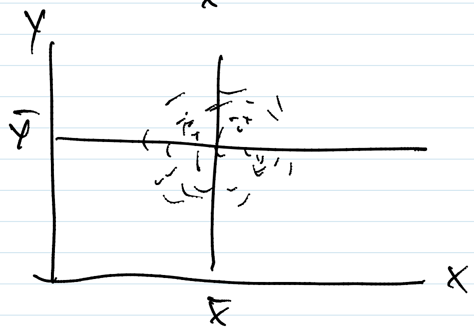
WHERE $x_i = X_i - \bar{X}$

$y_i = Y_i - \bar{Y}$



NEGATIVE CORRELATION

most of observation will fall in II AND IV



no ASSOCIATION

$$\sum_{i=1}^{n} X_i Y_i \quad X$$

$$\sum_{i=1}^{n} x_i y_i = \text{give a measure of association between } x_i \text{ and } y_i$$

and

$$\boxed{\sum_{i=1}^{n} x_i y_i = \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}$$

DRAWBACK OF THIS FORMULA:

1. IF # OF OBSERVATION INCREASES (n increases), $\sum_{i=1}^{n} x_i y_i$ WILL HAVE HIGHER NUMBER!
2. $\sum x_i y_i$ DEPENDS ON THE UNIT OF MEASUREMENT.

SO, HOW TO CORRECT THESE DRAWBACK?

SO, HOW TO CORRECT THESE DRAWBACK?

ANSWER: DEFINE $\boxed{r = \dfrac{\sum_{i=1}^{n} x_i y_i}{n \cdot S_x \cdot S_y}}$

WHERE $S_x = \sqrt{\dfrac{\sum_{i=1}^{n} x_i^2}{n}}$ $S_y = \sqrt{\dfrac{\sum_{i=1}^{n} y_i^2}{n}}$

NOTE: WEIGHTING THE $\sum_{i=1}^{n} x_i y_i$ WITH $n \cdot S_x \cdot S_y$ MAKES r UNITLESS!

SO
$$r = \frac{\sum_{i=1}^{n} x_i y_i}{n\sqrt{\frac{\sum x_i^2}{n}}\sqrt{\frac{\sum y_i^2}{n}}} = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} y_i^2}}$$

CORRELATION COEFFICIENT

IF X AND Y ARE POSITIVELY RELATED, $r > 0$,
IF X AND Y ARE NEGATIVELY RELATED, $r < 0$.

NOTE: BE AWARE THAT WE CANNOT TELL WHETHER WHICH VAR. DEPENDS ON WHICH ONE. (NOT LIKE ECONOMETRICS)

THIS IS JUST A MEASURE OF ASSOCIATION BET. TWO VARIABLES.

19 JAN 2012

A REVIEW OF SOME STATISTICAL CONCEPTS (CONTINUED)

SUMMATION AND PRODUCT OPERATORS

$$\boxed{\sum_{i=1}^{n} x_i = x_1 + x_2 + \dots + x_n ,}$$

PROPERTIES

① $\sum_{i=1}^{n} k = n \cdot k$ WHERE k IS CONSTANT.

EX: $\sum_{i=1}^{4} 5 = 4 \cdot 5 = 20.$

② $\sum_{i=1}^{n} k x_i = k \cdot \sum_{i=1}^{n} x_i$ WHERE k IS A CONSTANT.

③ $\sum_{i=1}^{n} (a + b x_i) = n \cdot a + b \sum_{i=1}^{n} x_i$ WHERE a AND b ARE CONSTANTS

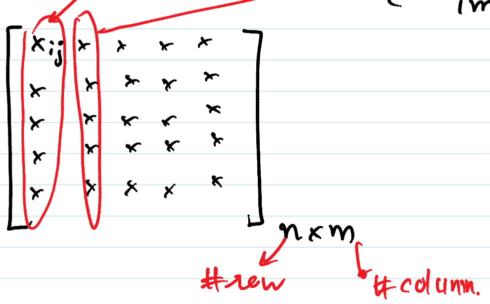
④ $\sum_{i=1}^{n} (x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$.

HOW ABOUT MULTIPLE SUMMATION?

$$\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} = \sum_{i=1}^{n} (x_{i1} + x_{i2} + \dots + x_{im})$$

row column

$$= (x_{11} + x_{21} + \dots x_{n1}) + (x_{12} + x_{22} + \dots + x_{n2}) + \dots + (x_{1m} + x_{2m} + \dots x_{nm}$$



PROPERTIES

① $\sum_{i=1}^{n} \sum_{j=1}^{m} x_{ij} = \sum_{j=1}^{m} \sum_{i=1}^{n} x_{ij}$

② $\sum_{i=1}^{n} \sum_{j=1}^{m} x_i y_j = \sum_{i=1}^{n} x_i \sum_{j=1}^{m} y_j$.

PROBABILITY DENSITY FUNCTION (PDF)

LET X BE A DISCRETE RANDOM VARIABLE THAT CAN TAKE DISTINCT VALUES $x_1, x_2, \dots, x_n, \dots$

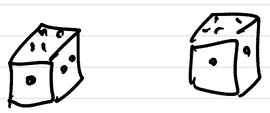
THEN THE FUNCTION $f(x) = P(X = x_i)$ FOR $i = 1, 2, \dots, n, \dots$

probability that the Rv. X take the value of x_i.

AND $f(x) = 0$ FOR $x \neq x_i$

IS CALLED (DISCRETE) PROBABILITY DENSITY FUNCTION (PDF)

EXAMPLE



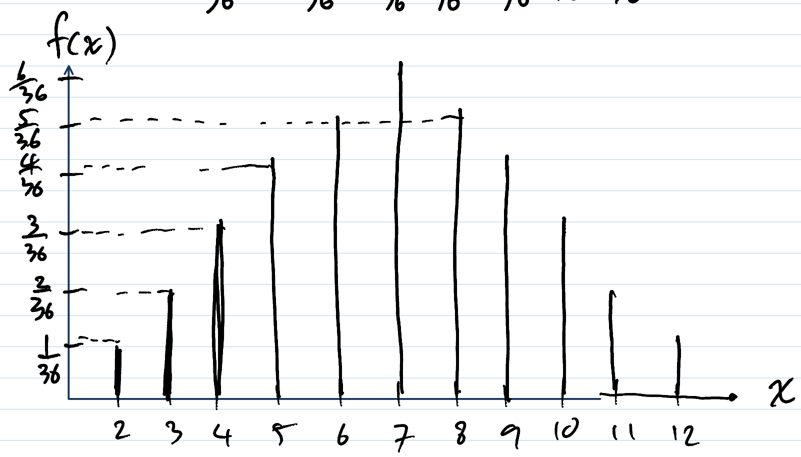
2 PIECES

SUPPOSE DEFINE X = THE SUM OF THE NUMBERS APPEARED ON TWO DICES

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

$$x = 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12$$

$$f(x) = \frac{1}{36}\ \ \frac{2}{36}\ \ \frac{3}{36}\ \ \frac{4}{36}\ \ \frac{5}{36}\ \ \frac{6}{36}\ \ \frac{5}{36}\ \ \frac{4}{36}\ \ \frac{3}{36}\ \ \frac{2}{36}\ \ \frac{1}{36}$$



DENSITY FUNCTION OF THE DISCRETE RV. X

PDF OF A CONTINUOUS RANDOM VARIABLE

LET X BE A CONTINUOUS RV.

THEN $f(x)$ IS SAID TO BE THE PDF OF X

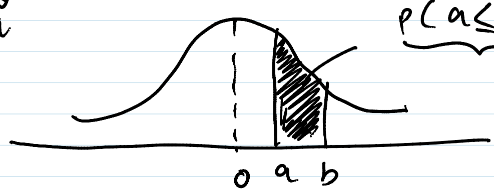
IF THE FOLLOWING CONDITIONS ARE HOLD:

$$f(x) \geq 0 \qquad \text{(non-negative)}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{a}^{b} f(x)\,dx = P(a \leq x \leq b)$$

$$\underbrace{p(a \leq x \leq b)}$$



PROBABILITY THAT x LIES IN THE INTERVAL a TO b

"JOINT" PROBABILITY DENSITY FUNCTION

DISCRETE JOINT PDF

LET X AND Y BE TWO DISCRETE RVs.
THEN THE FUNCTION

$$f(x, y) = \underbrace{P(X = x \text{ AND } Y = y)}$$

IS CALLED THE DISCRETE JOINT PROBABILITY DENSITY FUNCTION.

READ: JOINT PROBABILITY THAT X TAKES THE VALUE OF x AND Y TAKES THE VALUE OF y

EXAMPLE: JOINT PDF OF X AND Y

		X			
		-2	0	2	3
Y	3	0.27	0.08	0.16	0
	6	0	0.04	0.10	0.35

EX: THE CHANCE OR LIKELIHOOD THAT X TAKES THE VALUE OF 0 AND SIMULTENEOUSLY Y TAKES THE VALUE OF 6 IS 0.04

$$\sum f(x, y) = 1$$

MARGINAL PROBABILITY DENSITY FUNCTION

GIVEN $f(x, y)$ IS A JOINT PDF.

$f(x)$ AND $f(y)$ ARE CALLED "INDIVIDUAL" OR "MARGINAL" PDFs.

THAT IS,

$$\boxed{f(x) = \sum_y f(x, y)} \Rightarrow \text{MARGINAL PDF OF } X$$

SUM OVER ALL VALUES OF Y

$$\boxed{f(y) = \sum_x f(x, y)} \Rightarrow \text{MARGINAL PDF OF } Y$$

SUM OVER ALL VALUE OF X

EX:

$$f(x=-2) = \sum_{y} f(x=-2, y)$$
$$= 0.27 + 0 = 0.27$$
$$f(x=0) = \sum_{y} f(x,y)$$
$$= 0.08 + 0.04 = 0.12$$
$$f(x=2) = \sum_{y} f(x,y)$$
$$= 0.16 + 0.10 = 0.26$$
$$f(x=3) = \sum_{y} f(x,y)$$
$$= 0 + 0.35 = 0.35$$

BY THE SAME MANNER,

THE MARGINAL PDF OF Y CAN BE DER

$$f(y=3) = \sum_{x} f(x,y) = 0.27 + 0.08 + 0$$
$$= 0.51$$
$$f(y=6) = \sum_{x} f(x,y) = 0 + 0.04 + 0$$
$$= 0.49$$

NOTICE THAT:

$$\sum_{y} f(y) = 1.$$

SUM OVER ALL VALUES OF Y

$$\sum_{x} f(x) = 1.$$

SUM OVER ALL VALUES OF X

$\sum_{x} f(x) = 1$

IVED AS

$0.16 + 0$

$0.10 + 0.35$

$= 1$